

# Optimization Strategies for Integrating an Ejector into a Two-Stage Vapor Compression Refrigeration System

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**Abstract:** To further investigate how to optimize the coefficient of performance (COP) of a two-stage compression refrigeration system, this paper focuses on process optimization and theoretical calculations for an incomplete intercooling two-stage compression refrigeration cycle using refrigerant R410A. The model established for the ejector system was validated, showing an average calculation error of approximately 6%, indicating good agreement with the simulations. The study compared changes in the COP and ejector coefficient under variations in factors such as subcooling in the intercooler, superheating in the heat exchanger, and intermediate pressure. The ejector coefficient showed a maximum change of 72%, while the COP varied by up to 52%. Additionally, by comparing the refrigeration system with and without the ejector, it was found that the COP could be improved by up to 215%.

**Keywords:** Two-Stage Compression Refrigeration; Process Optimization; Ejector; R410a

## 1. Introduction

Compared to the conventional single-stage compression refrigeration cycle, the two-stage compression refrigeration cycle offers several advantages, including higher refrigeration efficiency, lower compression ratio, reduced compressor discharge superheat, the ability to produce lower temperatures, and significant energy-saving effects [1]. However, its main drawback is that under high condensing temperatures and low evaporating temperatures, its COP decreases significantly. To explore whether the two-stage compression refrigeration cycle can be further optimized, this study conducts theoretical calculations on an incomplete intercooling two-stage compression refrigeration cycle with single throttling, using

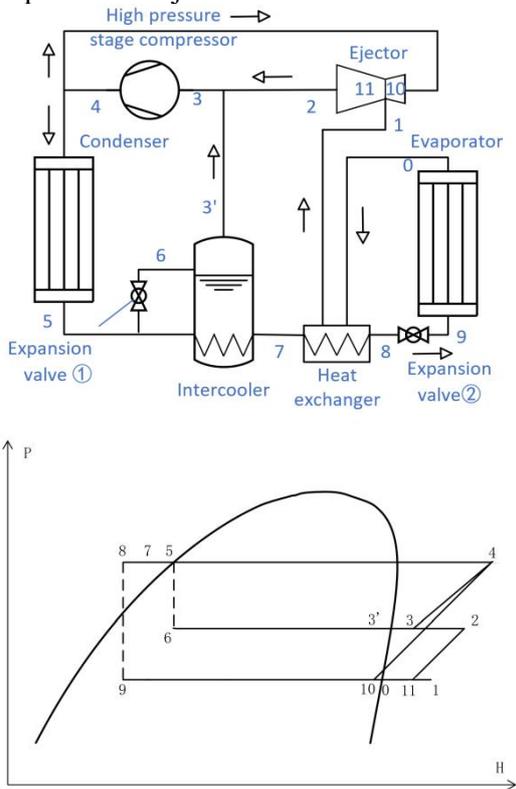
R410A as the refrigerant.

An injector is a device that utilizes high-pressure fluid to drive low-pressure fluid, forming a mixed fluid at an intermediate pressure [2]. Since it does not directly consume mechanical energy, it can be driven by low-grade heat sources, thereby reducing electricity usage and achieving energy savings. Incorporating it into other refrigeration systems can enhance the system's refrigeration efficiency [3]. Consequently, researchers have begun to conduct in-depth studies on jet refrigeration cycles, covering aspects such as refrigerant selection, optimization of injector geometric parameters [4], identification of suitable operating conditions for jet refrigeration cycles [5], and integration of jet systems with other refrigeration systems [3],[6]. These efforts significantly expand the application scenarios and popularity of jet refrigeration cycles. This study also opts to use an injector to optimize the traditional two-stage compression cycle with incomplete intermediate cooling. The final calculation results indicate that the injector indeed enhances the performance of this refrigeration cycle, thus providing new insights for the process optimization of subsequent two-stage compression refrigeration cycles.

## 2. System Introduction

As illustrated in the flowchart in Figure 1, the refrigerant absorbs heat from the cooled substance in the evaporator and evaporates. Subsequently, the low-pressure saturated vapor (point 0) passes through the regenerator, becoming superheated vapor (point 1), which serves as the ejector's driving fluid. This vapor then mixes with the working fluid, diverted from the high-pressure stage compressor, in the ejector until reaching point 2. It then combines with the saturated vapor (point 3') separated by the intermediate cooler to form point 3, which enters the high-pressure stage compressor. After compression, the refrigerant reaches state point 4,

where part of it is diverted back to the ejector as the working fluid, while another portion is cooled in the condenser to state point 5. From this cooled saturated liquid, a fraction undergoes throttling to produce flash vapor (point 6), which continues to mix with the fluid exiting the ejector before entering the high-pressure stage compressor. Another portion of the refrigerant passes through the intermediate cooler, resulting in a liquid with a certain degree of subcooling (point 7). This liquid further increases in subcooling as it passes through the regenerator to point 8, and is then throttled through the expansion valve to state point 9, where it enters the evaporator to absorb heat. This completes the two-stage compression cycle with incomplete throttling and intermediate cooling that incorporates the ejector.



**Figure 1. Schematic Diagram of the Refrigeration Cycle along with Its Pressure-Enthalpy Diagram**

At the intercooler, the heat balance equation can be expressed as:

$$c(h_3 - h_6) = a(h_5 - h_7) \tag{1}$$

In the equation,

*a*----- Refrigerant Mass Flow Rate of Working Fluid kg/s

*c*----- Refrigerant Mass Flow Rate in the Low-Stage Compressor kg/s

The isentropic efficiency of the high-stage compressor in this system can be determined using the formula provided in the literature[9].

$$\alpha \frac{h_4 - h_{3'}}{h_4 - h_3} = 0.874 - 0.0135 \frac{p_k}{p_0} \tag{2}$$

In the equation,

*h*<sub>3'</sub>-----The point obtained by the intersection of the isentropic line at the compressor outlet (state point 4) and the isobaric line at the intermediate pressure is defined as: *S*<sub>3'</sub>=*S*<sub>4</sub>

*α*-----Correction Factor, *α*=0.92

The COP of this refrigeration cycle system can be expressed as:

$$\varepsilon = \frac{Q_e}{W_c} = \frac{a(h_0 - h_9)}{(a + b + c)(h_4 - h_3)} \tag{3}$$

In the equation,

*ε* -----The COP of this system

*Q*<sub>e</sub> -----Evaporator Cooling Capacity kW

*W*<sub>c</sub> -----Compressor Power Consumption kW

*b*-----Refrigerant Mass Flow Rate of Jet Fluid kg/s

For all processes within the ejector, both mass conservation and energy conservation principles are upheld.

Mass conservation:

$$G_P + G_H = G_C \tag{4}$$

In the equation,

*G*<sub>P</sub> -----Refrigerant Mass Flow Rate of Working Fluid kg/s

*G*<sub>H</sub> -----Refrigerant Mass Flow Rate of Jet Fluid kg/s

*G*<sub>C</sub> -----Refrigerant Mass Flow Rate of Mixed Fluid kg/s

Energy conservation:

$$bh_4 + ah_1 = (a + b)h_2 \tag{5}$$

### 2.1 Flow Process of Working Fluid in the Working Nozzle

The working fluid enters the injector and initially undergoes an adiabatic expansion process within the working nozzle (4-10). During this process, the enthalpy of the working fluid decreases while the flow velocity increases, in accordance with the principle of energy conservation.

$$h_4 + \frac{v_4^2}{2} = h_{10} + \frac{v_{10}^2}{2} \tag{6}$$

In the equation,

*v*<sub>4</sub>-----Flow velocity of the working fluid at

the nozzle inlet m/s

$v_{10}$ -----Flow velocity of the working fluid at the nozzle outlet m/s

## 2.2 Mixing Process

The working fluid and the injected fluid mix at a specified pressure to form a mixed fluid. This process is governed by the principles of mass conservation, momentum conservation, and energy conservation.

Mass conservation

$$G_p + G_H = G_C \quad (7)$$

Momentum conservation

$$bv_{10} + av_1 = (a+b)v_{11} \quad (8)$$

In the equation,

$v_1$ -----Flow velocity of the injected fluid at the injector inlet ,which is negligible in comparison to the flow velocity of the working fluid at the nozzle outlet,  $v_1=0$ m/s;

$v_{11}$ -----Flow velocity of the mixed fluid at the isobaric surface m/s

Energy conservation

$$b\left(h_{10} + \frac{v_{10}^2}{2}\right) + a\left(h_1 + \frac{v_1^2}{2}\right) = (a+b)\left(h_{11} + \frac{v_{11}^2}{2}\right) \quad (9)$$

## 2.3 Flow of the Mixed Fluid within the Diffuser

This process is governed by the principle of energy conservation:

$$h_{11} + \frac{v_{11}^2}{2} = h_2 + \frac{v_2^2}{2} \quad (10)$$

In the equation,

$v_2$ -----Flow velocity of the mixed fluid at the nozzle outlet m/s

After determining the states point 2, the state point 3 can also be derived using the principle of energy conservation.

$$(a+b+c)h_3 = (a+b)h_2 + ch_3, \quad (11)$$

## 3. Modeling and Verification

### 3.1 System Modeling

When establishing a computational model for the injector, there are two assumptions regarding the mixing process of the working fluid and the injected fluid: the constant pressure mixing assumption and the constant area mixing assumption [23]. The former indicates that the mixing process occurs under constant pressure conditions, while the latter implies that the effective cross-sectional area remains unchanged during the mixing process. Both assumptions are

made to simplify the computational process; however, in most cases, the accuracy of the results obtained using the constant pressure mixing assumption is higher than that of the results obtained using the constant area mixing assumption. Therefore, in this study, the constant pressure mixing assumption is selected for the systematic modeling and computation of the injector.

In order to simplify the thermodynamic analysis model for the two-stage compression refrigeration cycle with incomplete intermediate cooling incorporating the injector, the following assumptions are made [7]:

(1) The flow of the fluid within the injector is considered to be one-dimensional and steady-state.

(2) The refrigerant at the outlets of the intercooler, condenser, and evaporator is assumed to be in a saturated state.

(3) The flow velocities of the refrigerant at the inlet and outlet of the injector are negligible.

(4) The friction losses and mixing losses during the flow of the refrigerant within the injector are disregarded, and the isentropic efficiencies for the pressure increase (11-2) and pressure decrease (4-10) processes in the injector are assumed to be equal to 1.

(5) The mixing process of the working fluid and the injected fluid is considered to be a constant pressure mixing process.

(6) It is assumed that the actual pressure at the injector outlet reaches the ideal working pressure, which is the intermediate pressure [8].

(7) The injector is assumed to be adiabatic.

For the two-stage compression refrigeration cycle with incomplete intermediate cooling incorporating the injector, it is necessary to first assume the degree of superheat at the outlet of the high-pressure stage compressor. The state point 4 can be determined using REFPROP. The enthalpy at point 3 can be calculated based on the isentropic efficiency formula for the compressor [9]. Given the operating conditions, the degree of subcooling in the intercooler, and the degree of superheating in the regenerator, the enthalpy values at each state point can be obtained from REFPROP. Subsequently, the mass flow rates of the working fluid, injected fluid, and refrigerant split in the intermediate cooler can be derived by solving the system of equations. This allows for the calculation of the jet coefficient and the COP. If the jet coefficient does not meet the specified conditions, the

assumed degree of superheat at the compressor outlet can be adjusted, and the above process can be repeated. The specific flowchart is shown in the figure 2.

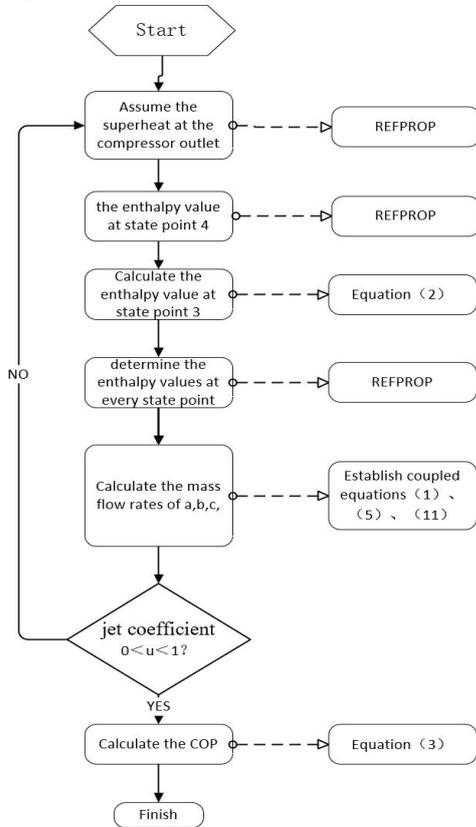


Figure 2.Theoretical Calculation Flowchart

3.2 Model Verification

3.2.1 Regarding the Correction Factor

The isentropic efficiency refers to the energy losses of the compressor during the compression process under the condition of constant entropy. The definition applied to this system is as follows:

$$\eta_s = \frac{h_{4'} - h_3}{h_4 - h_3} \tag{17}$$

In the equation,

$\eta_s$ -----The isentropic efficiency of the compressor

$h_{4'}$ -----The point obtained from the intersection of the isentropic line at state point 3 and the isobaric line at state point 4 ,  $S_{4'}=S_3$

Based on the formula for isentropic efficiency from reference [9], it can be determined that the isentropic efficiency of the compressor at this point is 0.7. In performing theoretical calculations for this system, it is necessary to derive state point 3 from state point 4. Directly using the above equation complicates the calculation process significantly; therefore, the author introduces a correction factor method. This involves multiplying the ratio of the isentropic line passing through state point 4 to the original process line by the correction factor. Verification shows that the error associated with this method is negligible and simplifies the calculation process. The equation is as follows:

$$\alpha \frac{h_4 - h_{3''}}{h_4 - h_3} = 0.7 \tag{18}$$

In the equation,

$h_{3''}$ -----The point obtained from the intersection of the isentropic line at the compressor outlet state point 4 and the isobaric line at the intermediate pressure ,  $S_{3''}=S_4$

$\alpha$ -----Correction Factor ,  $\alpha=0.92$

Based on the data in the table, the ratio of the isentropic line passing through state point 4 to the original process line stabilizes around 0.77. Therefore, a constant correction factor can be applied for adjustment.

Table 1.Calculation Results for the Verification of the Isentropic Efficiency Model

3' Superheat Degree(°C)	$h_3$ ·(kJ/kg)	$h_{4'}$ ·(kJ/kg)	$h_4$ ·(kJ/kg)	$h_3$ ·(kJ/kg)	$\eta_s$
0	419.28	469.36	490.82	435.31	0.77
5	424.48	476.28	498.48	441.08	0.77
10	429.55	483.1	506.05	446.79	0.77
15	434.48	489.68	513.34	452.32	0.77
20	439.31	496.11	520.45	457.75	0.77

3.2.2 Feasibility Evaluation

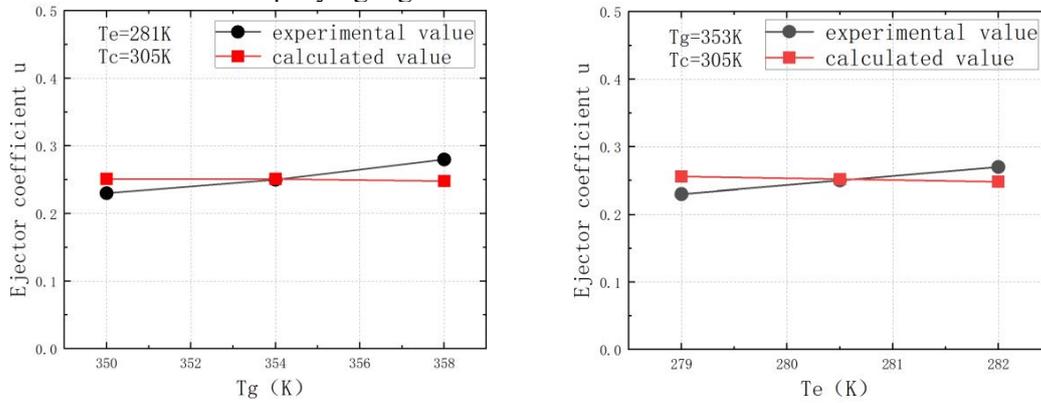
In the two-stage compression refrigeration cycle with incomplete intermediate cooling incorporating the injector, the working fluid is sourced from the superheated steam at the compressor outlet. In contrast, the working fluid in the literature generally originates from the refrigerant at the condenser outlet, which is either generated through the generator or

obtained from the steam separated by the throttling process in the vapor-liquid separator. Therefore, to assess the quality of the modeling in this study, it is crucial to verify the feasibility of the established injector model.

The refrigerant used in this system is R410A; however, experimental data related to the two-stage compression injection refrigeration cycle involving this refrigerant is insufficient.

Nevertheless, since the injector model exhibits universality for different refrigerants [10], this study utilizes the experimental data from reference [6], which focuses on R134a as the refrigerant, to validate the theoretical injector model established herein. The validation results are illustrated in the accompanying figures and

tables. From the deviation results, the average error between the data from this system and that from the literature is approximately 6%, demonstrating that the system has a high degree of accuracy and that the establishment of this injector model is feasible.



**Figure 3. Verification of Refrigeration Cycle Model**

**Table 1. Verification Results of the Refrigeration Cycle Model**

$T_c=281K$	$T_c=305K$		
$T_g(K)$	Experimental value	Calculated value	Deviation(%)
350	0.23	0.251	9.13
354	0.25	0.251	0.4
358	0.28	0.248	11.4
$T_g=353K$	$T_c=305K$		
$T_e(K)$	Experimental value	Calculated value	Deviation(%)
279	0.23	0.256	11.3
280.5	0.25	0.252	0.8
282	0.27	0.248	8.1

**4. Calculation Results and Analysis**

As shown in the figure 4, under operating conditions ranging from -50 to 60 °C, the effects of varying intermediate pressures, evaporation temperatures, condensation temperatures, and compressor superheat on the system's COP and the injector's jet coefficient are illustrated. The calculation results indicate that:

(1) As the intermediate pressure increases from 1 MPa to 2 MPa, the system's COP remains generally constant at around 1.02. However, the jet coefficient of the ejector gradually decreases from 1.294 to 0.364, a reduction of 72%. This is because the increase in intermediate pressure leads to a reduction in compressor power consumption and a decrease in the pressure ratio at the high-pressure side, which lowers the discharge temperature at the high-stage compressor outlet. This reduction in temperature negatively affects the entrainment capacity of the working fluid in the ejector, causing a

decrease in the jet coefficient. Consequently, the mass flow rate of the entrained fluid decreases, leading to a reduction in the system's cooling capacity.

(2) As the evaporation temperature increases from -50°C to -10°C, the COP rises from 1.026 to 1.429, an increase of 39%, while the jet coefficient decreases from 0.64 to 0.418, a reduction of 35%. This occurs because, as the evaporation temperature rises, the cooling capacity of the system increases, leading to an increase in the mixing pressure within the ejector. The enthalpy of the working fluid at the exit of the driving nozzle increases, resulting in a decrease in flow velocity and a reduction in entrainment capacity, which causes the jet coefficient to drop. However, due to the decrease in the pressure ratio at the low-pressure side, point 2 in Figure 1 lowers, leading to a reduction in power consumption.

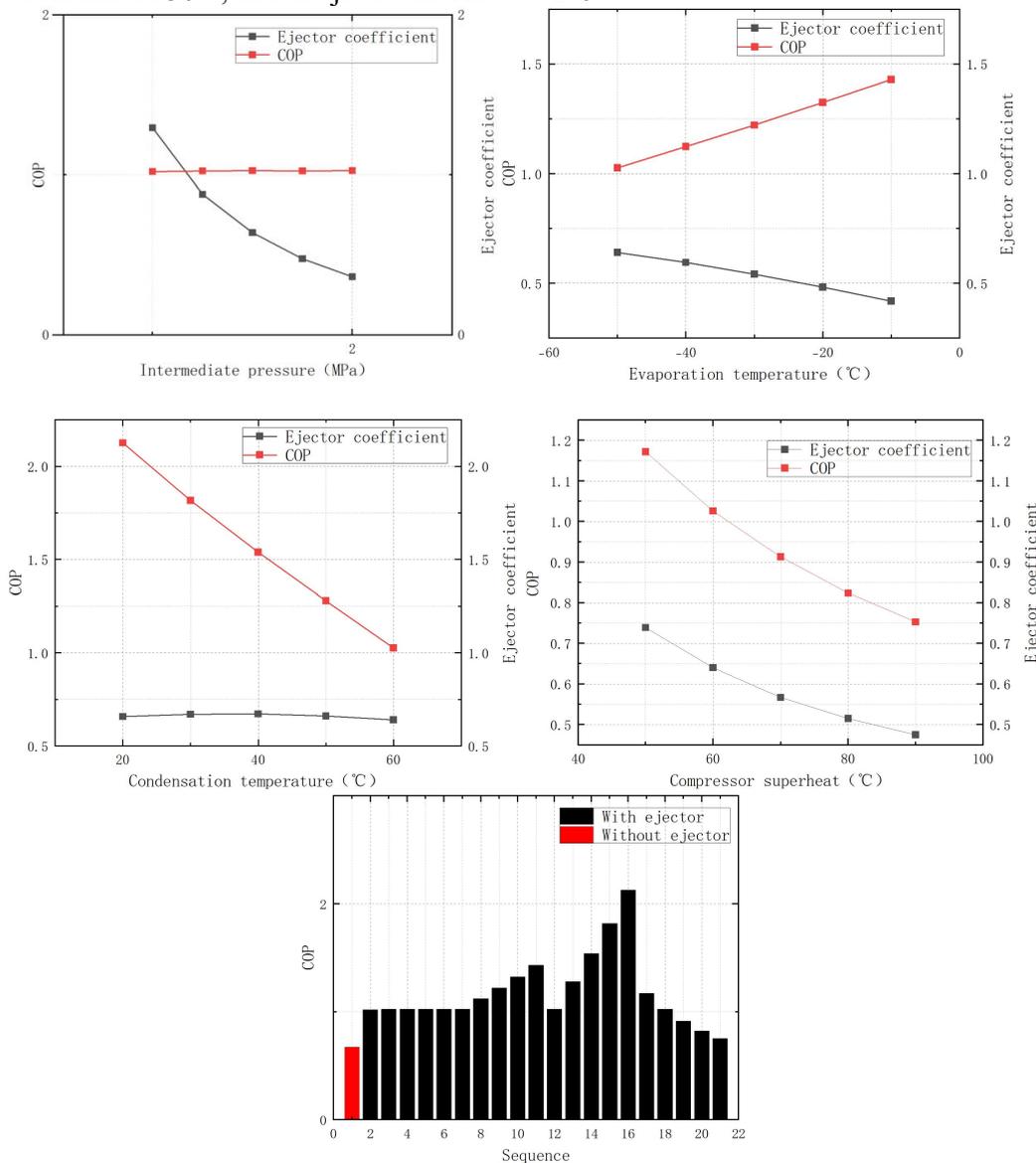
(3) As the condensation temperature increases from 20°C to 60°C, the COP decreases from

2.126 to 1.026, a reduction of 52%, while the jet coefficient remains roughly constant at around 0.66. This is because, as the condensation temperature rises, the pressure ratio of the high-stage compressor increases, causing the discharge temperature to rise and leading to an increase in system power consumption. At the same time, the entrainment capacity of the working fluid improves, resulting in a reduction in the mass flow rate of the working fluid. However, the intercooler diverts a portion of the refrigerant to serve as the working fluid, leading to a simultaneous increase in both the working and entrained fluid mass flow rates.

(4) As the compressor superheat increases from 50°C to 90°C, the COP drops from 1.172 to 0.753, a decrease of 36%, and the jet coefficient

falls from 0.739 to 0.475, also a 36% reduction. This occurs because higher compressor superheat increases power consumption. While a slight increase in superheat initially enhances the entrainment capacity of the working fluid, excessive compressor outlet superheat raises the pressure at the ejector's nozzle exit. As the pressure of the jet fluid remains constant, the pressure difference across the nozzle decreases, reducing the ejector's suction capacity and leading to a drop in the jet coefficient.

(5) The COP of the two-stage compression refrigeration cycle with incomplete intermediate cooling, after the incorporation of the injector, is generally greater than that of the system without the injector, with a maximum improvement of 215%.



**Figure 4. Variation of COP and Ejector Coefficient with Various Factors and Comparison of COP Before and After Ejector Introduction**

## 5. Conclusion

To thoroughly investigate the optimization potential of two-stage compression refrigeration, this study conducts flow optimization and theoretical calculations for a two-stage compression refrigeration cycle with incomplete intermediate cooling (refrigerant R410A). The model established for the injector system is validated, yielding an average calculation error of approximately 6%, indicating a good fit with the simulations. This study compares the changes in the coefficient of performance (COP) or jet coefficient of the system as factors such as the degree of subcooling in the intercooler, the degree of superheat in the regenerator, and the intermediate pressure vary, leading to the following conclusions:

As the intermediate pressure increases, the COP of the system remains relatively constant, while the jet coefficient of the injector gradually decreases. Conversely, as the evaporation temperature decreases, the COP of the system also declines, but the jet coefficient slowly increases. Additionally, when the condensation temperature rises, the system's COP sharply decreases, whereas the jet coefficient remains largely unchanged. An increase in compressor superheat results in a gradual decline in both the COP and the jet coefficient. The jet coefficient varies by up to 72%, while the COP changes by up to 52%.

The COP of the two-stage compression refrigeration cycle with incomplete intermediate cooling and the inclusion of the injector is generally greater than that of the corresponding cycle without the injector, with a maximum improvement of 215%.

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