## **Thermoelastic Dynamic Analysis of Microbeams Under Laser Pulse**

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**Abstract: In this paper the thermoelastic dynamic response of microbeam under laser pulse and the influence by noise interference are investigated for the first time. the governing equations** of the microbeam are  $(1 + \tau_q D_q^a)q = -K(1$ **established by the fractional order twophase hysteresis model and the Kelvin-Voigt model, and are solved using the shifted Chebyshev polynomials algorithm. Finally, some numerical simulations are provided, which demonstrate the validity the efficiency and robustness by the proposed method.**

### **Keywords: Microbeams; Fractional Order; Shifted Chebyshev Polynomials; Thermoelasticity**

### **1. Introduction**

**1. Introduction**<br>Microbeams are significant on the micronanoscale, and are employed extensively in a number of fields, including sensing, actuation and precision control [1]. Although the mechanical properties of viscoelastic materials have been well studied, research on their thermoelastic behaviour remains relatively understudied. Along with the accelerated advancement of laser technology, there has been a growing focus on the utilisation of laser load in viscoelastic structures. Putting a specific degree of laser loading on the microbeam causes increasing the temperature of the microbeam structure, resulting in thermal expansion and subsequent changes in strain and deformation of the microbeam [2].

Fourier's law of heat conduction is a foundational law that describes the heat conduction behaviour in solids. However, it has been observed that the law may have limitations in describing the heat conduction to a load free behaviour of complex materials. To address form of behaviour of complex materials. To address this, two-phase hysteresis model [3] and

fractional order have been introduced to better describe the dynamic behaviour of these materials [4-5].

$$
(1 + \tau_{\alpha} D_{t}^{\alpha}) q = -K \left(1 + \tau_{\tau} D_{t}^{\alpha}\right) \nabla T \tag{1}
$$

where  $K$  is the thermal conductivity of the material.

nent (ISSN: 2959-0612) Vol. 2 No. 3, 2024<br> **crobeams Under Laser**<br> **crobeams Under Laser**<br> *china*<br> *D*<br> *D*<br> **c**<br> **c**<br> **c**<br> **c**<br> The fractional order two-phase hysteresis model is a more comprehensive analytical tool for examining the temperature change and heat transfer behaviour of materials subjected to thermal shock. In this paper, Caputo fractionalorder derivatives [6-7] are employed to more accurately capture the dynamic response properties of the materials. the shifted Chebyshev polynomial algorithm is used to solve governing equations of the viscoelastic microbeam in the time domain directly and discretises it into a set of linear algebraic Subsequently, numerical simulations of the microbeams are carried out using the MATLAB programme. et is a more comprenents we analytical too<br>examining the temperature change and heat<br>for behaviour of materials subjected to<br>mal shock. In this paper, Caputo fractional-<br>retervatives [6-7] are employed to more<br>arately cap Imming the temperature change and near<br> *t* r behaviour of materials subjected to<br>
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### **2. Modeling**

The equations of motion and fractional order heat transfer equations are derived as follows by establishing the thermal stresses defined by the Kelvin-Voigt model [8], bending moment equations, the law of conservation of energy and the fractional order two-phase hysteresis model. quations. Subsequently, numerical<br>imulations of the microbeams are carried out<br>sing the MATLAB programme.<br> **.. Modeling**<br>
The equations of motion and fractional order<br>
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equations of motion and fractional order<br>
transfer equations are derived as follows<br>
stablishing the thermal stresses defined by<br>
Kelvin-Voigt model [8] bending moment<br>
trions, the microbeam in the time domain directly and<br>discretises it into a set of linear algebraic<br>equations. Subsequently, numerical<br>equations of the microbeams are carried out<br>simulations of the microbeams are carried out<br>sing the

$$
\left(1+\tau_{d}\frac{\partial}{\partial t}\right)\left[\frac{\partial^{4}\omega}{\partial x^{4}}+\frac{24\alpha_{r}}{h\pi^{2}}\frac{\partial^{2}T}{\partial x^{2}}\right]+\frac{\rho A}{EI}\frac{\partial^{2}\omega}{\partial t^{2}}+\frac{\sigma_{0}A}{EI}\frac{\partial^{2}\omega}{\partial x^{2}}=0, (2)
$$
\n
$$
\left(1+\tau_{r}D_{t}^{\alpha}\right)\left(\frac{\partial^{2}}{\partial x^{2}}-\frac{\pi^{2}}{h^{2}}\right)T+\frac{\pi^{2}}{2Kh^{2}}\left(1+\tau_{q}D_{t}^{\alpha}\right)\int_{-2/h}^{2/h}zFdz=\left(1+\tau_{q}D_{t}^{\alpha}\right)\left[\frac{\rho C}{K}\frac{\partial T}{\partial t}-\frac{\alpha_{r}\pi^{2}hT_{0}E}{24K}\left(1+\tau_{d}\frac{\partial}{\partial t}\right)\left(\frac{\partial^{3}\omega}{\partial t\partial x^{2}}\right)\right]
$$
\n(3)

It is assumed that the microbeam is subjected to a load from a heat source applied in the  $laser$  pulse  $[9-10]$  $F = F_0 \delta(x - vt) F(t)$ , where  $F_0$  represent the

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power density.

### **3. Solution Process**

### **3.1 Shift Chebyshev Polynomials**

The shifted Chebyshev polynomials is derived<br>the recurrence relation in the interval  $[0, 1]$   $[1]$ <sup>11</sup> the recurrence relation in the interval  $[0, L]$   $[1]$ <sup>:</sup>  $\qquad \qquad$   $\qquad$   $\qquad$ :

$$
D_{i+1}(t) = 2\left(\frac{2t}{L} - 1\right)D_i(t) - D_{i-1}(t), \quad i \in N_+.\tag{4}
$$

and of Industry and Engineering Management (ISSN: 2959-0612) Vol. 2<br>
wer density.<br> **Definition 2**<br> **Solution Process**<br> **Solution Process**<br> **Solution Process**<br> **Solution Process**<br> **Solution Process**<br> **Solution Process**<br> **S** The vector consisting of the shifted Chebyshev polynomials is given  $a^{\alpha}$ by the contract of the contrac

$$
\Phi_n(t) = [D_0(t), D_1(t), \dots, D_n(t)]^T = KG_n(t) ,
$$

where 
$$
k_{ii} = 1
$$
 if  $i = j = 0$ 

where 
$$
G_n(t) = [1, t, t^2, \dots, t^n]^T
$$
,  $K = [k_{ij}]_{i,j=0}^n$ ,  
\nwhere  $k_{ij} = 1$  if  $i = j = 0$ ;  
\nwhere  $k_{ij} = 0$  if  $i < jori < 0ori < 0$ ;  
\n $k_{ij} = 2\left(\frac{2}{L}k_{i-1,j-1} - k_{i-1,j}\right) - k_{i-2,j}$  else.  
\n3.2 Function Approximation  
\nThe truncated sequence of two-dimensional  
\ncontinuous function  $\omega(x, t)$  can be expressed:

### **3.2 Function Approximation**

The truncated sequence of two-dimensional

$$
\Phi_n(t) = [D_0(t), D_1(t), ..., D_n(t)] = KG_n(t) ,
$$
\nwhere  $G_n(t) = [1, t, t^2, ..., t^n]^T$ ,  $K = [k_{ij}]_{i,j=0}^n$ ,  
\nwhere  $k_{ij} = 1$  if  $i = j = 0$  ;  
\nwhere  $k_{ij} = 0$  if  $i < jori < 0ori < 0$  ;  
\n $k_{ij} = 2\left(\frac{2}{L}k_{i-1,j-1} - k_{i-1,j}\right) - k_{i-2,j}$  else.  
\n3.2 Function Approximation  
\nThe truncated sequence of two-dimensional  
\ncontinuous function  $\omega(x, t)$  can be expressed:  
\n $\omega(x, t) \approx \sum_{i=0}^n \sum_{j=0}^n u_{ij}D_i(x)D_j(t) = \Phi_n^T(x)U\Phi_n(t)$ ,  
\n(5)  
\nwhere  $U = [u_{ij}]^{n,n}$ 

where 
$$
k_{ij} = 0
$$
 if  $i < 3$  for  $\leq 6$  of  $k_{ij} = 2\left(\frac{2}{L}k_{i-1,j-1} - k_{i-1,j}\right) - k_{i-2,j}$  else.

\n3.2 Function Approximation

\nThe truncated sequence of two-dimensional continuous function  $\omega(x, t)$  can be expressed:

\n
$$
\omega(x, t) \approx \sum_{i=0}^{n} \sum_{j=0}^{n} u_{ij} D_i(x) D_j(t) = \Phi_n^T(x) U \Phi_n(t)
$$
\n(5)

\nwhere  $U = \left[u_{ij}\right]_{i,j=0}^{n,n}$ ,  $+\frac{\alpha_T}{T}$ 

\n
$$
u_{ij} = \frac{1}{k_i k_j} \int_0^L \int_0^T \omega(x, t) D_i(x) D_j(t) \omega_L(x) \omega_T(t) \, dt \, dx
$$
\nSimilarly,

\n
$$
T(x, t) \approx \sum_{i=0}^{n} \sum_{j=0}^{n} w_{ij} D_i(x) D_j(t) = \Phi_n^T(x) W \Phi_n(t)
$$
\n3.3 Differential Operator Matrices of Shifted Chebyshev Polynomials

\nDefinition 1 The first order differential

\n1,  $1$ 

$$
T(x,t) \approx \sum_{i=0}^{n} \sum_{j=0}^{n} w_{ij} D_i(x) D_j(t) = \Phi_n^T(x) W \Phi_n(t) \cdot
$$

# **Shifted Chebyshev Polynomials**

**Definition 1 The** first order differential operator matrix  $H^1$  of the shifted Chebyshev

polynomials  $\Phi_{n}(x)$  is defined as follows:

Similarly,  
\n
$$
T(x,t) \approx \sum_{i=0}^{n} \sum_{j=0}^{n} w_{ij}D_i(x)D_j(t) = \Phi_n^T(x)W\Phi_n(t)
$$
.\n\n**According to the variabola** Accordin-  
\n**Shifted Chebyshev Polynomials**  
\n**Definition 1 The first order differential**  
\n $Q_n(x)$  is defined as follows:  
\n $\Phi_n(x) = (KG_x(x))' = K(G_x(x))' =$   
\n $K(K_x^{-1}\Phi_n(x))' = KPK^{-1}\Phi_n(x) = H_x^1\Phi_n(x)$ , (6)  
\n $\Phi_n(y) = 0$ .\n\nwhere  $P = \begin{bmatrix} p_{ij} \end{bmatrix}_{i,j=0}^{n}$ , where  $p_{ij} = i$  if  $i = j+1$ .\n\n $P_{ij} = 0$  else.  
\nThe integer differential operators matrix of the shifted Chebyshev polynomials is defined  
\n $\Phi_n(y) = 0$  is the sum of the  
\n*suber* of the  
\n*sub*

The integer differential operators matrix of the shifted Chebyshev polynomials is defined by:  $\Phi_n^{(m)}(x) = (KPK^{-1})^m \Phi_n(x) = H_{x}^m \Phi_n(x)$ . the microbeam

(Journal of Industry and Engineering Management (ISSN: 2959-0612) Vol. 2 No. 3, 2024<br>
2000 by down density.<br>
3. **Solution Process**<br>
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3. **Colution Process al** of Industry and Engineering Management (ISSN: 2959-0612) Vol. 2 No. 3, 2024<br> **D** eff antition **Process**<br> **D** the variable fractional order<br> **differential operator's matrix**  $H_i^{\alpha}(t)$  **of the<br>
<b>hift Chebyshev Polynomia** of Industry and Engineering Management (ISSN: 2959-0612) Vol. 2 No. 3, 2024<br>
density.<br> **Definition 2 The** variable fractional<br>
differential operator's matrix  $H_i^{\alpha}(t)$  o<br>
shifted Chebyshev polynomials<br>
defined as follows mal of Industry and Engineering Management (ISSN: 2959-0612) Vol. 2 No. 3, 2024<br>
Ver density.<br> **Definition 2 The** variable fractional order<br>
differential operator's matrix  $H_i^s(f)$  of the<br>
shifted Chebyshev polynomials<br>
s *<sup>n</sup> <sup>n</sup> <sup>n</sup> t t D t <sup>D</sup> D t K t <sup>G</sup>* , where <sup>2</sup> ( ) [1, , , , ] *n T G t t t t <sup>n</sup>* , , 0 *Khanagement (ISSN: 2959-0612)* Vol. 2 No. 3, 2024<br> **IPhritical Operator's matrix**  $H_i^{\alpha}(t)$  **of the differential operator's matrix**  $H_i^{\alpha}(t)$  **of the shifted Chebyshev polynomials**  $\Phi_{\alpha}(x)$  **is defined as follows:<br> D\_i^{\ Definition 2 The** variable fractional order 59-0612) Vol. 2 No. 3, 2024 19<br> **Definition 2 The** variable fractional order<br>
differential operator's matrix  $H_t^{\alpha}(t)$  of the<br>
shifted Chebyshev polynomials  $\Phi_n(x)$  is<br>
defined as follows:<br>  $D_t^{\alpha}\Phi_n(t) = KD_t^{\alpha}(G_t(t)) = KP^{\alpha}G_t$ differential operator's matrix  $H_t^{\alpha}(t)$  of the 59-0612) Vol. 2 No. 3, 2024 19<br> **Definition 2 The** variable fractional order<br>
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shifted Chebyshev polynomials  $\Phi_n(x)$  is<br>
defined as follows:<br>  $D_i^{\alpha}\Phi_n(t) = KD_i^{\alpha}(G_i(t)) = KP^{\alpha}G_i$  $\Phi_n(x)$  is defined as follows: 1. 2 No. 3, 2024 19<br>
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operator's matrix  $H_t^{\alpha}(t)$  of the<br>
debyshev polynomials  $\Phi_n(x)$  is<br>
follows:<br>  $D_t^{\alpha}\Phi_n(t) = KD_t^{\alpha}(G_t(t)) = KP^{\alpha}G_t(t) =$ <br>  $\Phi_n(t) = H_t^{\alpha}\Phi_n(t)$ , (7)<br>  $p^{\alpha} = \left[p_{ij}^{\alpha}\right]_{i,j=0}^{$ 19<br>
ble fractional order<br>
atrix  $H_t^{\alpha}(t)$  of the<br>
nomials  $\Phi_n(x)$  is<br>  $(G_t(t)) = KP^{\alpha}G_t(t) =$ <br>  $t$ ), (7)<br>  $\int_{i,j=0}^{n}$ , where<br>  $= j = 0$  ; where 19<br>
al order<br>
of the<br>  $(x)$  is<br>  ${}^{\alpha}G_t(t) =$ <br>  $(7)$ <br>
where<br>
where *Vol.* 2 No. 3, 2024 19<br> **nn 2 The** variable fractional order<br>
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Chebyshev polynomials  $\Phi_n(x)$  is<br>
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le fractional order<br>
rix  $H_t^{\alpha}(t)$  of the<br>
omials  $\Phi_n(x)$  is<br>  $G_t(t) = KP^{\alpha}G_t(t) =$ <br>
,, (7)<br>
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,j=0 ; where<br>  $j = 0$  ; where *i*<sub>0</sub>. 3, 2024 19<br> **The** variable fractional order<br>
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shev polynomials  $\Phi_n(x)$  is<br>
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ws:<br>  $n(t) = KD_i^{\alpha}(G_t(t)) = KP^{\alpha}G_t(t) =$ <br>  $\hat{H}_i^{\alpha} \Phi_n(t)$ , (7)<br>  $p^{\alpha} = \left[p_{ij}^{\alpha}\right]_{i,j=0}^{n}$ , where<br>  $i \ne$ . 3, 2024 19<br>
he variable fractional order<br>
ator's matrix  $H_t^{\alpha}(t)$  of the<br>
ev polynomials  $\Phi_n(x)$  is<br>
s:<br>  $(t) = KD_t^{\alpha}(G_t(t)) = KP^{\alpha}G_t(t) =$ <br>  $= H_t^{\alpha}\Phi_n(t)$ , (7)<br>  $\alpha = \left[ p_{ij}^{\alpha} \right]_{i,j=0}^{n}$ , where<br>  $\neq j$  or  $i = j = 0$  ; where 3, 2024 19<br>
2 variable fractional order<br>
br's matrix  $H_t^{\alpha}(t)$  of the<br>
7 polynomials  $\Phi_n(x)$  is<br>  $= KD_t^{\alpha}(G_t(t)) = KP^{\alpha}G_t(t) =$ <br>  $H_t^{\alpha}\Phi_n(t)$ , (7)<br>  $= \left[P_{ij}^{\alpha}\right]_{i,j=0}^{n}$ , where<br> *j* or  $i = j = 0$  ; where<br>
else. 0612) Vol. 2 No. 3, 2024<br> **if inition 2** The variable fractional order<br>
fferential operator's matrix  $H_i^{\alpha}(t)$  of the<br>
fferential operator's matrix  $H_i^{\alpha}(t)$  of the<br>
fifted Chebyshev polynomials  $\Phi_n(x)$  is<br>
fined as fol bl. 2 No. 3, 2024 19<br>
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1 operator's matrix  $H_i^a(t)$  of the<br>
hebyshev polynomials  $\Phi_n(x)$  is<br>
follows:<br>  $D_i^{\alpha} \Phi_n(t) = K D_i^{\alpha}(G_t(t)) = K P^{\alpha} G_t(t) =$ <br>  $1 \Phi_n(t) = H_i^{\alpha} \Phi_n(t)$ , (7)<br>  $p^{\alpha} = \left[ p_{ij}^{\alpha}$ (0612) Vol. 2 No. 3, 2024<br> **efinition 2 The** variable fractional order<br>
fferential operator's matrix  $H_i^{\alpha}(t)$  of the<br>
difted Chebyshev polynomials  $\Phi_n(x)$  is<br>
fined as follows:<br>  $D_i^{\alpha}\Phi_n(t) = K D_i^{\alpha}(G_i(t)) = K P^{\alpha}G_i(t) =$ <br>  $K P$ 2) Vol. 2 No. 3, 2024<br>
19<br> **tion 2 The** variable fractional order<br>
mtial operator's matrix  $H_i^a(t)$  of the<br>
L Chebyshev polynomials  $\Phi_n(x)$  is<br>
d as follows:<br>  $D_i^a \Phi_n(t) = K D_i^a(G_t(t)) = K P^a G_t(t) =$ <br>  ${}^a K^{-1} \Phi_n(t) = H_i^a \Phi_n(t)$ , (7)

$$
D_t^{\alpha} \Phi_n(t) = K D_t^{\alpha} (G_t(t)) = K P^{\alpha} G_t(t) = K P^{\alpha} K^{-1} \Phi_n(t) = H_t^{\alpha} \Phi_n(t), \tag{7}
$$

where  $p^{\alpha} = \left[p_{ii}^{\alpha}\right]_{\alpha}^{n}$ , where

 $p_{ij}^{\alpha} = 0$  if  $i \neq j$  or  $i = j = 0$  ; where  $p_{ii}^{\alpha} = \frac{\Gamma(i+1)}{\Gamma(i+1)}$  else.  $=\frac{\Gamma(i+1)}{\Gamma(i+1)}$  else.

$$
p_{ij}^{\alpha} = \frac{1 + (i + 1)}{\Gamma(i + 1 - \alpha)}
$$
 else.

where

### *n* **3.4 Equation Discretisation**

shifted Chebyshev polyno<br>
defined as follows:<br>
derived<br> *I*<sub>c</sub><sup>*i*</sup> *i*<sub>*i*</sub> *I*<sup>c</sup><br> *I*<sub>c</sub><sup>*i*</sup> *i*<sub></sub> *I*<sup>c</sup><br> *i*<sub>*i*</sub> *i i i i p*<sup>α</sup> = *I*<sub>c</sub><sup>*i*</sup> *i*<sub>*i*</sub><br>
byshev<br>  $p_{ij}^{\alpha} = 0$  if  $i \neq j$  or  $i =$ <br>
given<br>  $p_{ij}$ Based on the above differential operator matrixs, the microbeam control equations (2) and (3) can be transformed into the following operator matrix forms:

Journal of Industry and Engineering Management (ISSN: 2959-0612) Vol. 2 No. 3, 2024  
\npower density.  
\n3. Solution Process  
\n3. Solution Chebyshev Polynomials  
\n3.1 Shift Chebyshev Polynomials  
\nThis child Chebyshev polynomials  
\nFhe shifted Chebyshev polynomials  
\nFhe estimated Chebyshev polynomials  
\n
$$
D_{n,1}(t) = \int_{t}^{2} (2t-1) D_{n}(t) = D_{n,1}(t)
$$
\n
$$
D_{n,2}(t) = \int_{t}^{2} (2t-1) D_{n,1}(t) = N_{n,2}(t)
$$
\n
$$
D_{n,3}(t) = \int_{t}^{2} (2t-1) D_{n,3}(t) = N_{n,3}(t)
$$
\nwhere  $U_{n,1} = \int_{t}^{2} (2t-1) D_{n,3}(t) = N_{n,3}(t)$   
\nwhere  $U_{n,1} = \int_{t}^{2} (2t-1) D_{n,1}(t) = N_{n,1}(t)$   
\n
$$
D_{n,2}(t) = \left[ \frac{2t-1}{t} \right] D_{n,1}(t) = D_{n,2}(t)
$$
\nwhere  $P_{n} = 0$  if  $i \neq j$  or  $i = j = 0$ ;  
\nwhere  $V_{n} = 0$  if  $i = j = 0$   
\n
$$
D_{n} = 0
$$
 if  $i \neq j$  and  $i$  (3) can be transformed into the following  
\nwhere  $k_{n} = 0$  if  $i < j \neq j$ .  
\n
$$
k_{n} = 0
$$
 if  $i < j \neq j$  else.  
\n
$$
k_{n} = 0
$$
 if  $i < j \neq j$  else.  
\n
$$
k_{n} = 0
$$
 if  $i < j \neq j$  else.  
\n
$$
k_{n} = 0
$$
 if  $i < j \neq j$  else.  
\n
$$
k_{n} = 0
$$
 if  $i < j \neq j$  else.  
\n
$$
k_{n} = 0
$$
 if  $i < j \neq j$  else.  
\n
$$
k_{n} = 0
$$

(a)  $L = [u_{ij}]_{i,j=0}^{n,m}$ <br>  $V = [u_{ij}]_{i,j=0}^{n,m}$ <br>  $+ \frac{\alpha_7 \pi^2 h T_0 E}{24K} \tau_0 (\theta_5^n)(H_5^n)UH_7^H H_7^H \theta_n(t)$ <br>  $+ \frac{\alpha_7 \pi^2 h T_0 E}{24K} \tau_0 (\theta_5^n)(H_5^n)UH_7^H H_7^H \theta_n(t)$ <br>  $+ \frac{\alpha_7 \pi^2 h T_0 E}{24K} \tau_0 (\theta_5^n)(H_5^n)UH_7^H H_7^H \theta_n(t)$ <br>  $+ \frac{\alpha_$ *n*  $L = \frac{1}{k} \int_0^k \int_0^{\pi/2} \omega(x) D_i(x) D_i(x) \omega_x(x) \omega_x(x) dx$ <br>  $= \frac{1}{k} \int_0^k \int_0^k \int_0^k \omega(x, D_i(x) D_i(x)) \omega_x(x) \omega_x(x) dx$ <br>  $= \frac{1}{k} \int_0^k \int_0^k \int_0^k \omega(x, D_i(x)) D_i(x) D_i(x) \omega_x(x) \omega_x(x) dx$ <br>  $= \frac{1}{k} \int_0^k \int_0^k \omega(x, D_i(x)) U_i(x) D_i(x) \omega_x(x) \omega_x(x) dx$ <br>  $= \frac$ According to the method of matching points,  $\frac{2i-1}{i}$  i – 0.1.2 ... m.  $2m$  and  $2m$  and  $2m$ , = 0,1,2, ⋯, <sup>−</sup> 1,  $t_j = \frac{2j-1}{2m}T, j = 0,1,2,\dots, m-1$ . Th  $2m$   $\sim$   $2m$  $\varphi_n^1(x)(H_x^2)^t U H_t^{\alpha} H_t^{\alpha} \varphi_n(t)$ <br>  $+ \tau_d \varphi_n^T(x) (H_x^2)^T U H_t^{\alpha} H_t^2 \varphi_n(t)$ <br>  $\frac{\pi}{\kappa} \varphi_n^T(x) W H_t^{\alpha} H_t^1 \varphi_n(t) - \frac{\pi^2}{2\kappa h^2} (1 + \tau_q D_t^{\alpha}) \int_{-2/h}^{2/h} z F dz = 0.$  (9)<br>
ne method of matching points,<br>  $x, t$  is discretised into numerical solution of the control equations (2) and (3) is then obtained through the MATLAB programme. +  $\frac{\alpha_{\tau}\pi^2 hT_0 E}{24K} + \frac{1}{4}e^2 \kappa_x (x) (n_x^2)^T U H_i^{\alpha} H_i^1 \phi_n(t)$ <br>  $+ \frac{\alpha_{\tau}\pi^2 hT_0 E}{24K} \tau_q (\Phi_n^{\pi}(x) (H_i^2)^T U H_i^{\alpha} H_i^1 \phi_n(t) - \frac{\pi^2}{2Kh^2} (1 + \tau_q \Phi_n^{\pi}(x)) H_i^{\alpha} H_i^1 \phi_n(t) - \frac{\pi^2}{2Kh^2} (1 + \tau_q \Phi_n^{\alpha}(x)) \frac{1}{\sum_{\ell} n} z R dz = 0.$ 

### **4. Numerical Results**

The Lord-Shulman fractional order thermoelastic model (abbreviated as FVLS,  $\tau_a = 0$ ) and the fractional order thermoelastic two-phase lag model (abbreviated as FVDPL,  $\tau_{\theta} > 0$ ) are considered [<sup>13</sup>] the parameters of

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 $\rho = 1930$ *kgm*<sup>-3</sup>,  $\alpha = 2.59K^{-1}$ ,  $T_0 = 293K$  [12] .

### **4.1 Comparison of Microbeams Deflection Response Based Two Models**

Figure 1 demonstrates the trends of the FVLS  $\frac{2}{5}$  5 and FVDPL models are similar, the peak value of the deflection is significantly higher in the  $\frac{1}{2}$ FVDPL model  $[13]$  Specifically, Figure 1(a) shows the deflection variation of the FVLS model with noise interference and no  $-5\frac{1}{0}$ interference conditions, with almost the same results; Figure 1(b) further validates this conclusion and proves the stability of the  $4 \times 10^{-7}$ 





**(b) Deflection variation with FVDPL model. Figure 1. Comparison of Microbeam Deflection Under Two Models**

**4.2 Comparison of Microbeams Temperature Response Based Two Models** Figure 2 illustrates the temperature variation trend is similar for both models, but the peak temperature value is significantly higher under the FVLS model. Figure 2(a) illustrates that the temperature variation of the FVLS model is essentially identical under both noise-disturbed and non-disturbed conditions  $[14]$  $\frac{14}{14}$  This  $\frac{56}{102}$ 

outcome serves to reinforce the veracity of the



 $t(s)$ **(b) Temperature variation with FVDPL model Figure 2. Comparison of Microbeam Temperatures Under the Two Models.**

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 $0.5$ 

### **5. Conclusion**

 $-1$ 

 $\overline{0}$ 

(1) The peak of the microbeam deflection under the FVDPL model is markedly higher than that under the FVLS model, whereas the peak of the temperature under the FVLS model is significantly higher than that under the FVDPL model.

 $(2)$  The algorithm proposed in this paper has stability and reliability that can effectively resist noise interference.

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