Teaching Design of the First Kind of Curve Integral

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Abstract: This paper designs the teaching of the first kind of curve integral. Firstly, based on the physical background of the first kind of curve integral, using "the methods of elementary mathematics +the thinking method of limit" and combining the historical background of the differential calculus symbols, the definition of the first kind of curve integral is strengthened in the intuitive thinking; secondly, based on the definition form of the first kind of curve integral, three small problems in the calculation are analyzed from a symbolic perspective, and the basic calculation method of the first kind of curve integral is illustrated with examples; finally, the definition form of the first kind of curve integral is used to generalize and solve the surface area problem of a class of rotating surfaces, deepening the understanding of the application of the first kind of curve integral. This paper aims to provide new inspiration to advanced mathematics enthusiasts and to provide better mathematical logical thinking abilities.

Keywords: Physics Background; Intuitive Thinking; The First Kind of Curve Integral; Calculation Understanding; Area

1. Introduction

The first kind of curve integral is a very important knowledge point in the integration of higher mathematics. In the actual teaching process, in view of the characteristics of mathematics and the preciseness of logical thinking [1], most students have different problems from concept understanding to actual calculation, and to a certain extent, they have difficulties in learning, and do not play a connecting role between the past and the future [2][3].

As teaching practitioners, we need to re-design the teaching design of this knowledge point in order to achieve better teaching effect through concise mathematical thinking and formalization [4,5]. Firstly, in the teaching of concept definition, combining with the physical background of the first kind of curve integral, using "the method of elementary mathematics+ the thinking method of limit" and combining the historical background of the differential symbol, we can strengthen the understanding of the definition of the first kind of curve integral in the intuitive thinking; secondly, in the teaching of calculation, the definition of the first kind of curve integral is analyzed in depth from the form, that is, three small problems in the calculation are analyzed from the perspective of symbol, and the basic calculation methods of the first kind of curve integral are solved one by one and illustrated with examples; finally, the surface area problem of a kind of rotating surface is generalized and solved by using the definition form of the first kind of curve integral, which not only deepens the application understanding of the first kind of curve integral, but also makes a useful supplement to the previous understanding of the concept understanding.

2. Physical Background and Mathematical Concept of the First Kind of Curve Integral

2.1 Physical Background

In elementary mathematics, the quality of components with uniform material and line shape is publicized as follows: $M=\rho\cdot s$,where ρ is density and s is length. Now let's consider the following question: Let L be a curved component of a certain material on the plane, and $\rho(x,y)$ be a continuous linear density function on the component L . now how can we find the mass M of this curved component. In order to better describe the application of the idea of Riemann integral [6] to such problems, we describe the two

relatively "thorny" problems faced in this practical application as follows:
(a) The density of curved members is non-uniform, that is, the density at any point is not the same;
(b) How to calculate the length of components with non-linear shape?
Based on the idea of elementary mathematics, we first adopt the method of "segmentation", that is, the curved component L is divided into small curve segments $L_i(i=1,2,\cdots,n)$, and the length of each L_i is recorded as Δs_i. Because $\Delta s_i \to 0$,the non-uniform small component L_i can be approximately understood as a small uniform component, and any point (ξ_i,η_i) in the above L_i can be taken to describe the density of the small component $\rho(\xi_i,\eta_i)$, then the mass ΔM_i of the small component L_i can be approximated as $\Delta M_i=\rho(\xi_i,\eta_i)\Delta s_i$, and the idea is applied to the whole curved component L, so $M=\sum_{i=1}^{n}\Delta M_i$, and the above process is a simple reappearance of "segmentation, approximation and summation" in Riemann integral idea.
In order to obtain more accurate mass M of the curved component L, it is obvious that the more quantity of the $L_i(i=1,2,\cdots,n)$ better, so how much is the quantity good and accurate? The answer is self-evident, $n\to\infty$ is the best, but there is no way to really realize ∞, and we naturally come up with the thought of the limit. Note $\lambda=\max\limits_{1\le i\le n}\Delta s_i$, when the $\lambda\to 0$, (and when the segmentation becomes more and more detailed, that is $n\to\infty$), If the above sum formula $M=\sum_{i=1}^{n}\Delta M_i$ has a limit, then this limit is the true mass M of the curved component L, that is:

$$M=\lim_{n\to\infty}\sum_{i=1}^{n}\Delta M_i=\lim_{n\to\infty}\sum_{i=1}^{n}\rho(\xi_i,\eta_i)\Delta s_i \quad (1)$$

Obviously, the value of M has nothing to do with the method of division and the method of point (ξ_i,η_i) , so we solve this practical problem completely through the idea of "segmentation, approximation and summation and limit".

2.2 Understanding the Definition of the First Kind of Curve Integral with Intuitive Thinking

Let's first review some commonly used integral and differential symbols: in 1675, Leibniz introduced dx and dy to represent the differential of x and y respectively. The differential symbol d takes the first letter of English differential (the meaning of gap, difference), which is highly consistent with the symbol " Δ " of " Δs_i " in the small segment L_i described by "division"; in addition, Leibniz also uses " $omn.I$ " to represent the sum of I, where omn is the abbreviation of omnia (i. e. all, all), and then he rewrites it as $\int$, and represents the sum of all I (Summa) with " $\int I$ ", which is the same as the symbol " $\sum$ " in the "sum" process. The " $\sum$ " is being stretched to the symbol " $\int$ ". Thus, equation (1) can be understood as the following familiar expression of the first type of curve integral (herein written $\rho(x,y)=f(x,y)$):

$$\begin{aligned} M&=\lim_{n\to\infty}\sum_{i=1}^{n}\Delta M_i \\ &=\lim_{n\to\infty}\sum_{i=1}^{n}\rho(\xi_i,\eta_i)\Delta s_i \\ &=\int_L\rho(x,y)ds=\int_L f(x,y)ds \end{aligned} \quad (2)$$

Here, we will explain the symbols in the practical sense in the formula (2):
(a) In the "segmentation" step, the length of each L_i is denoted as Δs_i. Since Δs_i is the description of one-dimensional variable "length", only one integral symbol is required;
(b) The integrand function $\rho(x,y)$ is the reproduction of $\rho(\xi_i,\eta_i)$ the non-uniform density function of the small segment L_i in the original problem, in which $\rho(\xi_i,\eta_i)\Delta s_i$ will be understood as $\rho(x,y)ds$ or $f(x,y)ds$ in the integral expression, that is, the mass of the uniform material component approximately

represents the mass of the non-uniform material component.

In short, integral is the product of "the method of elementary mathematics (regular method) +the thinking method of limit ". It is also applicable to the understanding and understanding of integral concepts such as definite integral, quadratic integral, curve integral, surface integral and other integral concepts. It can also improve the students' extensive application ability of integral and provide more support and help for practical teaching.

3. The Calculation of the First Kind of Curve Integral

3.1 Comprehension of the First Kind of Curve Integral Calculation and Arc Differential Formula

Combining the expression of the first kind of curve integral $\int_L f(x,y)ds$, we find that this expression has three characteristics:

(a) The integrand is a binary function.

(b) How to transform the relationship between the integrand s and the two independent variables (x,y) in the integrand.

(c) there is only one integral sign " $\int$ ", that is, it can be integrated only once.

Recognizing the above three points, the focus of our understanding the calculation method is to simplify it the following two problems:

(a) Because only one integration can be performed, it is necessary to reduce the integrand from the binary function to the unary function.

(b) how to transform the integral variable into the independent variable in the univariate integrand function after processing. To sum up, how to convert the first kind of curve integral into definite integral [7]?

For the first problem, because $f(x,y)$ is a continuous function defined on L, we can use the curve equation form of L to replace the integrand $f(x,y)$ to unify the binary independent variable into the univariate independent variable, and make it become the univariate function we need. This is what we have repeatedly emphasized: whether the curve equation of L is in the form of rectangular coordinate, parameter equation, polar coordinate, etc., the curve equation of L must be substituted into integrand function.

For the second problem, the integral variable s appears in the form of differential ds in the integral expression, that is, it represents the length of the small curve L_i . Although the shape of the small curve L_i is “curved”, due to the small enough division, the length of the arc is approximately the length of the straight line. According to the Pythagorean theorem, it can be obtained as follows: $\Delta s_i \approx \sqrt{\Delta x_i^{\,2} + \Delta y_i^{\,2}}$. Combined with the idea of differential sign and the thinking method of limit, we have, $ds = \sqrt{dx^2 + dy^2}$ that is, the arc differential expression is connected in series with the relationship between the unary integral variable s and the independent variable of two variables (x,y) in the integral function.

In view of the third problem, there is only one integral symbol, so in the equation $ds = \sqrt{dx^2 + dy^2}$, it is necessary to transform the binary integral variable (x,y) into a new unary integral variable, which involves the operation of changing "binary" into "univariate"; At the same time,, combined with the physical background of the first kind of curve integral, the upper limit of the integral variable must be greater than the lower limit after it is transformed into the definite integral of a function of one variable,, so as to ensure that the final integral result is a non-negative.

To sum up, the core of the three problems is the change of binary to unary. The first problem is solved with the help of the form of curve equation L , so the second and third problems are also inseparable from the curve equation L [8]. Next, we understand the solutions to the second and third problems according to the different forms of the curve equation L:

(a) Let the parametric equation of a plane curve be $\begin{cases} x = \varphi(t) \\ y = \psi(t) \end{cases}$, where $\alpha \le t \le \beta$, and $\varphi(t)$, $\psi(t)$ has a first-order continuous derivative on $[\alpha,\beta]$, and $\varphi'^2(t) + \psi'^2(t) \ne 0$, then there is $ds = \sqrt{dx^2 + dy^2} = \sqrt{\varphi'^2(t) + \psi'^2(t)}dt$;

(b) If the equation of a curve L is a rectangular coordinate equation $y=\varphi(x)$, where $a \le x \le b$, and $\varphi(x)$ has a continuous derivative on $\varphi(x)$, at the same time $dy = d\varphi(x) = \varphi'(x)dx$, then there is $ds=\sqrt{dx^2+dy^2}=\sqrt{1+\varphi'^2(x)}dx$;

In addition, if the equation of the curve L is a rectangular coordinate equation $x=\psi(y)$, where $c \le y \le d$, and $\psi(y)$ has a continuous derivative on $[c,d]$, at the same time $dx = d\psi(y) = \psi'(y)dy$, then, there is $ds=\sqrt{dx^2+dy^2}=\sqrt{1+\psi'^2(y)}dy$;

(c) If the equation for curve L is determined by a polar equation $\rho=\rho(\theta)$, where $\alpha \le \theta \le \beta$,the curve can be expressed here θ as a parametric equation with parameters here:

$$\begin{cases} x=\rho(\theta)\cos\theta \\ y=\rho(\theta)\sin\theta \end{cases} \tag{3}$$

The arc differential is thus expressed as:

$$\begin{aligned} ds &= \sqrt{dx^2+dy^2} \\ &= \left[(\rho'(\theta)\cos\theta-\rho(\theta)\sin\theta)^2 \right. \\ &\quad \left. +(\rho'(\theta)\sin\theta+\rho(\theta)\cos\theta)^2\right]^{\frac{1}{2}} d\theta \end{aligned} \tag{4}$$

Then there is: $ds=\sqrt{\rho^2(\theta)+\rho'^2(\theta)}d\theta$.

3.2 Example of Calculation

Example 1: Let L be given as follows: $\begin{cases} x=a\cos(t) \\ y=a\sin(t) \end{cases}$, here $0 \le t \le \frac{\pi}{2}$, Try to calculate the first kind of curve integral $\int_L (x^2+y^2)ds$.

Solution: Due to,

$$\begin{aligned} ds &= \sqrt{dx^2+dy^2} \\ &= \sqrt{a^2\sin^2(t)+a^2\cos^2(t)}dt = adt \end{aligned} \tag{5}$$

Therefore,

$$\int_L (x^2+y^2)ds = \int_0^{\frac{\pi}{2}} a^2 \cdot adt = \frac{\pi}{2}a^3 \tag{6}$$

Example 2: Calculation $\oint_L (x+y)ds$, where L is the part enclosed by straight line segments connecting: $O(0,0)$, $A(0,1)$, $B(1,1)$.

Solution: As the expressions of the three segments are not uniform, they are calculated as follows:

OA : $x=0$, $0 \le y \le 1$, $ds=dy$; OB : $y=x$, $0 \le y \le 1$, $ds=\sqrt{2}dx$; AB : $y=1$, $0 \le x \le 1$, $ds=dx$,; then:

$$\begin{aligned} \oint_L (x+y)ds &= \int_{OA+OB+AB}(x+y)ds \\ &= \int_0^1 ydy + \int_0^1 (x+x)\sqrt{2}dx + \int_0^1 (x+1)dx \\ &= 2+\sqrt{2} \end{aligned} \tag{7}$$

4. Popularization and Application of the First Kind of Curve Integral

First, let's look at a simple problem [8]. Let the curve segment l : $y=f(x)$ be the curve segment on the plane xoy, where $a \le x \le b$, where $f(x)$ is a continuous first derivative, and the distance between the curve segment l and the coordinate axis x is $D=|y|=|f(x)|$, then the area of the rotating surface obtained by rotating the curve segment l around the coordinate axis x is calculated and analyzed as follows:

The curve segment l is divided into some n small curve segments $l_i (i=1,2,\cdots,n)$, and the length of each l_i is marked as Δs_i, when $\Delta s_i \to 0$, the small surface of each l_i rotation is approximately rectangular. If any point ξ_i is taken from l_i, so $D_i(\xi_i)=|f(\xi_i)|$ is the radius of the approximate circle formed by the l_i rotation, and the area of the small surface can be approximately expressed as follows: $2\pi D_i(\xi_i)\Delta s_i$, then the area of the rotation surface obtained by rotating the whole curve section l once around the coordinate axis x is approximately $\sum_{i=1}^{n} 2\pi D_i(\xi_i)\Delta s_i$, if $\lim_{n\to\infty}\sum_{i=1}^{n} 2\pi D_i(\xi_i)\Delta s_i$ exists, we can obtain the area formula of the rotation surface obtained by rotating the curve section l once around the coordinate axis x in combination with the first type of curve integral:

$$\begin{aligned} s &= \int_l 2\pi D ds = \int_a^b 2\pi|f(x)|ds \\ &= \int_a^b 2\pi|f(x)| \cdot \sqrt{1+f'^2(x)}dx \end{aligned} \tag{8}$$

Similarly, we can deduce:

Curve Segment l: $y=f(x)$ is a curve segment on a plane xoy, where $a \le x \le b$, in which $f(x)$ has a continuous first derivative, and the surface area of revolution formed by the rotation of the curve segment l about a defined line $L: Ax+By+C=0$ (a line on the plane xoy) is:

$$\begin{aligned} s &= \int_a^b 2\pi \frac{|Ax+By+C|}{\sqrt{A^2+B^2}} ds \\ &= \int_a^b 2\pi \left[\frac{|Ax+Bf(x)+C|}{\sqrt{A^2+B^2}} \right. \\ &\left. \times \sqrt{1+f'^2(x)} \right] dx \end{aligned} \quad (9)$$

Example 4 Find the area of the surface of revolution formed by the line segment $y=2x$ ($0 \le x \le 1$) rotating one revolution around a straight line $2x+y-1=0$.

Solution: From Formula (4), we can get:

$$\begin{aligned} s &= \int_0^1 2\pi \frac{|2x+y-1|}{\sqrt{2^2+1^2}} ds \\ &= \int_0^1 2\pi \frac{|2x+2x-1|}{\sqrt{5}} \cdot \sqrt{1+4} dx \\ &= \frac{9\pi}{2} \end{aligned} \quad 10)$$

5. Conclusion

By solving the quality problem of curved non-uniform material components, and combining the essence of Riemann integral thought, that is, “elementary mathematical method (regular method)+the thinking method of limit, the definition form of the first kind of curve integral is intuitively and vividly understood; from the symbolic analysis of the expression of the first kind of curve integral , the essence of calculating the first kind of curvilinear integral is emphasized, that is, the integrand function and arc differential are classified a the definite integral of one variable through the given expression of curvilinear equation; at the same time, the idea of the first kind of curve integral is used to solve the area of a kind of rotating surface. There are few examples involved in this paper, and there are many specific and special calculation or concise methods for the first kind of curve integral [9,10], which are not given here. In a word, we hope that in that design of mathematics practice teaching, we should pay attention to the commonness of thinking methods, strengthen the guidance of feeling or understanding the mathematical thought method hidden in the concept form, deepen the understanding of concepts and calculations, and cultivate better mathematical logic thinking ability, hoping to have some inspiration for the majority of mathematics lovers.

Acknowledgements

The relevant work of this paper has been supported and funded by the Science Research Fund Project of Yunnan Provincial Department of Education (No.2022J1097).

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