A Novel Support Vector Regression Algorithm for Processing Experimental Data with Perturbation

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Abstract: Processing experimental data with uncertainty is a huge challenge for robust regression modeling and high-accuracy data forecasting. The objective of the present study is the development of a robust support vector regression model for processing observed data of independent variables containing polyhedral perturbation and achievement of high forecasting precision. Firstly, the conception of the data collection of polyhedral perturbation is given and analyzed. Secondly, a novel robust support vector regression model is constructed by replacing the original independent variables in a linear support vector regression model with the independent variables containing polyhedral perturbation. Thirdly, the novel robust linear support vector regression method is also expanded to the non-linear regression model. Both the two models are validated by the linear and nonlinear numerical regression experiments. Comparison of the experimental results show that the proposed method provide more accurate predictions than the traditional regression method for processing data with polyhedral perturbation.

Keywords: Polyhedral Perturbation; Robust Support Vector Regression; Convex Quadratic Programming; Prediction Accuracy

1. Introduction

Regression analysis is a numerical modeling and analysis technique for estimating the quantitative relationship between one dependent variable and single independent variable or multiple independent variables. In recent years, it has became the best-known and most important data processing method owing to the increased importance of data prediction in engineering fields. However, poor and insecure prediction precision of the traditional regression algorithms has limited the development of the data processing methods for a long time. Some machine learning modeling methods such as least squares regression, decision tree learning, the Bayesian method, support vector regression (SVR), artificial neural networking and deep learning have been developed to improve their forecasting accuracy [1]. SVR is particularly a powerful tool for solving small-sample learning problems, and is used for data prediction and optimization of process parameters in engineering applications. Nevertheless, the regression hyperplane of the traditional support vector machine is sensitive to noise. In addition, the uncertainty of the independent variables has always been ignored. Only few previous studies considered the effect of the perturbed independent variables on the existence and optimality of the solution of the regression model [2]. It thus becomes necessary to develop a robust regression modeling technique that could be used to process observed data of the independent variables containing noise and possesses favourable and stable prediction precision.

Many researchers have put great attentions on development of novel support vector machine for processing data with perturbation. In Luo's study [3], a kernel-free quadratic surface support vector regression model based on optimal margin distribution was built. It minimizes the variance of the functional margins of all data points to achieve better generalization capability. When the data points exhibit uncertainty, the covariance information of noise is employed to construct a robust model which ensures its worst-case performance. The probabilistic constraints in the proposed model were proven to be equivalently reformulated as second-order cone constraints for efficient implementation. A new relaxed support vector regression is constructed by Panagopoulos [4] based on the concept of constraint relaxation which leads to increased robustness in datasets with outliers. The model is formulated using both linear and quadratic loss functions. Numeric experiments proved that the novel method achieves better overall performance than support vector regression. Alzalg [5] derived an infeasible interior-point algorithm for the same stochastic optimization problem by utilizing the deterministic symmetric cone programs. It was showed that the infeasible interior-point algorithm has less complexity and more efficiency than that of the homogeneous self-dual algorithm. A novel second-order cone programming formulation is proposed by Dong [6] for the soft-margin support vector machine.

Actually, all the modeling and solving issues of the robust support vector machines are the programming problems in mathematics. The rapid development of robust optimization tecnology was independently promoted by Robert W. Hanks, Mir Saman Pishvaee, Mhammadreza Chamanbaz, Milan Hladík, and Julio López [7–11]. The works of these researchers focused on the need for the execution of more complex computations in robust second-order cone programming (SOCP), robust semi-definite programming (SDP) and non-deterministic polynomial-time (NP) hard problems, which were respectively derived from robust linear programming (LP), SOCP and SDP through the introduction of the conception of general perturbed data set [12–15]. Ye [16] established explicit formulas for the proximal, regular and limiting normal cone of the second-order cone complementarity set. The second-order optimality conditions for the mathematical program with semidefinite cone complementarity constraints were researched by Liu Yulan [17]. By using generalized differential tools of second-order variational analysis, Hang [18] formulated the corresponding version of second-order sufficiency and gave the uniform second-order growth condition for the augmented Lagrangian. A sensitivity result for quadratic second-order cone programming under the weak form of second-order sufficient condition was present in Zhao Qi's research [19]. Xu Zhijun [20] proposed a enhanced second order cone programming relaxation using the simultaneous matrix diagonalization technique for the linearly constrained quadratic fractional programming problem. To overcome the difficulty of obtaining the exact solutions of the subproblems, a proximal alternating direction method of multipliers whose regularization matrix in the proximal term is generated by BFGS at every iteration was proposed by Mu Xuewen [21]. By establishing an abstract result on second-order optimality conditions for a multi-objective mathematical programming problem, Toan [22] derive second-order necessary and sufficient optimality conditions for a multi-objective discrete optimal control problem. Generic non-convex quadratic ε-insensitive loss function was proposed by Ye Yafen [23]. Support vector regression method with this loss was proved with robustness and generalization ability. In order to reduce the influence of noise or outliers on the performance of TSVR, Anagha [24] gave a novel robust twin support vector regression with smooth truncated loss function. A concave-convex programming was employed to solve the nonconvex optimization problem in the primal space. In addition, the convergence of the squared pinball twin support vector regression is also proved. The effectiveness, convergence and robustness of the proposed method were verified by the experiments based on some artificial datasets and UCI datasets with noise and without noise.

Some scholars also investigated the fabric characteristics of the perturbed data sets and researched on calculation of the generalized SVR model for particular structural data. For example, a new correlated polyhedral uncertainty set is developed by Jalilvand [25]. In his study, the robust counterpart of a LP problem is developed under the proposed uncertainty set. The application of this method for solving sample robust problems are discussed. Qui [26] studied on solution stability of generalized equations over polyhedral convex sets. An exact formula for computing the Mordukhovich coderivative of normal cone operators to nonlinearly perturbed polyhedral convex sets is established based on a chain rule for the partial second-order subdifferential. This formula leads to a sufficient condition for the local Lipschitz-like property of the solution maps of the generalized equations under nonlinear perturbations [27]. However, there is no report on development and application of specialized SVR model which is built for resolving the challenging problem of the optimal hyperplane with consideration of the influence of the polyhedral convex data sets.

In the present study, the structural features of the polyhedral perturbation of the independent variables were analyzed. The objective of this paper was to develop a robust support vector regression modeling technique for processing observed data of the independent variables with polyhedral perturbation specifically. The proposed modeling technique is intended for data prediction in engineering applications.

2. Materials and Methods

The key problem of modeling of support vector regression (SVR) is how to obtain an optimal hyper-plane that is the nearest to all the sample data. As discussed in this section, the process of obtaining this hyper-plane comprises three steps: (1) Setting the training data collection and its polyhedral perturbation collection. (2) Defining the original optimization problem. (3) Giving the linear and nonlinear robust regression algorithm procedures.

2.1. Training Data Collection and Polyhedral Perturbation

Training data is used to train learning machine and search for the optimal parameters of the regression model that define the optimal hyper-plane. Here, the training data collection T is described as following formula:

$$\mathrm{T} = \{(\chi_1, y_1), (\chi_2, y_2), \cdots, (\chi_i, y_i), \cdots, (\chi_m, y_m)\}, i = 1,2, \cdots, m, \quad (1)$$

The independent variables χ_i are determined by the perturbation centers $[\chi_i]_j$, the perturbation amplitude $[\Delta\chi_i]_j$ and the parameter Ω, as follows:

$$\begin{aligned} \chi_i = \{\tilde{x}_i | [\tilde{x}_i]_j = [x_i]_j + [\Delta x_i]_j \cdot [z_i]_j, j = 1,2, \cdots, n, \\ z_i = \left([z_i]_1, \cdots, [z_i]_j, \cdots, [z_i]_n\right)^T, \|z_i\|_1 \le \Omega\}, x_i, \Delta x_i \epsilon R^n, \end{aligned} \quad (2)$$

where

χ denotes the independent variables $(\chi_1, \chi_2, \cdots, \chi_i, \cdots, \chi_m)$, referred to as the input data of the training collection;

y denotes the dependent variables $(y_1, y_2, \cdots, y_i, \cdots, y_m)$, referred to as the output data of the training collection;

$[\chi_i]_j$, denotes the perturbation center of the input data point χ_i;

$[\Delta\chi_i]_j$, denotes the perturbation amplitude of the input data point χ_i;

Ω is a weight factor, which is a given real number, used to adjust the region size of the perturbation (generally, Ω=1);

$[z_i]_j$ denotes the components of the weight factor Ω, where z is an n×m matrix, with the 1-norm of the column vector z_i less than or equal to Ω;

m is the number of training data in the collection T;

n is the dimensionality of the independent variables χ_i, namely, the number of independent variables.

The sufficient and necessary conditions for $\tilde{x}_i \epsilon \chi_i$ is

$$\sum_{j=1}^{n} \left| \frac{[\tilde{x}_i]_i - [x_i]_j}{[\Delta x_i]_j} \right| \le \Omega \quad (3)$$

To define the polyhedral perturbation problem, the case of a two-dimensional perturbed polyhedron, which is actually a rhombus, is shown in figure 1. Here, $([x_1]_1, [x_1]_2), ([x_2]_1, [x_2]_2)$, and $([x_3]_1, [x_3]_2)$, are respectively the coordinates of the perturbation center points of the observed values of the independent variables χ_1, χ_2 and χ_3, while $([\Delta x_1]_1, [\Delta x_1]_2), ([\Delta x_2]_1, [\Delta x_2]_2)$, and $([\Delta x_3]_1, [\Delta x_3]_2)$ are the corresponding perturbation amplitudes of the observed values. Ω_1 and Ω_2 are given real numbers, where $\Omega_1 > \Omega_2$; $S_1$and S_2 are respectively the areas of the perturbed rhombuses of the independent variables x_1 and x_2.

Figure 1 shows that: (1) When $\Omega_1 > \Omega_2$, then $S_1 > S_1$. This indicates that the given real number Ω, which is related to the measurement precision of the independent variables, determines the area of the rhombus which is formed as a result of data perturbation. (2) If the perturbation amplitude $[\Delta x_1]_1$ in the direction of the $[x_1]$ axis is larger than the amplitude $[x_1]_2$ in the direction of the $[x_1]$ axis, the rhombus would be horizontal. However, if the perturbation amplitude $[\Delta x_3]_1$ in the direction of the $[x_1]$ axis is smaller than the amplitude $[\Delta x_3]_2$ in the direction of the $[x_1]$ axis, the rhombus would be vertical. Obviously, the ratio of the perturbation amplitude of one independent variable to the amplitude of the other independent variable is the key determinant of the direction of the rhombus.

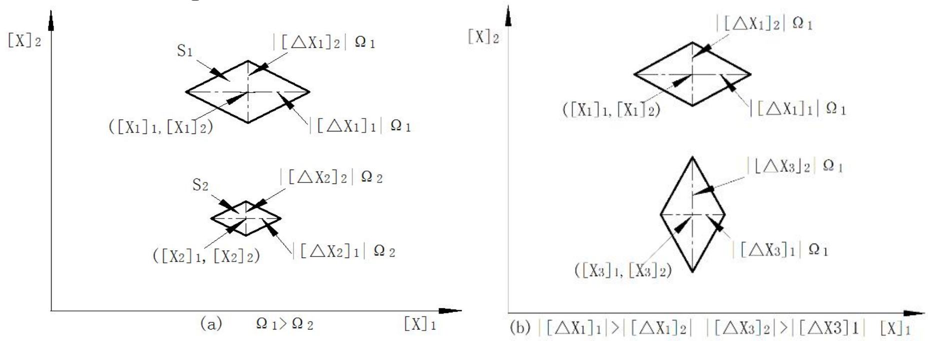

Figure 1. Polyhedral Perturbation

2.2. Robust Regression Algorithm

2.2.1. Linear robust regression algorithm

Based on the mathematics model of the robust support vector regression (RSVR) for processing input data with polyhedral perturbation, the specific algorithm flow of the process is as follows:

The training data is provided in the form of equations (1) and (2).

The mathematical model of support vector regression for processing polyhedral perturbed data proposed in this paper is constructed as follow:

$$min_{w,b,\xi_i,\bar{\xi}_i} \frac{1}{2}\|w\|^2 + C \cdot \sum_{i=1}^{m} (\xi_i + \bar{\xi}_i), \quad (4)$$

$$\text{s.t.} \ \left(\left(w \cdot ([x_i]_j + [\Delta x_i]_j \cdot [z_i]_j)\right) + b\right) - y_i \le \varepsilon + \xi_i, \quad (5)$$

$$y_i - \left(\left(w \cdot ([x_i]_j + [\Delta x_i]_j \cdot [z_i]_j)\right) + b\right) \le \varepsilon + \bar{\xi}_i, \quad (6)$$

$$\xi_i, \bar{\xi}_i \ge 0, \|z_i\|_1 \le \Omega, i = 1,2,\cdots,m, and\ j = 1,2,\cdots,n, \quad (7)$$

where

w is the line normal to the regression hyperplane;

b is the intercept of the regression hyperplane;

ξ_i and $\bar{\xi}_i$ are slack variables;

C is the penalty parameter;

ε is the ε-band of the regression hyperplane.

The solution (w^*, b^*) to the model (4)–(7) can be expressed as

$$w^* = \frac{\delta^* \cdot \sum_{i=1}^{m} \left(x_i \cdot (\alpha_i^* - \beta_i^*)\right)}{\sum_{i=1}^{m} \left(\psi_i \cdot (\alpha_i^* + \beta_i^*)\right) - \delta^*} \quad (8)$$

A component α_k^* is sequentially selected from the data set α^*.

If $0 < \alpha_k^* < C$,

$$b1_k^* = \frac{y_k - \delta^* \cdot \sum_{i=1}^{m} \left((x_i \cdot x_k) \cdot (\alpha_i^* - \beta_i^*)\right)}{\sum_{i=1}^{m} \left(\psi_i \cdot (\alpha_i^* + \beta_i^*)\right) - \delta^*} + \Psi_k \cdot \delta^* + \varepsilon, k = 1,2,\cdots,m, \quad (9)$$

else, $b1_k^* = 0$.

A component β_k^* is sequentially selected from the data set β^*.

If $0 < \beta_k^* < C$,

$$b2_k^* = \frac{y_k - \delta^* \cdot \sum_{i=1}^{m} \left((x_i \cdot x_k) \cdot (\alpha_i^* - \beta_i^*)\right)}{\sum_{i=1}^{m} \left(\psi_i \cdot (\alpha_i^* + \beta_i^*)\right) - \delta^*} - \Psi_k \cdot \delta^* - \varepsilon, k = 1,2,\cdots,m, \quad (10)$$

else, $b2_k^* = 0$; and

$$b^* = \frac{\sum_{k=1}^{m} b1_k^* + \sum_{k=1}^{m} b2_k^*}{m_1 + m_2}, k = 1,2,\cdots,m, \quad (11)$$

Here, m_1 is the number of the components α_k^* that satisfy the condition $0 < \alpha_k^* < C$; and m_2 is the number of the components β_k^* that satisfy the condition $0 < \beta_k^* < C$.

The regression function is established as follows:

$$f(x) = \frac{\delta^* \cdot \sum_{i=1}^{m} \left((x_i \cdot x) \cdot (\alpha_i^* - \beta_i^*)\right)}{\sum_{i=1}^{m} \left(\psi_i \cdot (\alpha_i^* + \beta_i^*)\right) - \delta^*} + b^*, i = 1,2,\cdots,m, \quad (12)$$

2.2.2. Nonlinear robust regression algorithm

Using the kernel function, the linear robust regression algorithm can be easily converted into a nonlinear algorithm, as follows:

The inner products $(x_i \cdot x_k)$ and $(x_i \cdot x)$ in equations (8)–(12) are replaced with the kernel functions $\mathrm{K}(x_i \cdot x_k)$ and $\mathrm{K}(x_i \cdot x)$.

The variables ψ_i in equations (8)–(12) are replaced with R_i.

According to equation (2),

$$\|\tilde{x}_i - x_i\| = \left\|[\Delta x_i]_j \cdot [z_i]_j\right\| \le \psi_i, \quad (13)$$

It is thus obvious that

$$\begin{aligned} \|\Phi(\tilde{x}_i) - \Phi(x_i)\|^2 &= \left(\Phi(\tilde{x}_i) - \Phi(x_i)\right) \cdot \left(\Phi(\tilde{x}_i) - \Phi(x_i)\right) \\ &= \mathrm{K}(\tilde{x}_i \cdot \tilde{x}_i) - 2K(\tilde{x}_i \cdot x_i) + K(x_i \cdot x_i) \\ &= \mathrm{F}(\|\tilde{x}_i - x_i\|) \le R_i^2 \end{aligned} \quad (14)$$

Hence, $R_i = \left(F(\psi_I)\right)^{1/2}$, where F is a function of the kernel function. In the determination of R_i, the transformation from the linear area to the nonlinear area is denoted by $X = \Phi(x)$.

3. Experimental Results and Discussion

3.1. Linear Regression Experiment

To validate the robust linear support vector regression model proposed in section 2.4.1, linear function (y = 0.1 χ + 1) was considered as the objective function with the input variable χ which was a two-dimensional vector. Based on equation (2), the input set was described as $\chi_i = \{\tilde{x}_i = [\tilde{x}_i]_1 + [\tilde{x}_i]_2 | [\tilde{x}_i]_1 = [x_i]_1 + [\Delta x_i]_1 \cdot [z_i]_1, [\tilde{x}_i]_2 = [x_i]_2 + [\Delta x_i]_2 \cdot [z_i]_2\}, i = 1,2,\cdots,25$, where i is an index indicating the data-sample number. The center of the disturbed input data comprised a 25 × 2 matrix x = [1 2 … 25; 1 2 … 25]^T. The disturbing quantity Δx · z was also a 25 × 2 matrix. The value of $[\Delta x_i]_j$ was determined to be a random number in [-1 1] while the value of $[z_i]_j$ was a random number in [0 1], so $\|z_i\|_1 \leq \Omega$. The variable ψ was calculated by using the following inequality $\|[\Delta x_i]_j \cdot [z_i]_j\| \leq \psi$. The Gaussian kernel function was given by $K(X \cdot Y) = Exp\left(-\|X-Y\|^2\right)/2Q^2$, where Q=0.6.

Based on the standard linear support vector regression method and the novel robust linear support vector regression method, two different linear regression models were constructed and implemented using MATLAB (MathWorks, Inc., USA). For evaluating the prediction accuracy of those two models, a 5-fold cross-validation method was employed, and the same five sets of validation data are listed separately in Tables 1 and 2, wherein values of optimal parameters C and e of the robust regression model were listed. After running the computer programs of the two linear regression models on a computer, errors between the predicted results and the actual results as well as the corresponding mean absolute value (MAV) of predicted errors were obtained and listed in Tables 1 and 2.

Table 1. Prediction Errors of Linear RSVR

No.	Validation Data NO.	Parameters	Prediction Errors	MAV
1	1, 3, 5, 7, 9	$C=10^9$,$e=5\times10^{-14}$	-0.6072, 0.3409, 0.0223, 0.0361, -0.1773	0.2368
2	11, 13, 15, 17, 19	$C=10^8$,$e=5\times10^{-14}$	0.0272, 0.0492, 0.0540, 0.0551, 0.0873	0.0546
3	21, 23, 25, 2, 4	$C=10^8$,$e=5\times10^{-14}$	0.3308, -0.0709, 0.4894, 0.0709, -0.3308	0.2586
4	6, 8, 10, 12, 14	$C=10^{10}$,$e=5\times10^{-18}$	-0.0053, -0.0022, -0.0336, -0.0585, -0.0734	0.0346
5	16, 18, 20, 22, 24	$C=10^{10}$,$e=5\times10^{-22}$	0.0528, 0.0507, 0.0493, 0.0594, -0.0564	0.0537
Total				0.1276

Table 2. Prediction Errors of Linear SVR

No.	Validation Data NO.	Parameters	Prediction Errors	MAV
1	1, 3, 5, 7, 9	$C=10^{14}$,$e=5\times10^{-5}$	-1.2493, 0.0532, -0.0363, -0.0038, -0.0108	0.2707
2	11, 13, 15, 17, 19	$C=10^4$,$e=5\times10^{-7}$	-0.0635, 0.0048, 0.0262, 0.0417, 0.1573	0.0587
3	21, 23, 25, 2, 4	$C=10^4$,$e=5\times10^{-9}$	0.1595, 0.0341, 1.3103, -0.0340, -0.1594	0.3395
4	6, 8, 10, 12, 14	$C=10^4$,$e=5\times10^{-11}$	-0.1451, -0.0550, -0.0366, -0.0145, 0.0438	0.0590
5	16, 18, 20, 22, 24	$C=10^4$,$e=5\times10^{-3}$	0.0001, 0.0174, -0.0237, 0.0708, -0.1618	0.0548
Total				0.1565

As can be seen in Tables 1 and 2, using the 5-fold cross-validation methods, 10 sets of prediction errors in the RSVR and the SVR linear regression tests are listed. Furthermore, MAV of the 10 sets of prediction errors have also been calculated and presented. A comparison between MAV listed in Tables 1 and 2 clearly demonstrates that MAV of the prediction errors for RSVR method are smaller than MAV of the prediction errors for SVR method, even the same training data sets and validation data sets were employed in the prediction tests for RSVR method and SVR method.

After analyzing the prediction errors of the 25 validation data samples listed in Tables 1 and 2, statistical results indicate that the range of absolute values of prediction errors were reduced by 51.6% (from [0.0001 1.2493] to [0.0022 0.6072]) when the SVR method was replaced with the RSVR method. Moreover, by using the novel robust model, MAV of prediction errors in the linear regression tests was reduced by 18.5% (from 0.1565 to 0.1276) as well as the variance of absolute values of prediction errors decreased 78.7% (from 0.1585 to 0.0337). It is clear that the the MAV and variance of the prediction errors represent regression performance of the mathematical model. Therefore, the comparison between the linear regression tests for the RSVR method and the SVR method demonstrate that the RSVR algorithm possesses more advantages in high prediction accuracy than the traditional SVR algorithm.

For visually displaying the predicting performance of the novel robust regression model, the theoretical value of the linear function (y = 0.1 χ + 1) and the predicting results of the two different regression models are depicted in Figure 2, wherein the black line represents the theoretical values of the linear function while red asterisks and blue circles denote predictions of the SVR and the RSVR algorithms respectively. As can be clearly observed from the chart, similar high predicting accuracy occurred at validation data points of X coordination from $X = 4([x_1]_1 = 2, [x_1]_2 = 2)$ to $X = 48([x_{25}]_1 = 24, [x_{25}]_2 = 24)$ for the two models. However, there were significant difference of predicting results between the two models at validation data points of X coordination of $X = 2([x_1]_1 = 1, [x_1]_2 = 1)$ and $X = 50([x_{25}]_1 = 25, [x_{25}]_2 = 25)$. In general, forecasting performance of traditional machine-learning model heavily relies on empirical training datasets. So it is proved the SVM has a high forecast precision only if the prediction data fall in the training data sets. The linear regression experiments also indicate that the RSVM has better generalization ability even the fresh prediction samples beyond the training sample sets.

Blue circles are placed closer to the black line compared to red stars at $X = 2([x_1]_1 = 1, [x_1]_2 = 1)$ and $X = 50([x_{25}]_1 = 25, [x_{25}]_2 = 25)$. This indicates that the RSVR algorithm demonstrates higher prediction accuracy compared to the traditional SVR algorithm for data validation outside the scope of training data. This can be attributed to the superiority of RSVR in processing data with polyhedral perturbations.

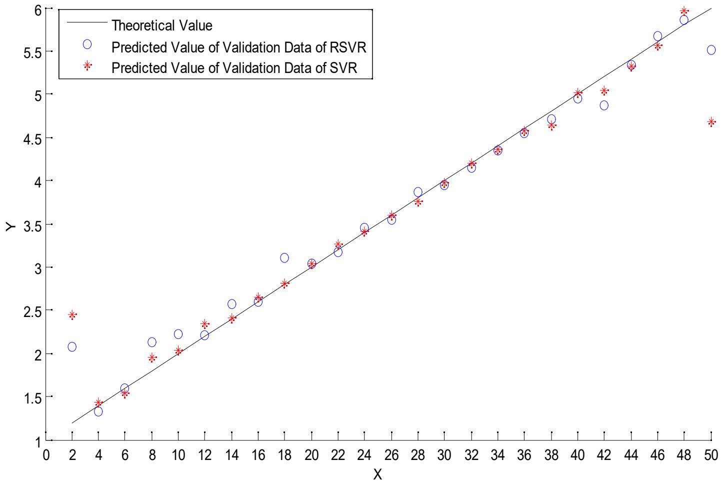

Figure 2. Linear Regression of RSVR and SVR

3.2. Nonlinear Regression Experiment

3.2.1 Aluminum alloy creep tests

It is widely known that the overwhelming majority of metallic materials including aluminium alloys intend to produce creep deformation under the combined effect of temperature, time and stress even as the load exerted on the metals is far less than its yield strength. To validate the present robust support vector regression algorithm, a series of creep tests for aluminum alloy 2124 were conducted to acquire a set of experimental data. The testing specimens and experimental equipment are shown in figure 3 and figure 4. The temperature measurement error is less than ±2°C, and the stress measurement accuracy is higher than 0.1. The perturbation amplitude of the independent variables are closely related to the measurement accuracy of the creep test equipment.

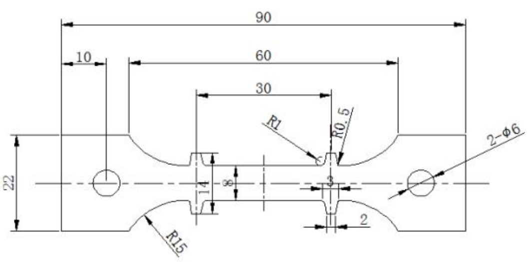

Figure 3. Test 2124 Aluminum Alloy Specimen (Length Unit: mm)

Figure 4. Creep Test Equipment

The primary processing parameters of the creep test include the creep temperature, time, and stress level. Some research revealed that the influence of the fluctuation of the time measurements on the creep deformation of aluminium alloy 2124 was significantly smaller than the fluctuation of creep temperature and stress measurements. The latter parameters were thus selected as the independent variables, while the plastic deformation was set as the dependent variable. It is known that a phenomenon of creep deformation of aluminum alloy 2124 occurs in the temperature range of 140°C –200°C under low stresses. As the yield strengthen of aluminum alloy 2124 is equal or greater than 310MPa, then the corresponding experimental scheme is designed and showed in Table 3.

Table 3. Experimental Scheme

Test No.	Temperature / °C	Stress Level / MPa	Test No.	Temperature / °C	Stress Level / MPa
1	140	100	19	180	200
2	150	100	20	190	200
3	160	100	21	200	200
4	170	100	22	140	250
5	180	100	23	150	250
6	190	100	24	160	250
7	200	100	25	170	250
8	140	150	26	180	250
9	150	150	27	190	250
10	160	150	28	200	250
11	170	150	29	140	300
12	180	150	30	150	300
13	190	150	31	160	300
14	200	150	32	170	300
15	140	200	33	180	300
16	150	200	34	190	300
17	160	200	35	200	300
18	170	200	/	/	/

For convenience of calculation and modelling, the independent variables with various units and values of different orders of magnitude were normalized. The normalized temperature and stress variables were respectively denoted by X_1 and X_2. And they were defined as follows:

$$X_1 = \frac{T-120}{20} \quad (52)$$

$$X_2 = \frac{\sigma}{100} \quad (53)$$

The normalized process parameters are present in Table 4.

Table 4. Original Values and Normalized Values of the Process Parameters

Temperature	**Original Value/°C**	140	150	160	170	180	190	200
	Normalized Value	1	1.5	2	2.5	3	3.5	4
Stress level	**Original Value/MPa**	100	150	200	250	300	/	/
	Normalized Value	1	1.5	2	2.5	3	/	/

The real values of the plastic deformation ε_0 for creep tests were observed by using strain gauge device. Testing results indicate that only few creep plastic deformation was measured at temperatures below 150 °C. For this reason, testing data marked as 1, 2, 8, 9, 15, 16, 22, 23, 29 and 30 were disregarded. The remaining 25 test data samples were used for modeling and validation of nonlinear SVR and nonlinear RSVR.

3.2.2 Cross-validation of SVR and RSVR

Based on equation (2), the independent variable set was denoted as $\chi_i = \tilde{x_i} = [\tilde{x}_i]_1 + [\tilde{x}_i]_2 | [\tilde{x}_i]_1 = [x_i]_1 + [\Delta x_i]_1 \cdot [z_i]_1, [\tilde{x}_i]_2 = [x_i]_2 + [\Delta x_i]_2 \cdot i = [z_i]_2, 1, 2, \cdots, 25$, where i is the index referring to the data-sample number. The center of the disturbed input data was a 25 × 2 matrix x = [2 2.5 3 3.5 4 2 2.5 3 3.5 4 2 2.5 3 3.5 4 2 2.5 3 3.5 4 2 2.5 3 3.5 4; 1 1 1 1 1 1.5 1.5 1.5 1.5 1.5 2 2 2 2 2 2.5 2.5 2.5 2.5 2.5 3 3 3 3 3]^T, and the disturbing quantity Δx · z was also represented by a 25 × 2 matrix. The value of $[\Delta x_i]_j$ is a random number in [-1 1] while the value of $[z_i]_j$ is a random number in [0 1], such that $\|z_i\|_1 \le \Omega$. The variable ψ was calculated using the equation $\left\|[\Delta x_i]_j \cdot [z_i]_j\right\| \le \psi$. The Gaussian kernel function was given by $K(X_1 \cdot X_2) = Exp \quad \left(-\|X_1 - X_2\|^2\right)/2Q^2$, where $Q = 0.4$.

Two procedures of the nonlinear SVR and the nonlinear RSVR were compiled and run on computer to analyze their regression property. Errors between actual values and predictions along with corresponding MAV of prediction errors were listed in Table 5 and Table 6.

Table 5. Prediction Errors of Nonlinear RSVR

No.	Validation Data NO.	Parameters	Prediction Errors	MAV
1	3, 5, 7, 11, 13	$C=10^{12}, e=5\times10^{-6}$	-0.0190, -0.0018, -0.0005, 0.0025, -0.0011	0.0050
2	17, 19, 21, 25, 27	$C=10^{12}, e=5\times10^{-5}$	0.0039, -0.0055, -0.0008, 0.0013, -0.0046	0.0032
3	31, 33, 35, 4, 6	$C=10^{12}, e=5\times10^{-13}$	0.0001, -0.0412, 0.0032, 0.0299, 0.0078	0.0164
4	10, 12, 14, 18, 20	$C=10^{11}, e=5\times10^{-6}$	-0.0021, 0.0015, -0.0043, 0.0020, -0.0058	0.0031
5	24, 26, 28, 32, 34	$C=10^{5}, e=5\times10^{-6}$	0.0013, -0.0039, 0.0147, 0.0103, 0.0113	0.0083
Total				0.0072

Table 6. Prediction Errors of Nonlinear SVR

No.	Validation Data NO.	Parameters	Prediction Errors	MAV
1	3, 5, 7, 11, 13	$C=10^{4}, e=5\times10^{-5}$	-0.0197, -0.0027, 0.0005, 0.0025, 0.0029	0.0057
2	17, 19, 21, 25, 27	$C=10^{4}, e=5\times10^{-5}$	0.0032, -0.0030, 0.0036, -0.0032, 0.0082	0.0042
3	31, 33, 35, 4, 6	$C=10^{4}, e=5\times10^{-5}$	-0.0073, -0.0149, 0.0565, -0.0038, -0.0033	0.0172
4	10, 12, 14, 18, 20	$C=10^{4}, e=5\times10^{-4}$	-0.0097, 0.0014, -0.0025, 0.0017, -0.0045	0.0040
5	24, 26, 28, 32, 34	$C=10^{4}, e=5\times10^{-4}$	-0.0112, -0.0059, -0.0071, 0.0212, -0.0091	0.0109
Total				0.0084

Ten sets of prediction errors of validation data for RSVR and SVR were listed in above tables by using the 5-fold cross-validation method in nonlinear regression experiments. Statistical results of prediction errors listed in Table 5 and Table 6 demonstrate that 26.6% reduction in the absolute-value range of prediction errors (from [0.0005 0.0565] to [0.0001 0.0412]) was realized by replacing the SVR with RSVR. Furthermore, the novel model decreased the MAV of the prediction errors by 14.3% reduction (from 0.0084 to 0.0072) while the variance in absolute values of prediction errors was reduced by 30.8% (from 0.00013 to 0.00009). It is obvious that smaller mean absolute value and variance absolute value of prediction errors prove that the RSVR model has the advantages of minor training sample data and high prediction accuracy.

For visually present the prediction performance of the novel model, testing data and prediction data from the two different regression models were showed in Figure 5, wherein the gray surface represent the creep deformation trend, the black nodes represent experimental data of creep deformation, and the blue short lines and the red short lines represent output values from the SVR and the RSVR respectively. In general, forecasting performance of traditional machine-learning model heavily relies on the training data samples. So it is easy to find that prediction points with large errors are entirely located on the edges of the gray surface. That is because those validation prediction points located on the edges actually exceed the training range of the machine learning model. Particularly, the prediction of the SVR model with maximal error is NO.35 validation sample which beyond the training range of the stress independent variable as well as the training range of the temperature independent variable.

In contrast, the RSVR has advantages in processing data with polyhedral perturbations and improving the generalization ability of the regression model.

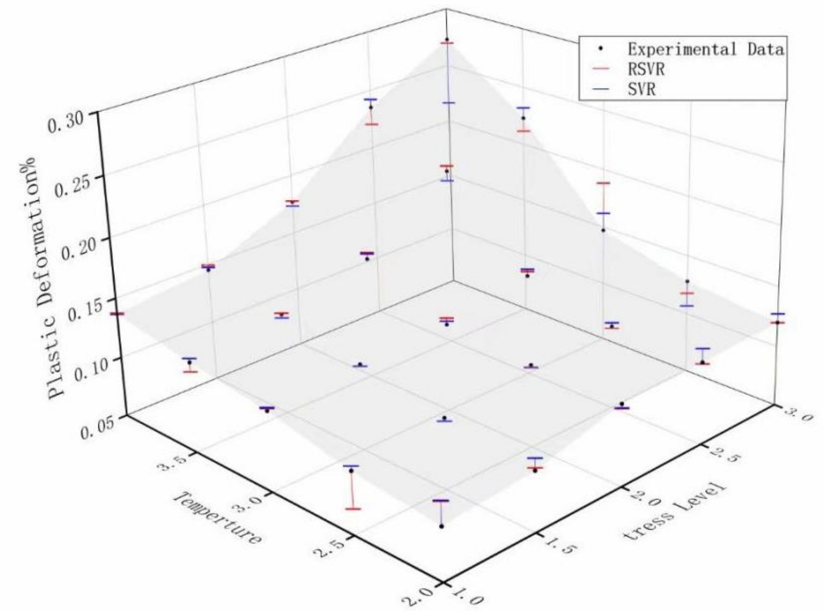

Figure 5. Nonlinear Regression of RSVR and SVR

4. Conclusions

(1) A robust support vector regression method for processing data with polyhedral perturbation was developed in this study. The original optimization model of support vector machine for processing experimental data with polyhedral perturbation is a convex quadratic program, and it shares its solution with the second-order cone program transformed from it. So, the convex quadratic program can be resolved by solving its dual problem.

(2) A linear robust regression modelling approach and a nonlinear robust regression modelling approach were present. To validate the proposed RSVR method, a linear regression experiment and a nonlinear regression experiment were performed. The experimental results showed that the proposed method has its advantages of high prediction accuracy and strong generalization ability for processing data with polyhedral perturbation.

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References

[1] Francisco. Pereira, Tom. Mitchell. Machine learning classifiers and fMRI: A tutorial overview. *NEUROIMAGE* 2009, 45, s199-s209. 10.1016/j.neuroimage.2008.11.007

[2] Vapnik V. N, Learning hidden information: SVM+. IEEE International Conference on Granular Computing, 2006.

[3] Luo Jian. Robust kernel-free support vector regression based on optimal margin distribution. KNOWL-BASED SYST 2022. 109477. 10.1016/j.knosys.2022.109477

[4] Panagopoulos; Orestis P. Relaxed support vector regression. ANN OPER RES 2019. 191-210. 10.1007/s10479-018-2847-6

[5] Alzalg Baha, Khaled Badarneh. An infeasible interior-point algorithm for stochastic second-order cone optimization. Journal of Optimization THEORY AND APPLICATIONS 2019 324-346. 10.1007/s10957-018-1445-8

[6] Dong Guishan, Xuewen Mu. A novel second-order cone programming support vector machine model for binary data classification. Journal of Intelligent & Fuzzy Systems 39.3 2020, 4505-4513. 10.3233/JIFS-200467

[7] Robert. W. Hanks, Jeffery. D. Weir. Robust goal programming using different robustness echelons via norm-based and ellipsoidal uncertainty sets. *EUR J OPER RES* 2017, 262, 636-646.

10.1016/j.ejor.2017.03.072

[8] Mir. Saman. Pishvaee, Mohamadreza. Fazli. Khalaf. Novel robust fuzzy mathematical programming methods. *APPL MATH MODEL* 2016, 40, 407-418. 10.1016/j.apm.2015.04.054

[9] Mohammadreza. Chamanbaz, Giuseppe.Notarstefano. Randomized Constraints Consensus for Distributed Robust Linear Programming. *IFAC-PapersOnLine* 2017, 50, 4973-4978. 10.1109/TCNS.2020.3024483

[10] Milan. Hladík. Robust optimal solutions in interval linear programming with forall-exists quantifiers. *EUR J OPER RES* 2016, 254, 705-714. 10.1016/j.ejor.2016.04.032

[11] Julio. López, Sebastián. Maldonado. Double regularization methods for robust feature selection and SVM classification via DC programming. *INFORMATION SCIENCES* 2018, 429, 377-389. 10.1016/j.ins.2017.11.035

[12] Julio. López, Sebastián. Maldonado. Group-penalized feature selection and robust twin SVM classification via second-order cone programming. *NEUROCOMPUTING* 2017, 235, 112-121. 10.1016/j.neucom.2017.01.005

[13] Julio. López, Sebastián. Maldonado. Multi-class second-order cone programming support vector machines. *INFORMATION SCIENCES* 2016, 330, 328-341. 10.1016/j.ins.2015.10.016

[14] Maryam. Babazadeh, Amin. Nobakhti. Robust controllability assessment via semi-definite programming. *SYST CONTROL LETT* 2016, 98, December 2016, 1-7. 10.1016/j.sysconle.2016.10.001

[15] T. C. E. Cheng, Y. Shafransky. An alternative approach for proving the NP-hardness of optimization problems. *EUR J OPER RES* 2016, 248, 52-58. 10.1016/j.ejor.2015.06.076

[16] Ye J J, Zhou J. Exact formulas for the proximal/regular/limiting normal cone of the second-order cone complementarity set. Mathematical Programming, 2017, 162: 33-50. 10.1007/s10107-016-1027-1

[17] Liu Yulan, Shaohua Pan. Second-order Optimality Conditions for Mathematical Program with Semidefinite Cone Complementarity Constraints and Applications. Set-Valued and Variational Analysis 30.2 2022, 373-395. 10.1007/s11228-021-00587-z

[18] Hang, Nguyen TV; Boris S. Mordukhovich, and M. Ebrahim Sarabi. Augmented Lagrangian method for second-order cone programs under second-order sufficiency. Journal of Global Optimization 82.1 2022, 51-81. 10.1007/s10898-021-01068-1

[19] Zhao Qi; Wenhao Fu; Zhongwen Chen. A sensitivity result for quadratic second-order cone programming and its application. Applications of Mathematics 66 2021, 413-436. 10.21136/AM.2020.0278-19

[20] Xu Zhijun; Jing Zhou. A global optimization algorithm for solving linearly constrained quadratic fractional problems. Mathematics 9.22 2021, 2981. 10.3390/math9222981

[21] Mu Xuewen; Yaling Zhang. An alternating directions method of multipliers for convex quadratic second-order cone programming. Pacific Journal of Optimization 14.2 2018, 369-380.

[22] Toan; Nguyen Thi; et al. Second-order KKT optimality conditions for multiobjective discrete optimal control problems. Journal of Global Optimization 79 2021, 203-231. 10.1007/s10898-020-00935-7

[23] Ye Yafen, et al. Robust support vector regression with generic quadratic nonconvex ε-insensitive loss. Applied Mathematical Modelling 82 2020, 235-251. 10.1016/j.apm.2020.01.053

[24] Anagha P, S. Balasundaram, Yogendra Meena. On robust twin support vector regression in primal using squared pinball loss. Journal of Intelligent & Fuzzy Systems 35.5 2018, 5231-5239. 10.3233/JIFS-169807

[25] Jalilvand Nejad, Amir. Robust optimization under correlated polyhedral uncertainty set. COMPUT IND ENG 2016. 82-94. 10.1016/j.cie.2015.12.006

[26] Qui; Nguyen Thanh. Stability of generalized equations under nonlinear perturbations. OPTIM LETT 2018. 799-815. 10.1007/s11590-017-1147-4

[27] Bertsimas D, Sim M. The price of robustness. Operations Research, 2004. 52(1): 35-53.

