Optimizing Dynamic Programming Algorithm Based on Bayesian Model to Solve Enterprise Decision-Making Problem

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Abstract: In the current highly competitive business landscape, enterprises face a series of complex decision-making challenges during the efficient production process of popular electronic products. Taking this as the entry point, this paper deeply analyzes these problems, aiming to build a scientific theoretical foundation for enterprise decision-making, optimize the production process, reduce costs, and enhance economic benefits. This paper first uses statistical hypothesis testing to design a sampling scheme to evaluate the defective rate of parts. Then, the dynamic programming algorithm is employed, and a defective rate optimization model based on Bayesian update is incorporated to comprehensively optimize the detection and disposal decisions of parts, finished products, and defective products in the enterprise production process. The research results show that the optimization model can effectively deal with the risk of defective rate fluctuations and ensure the consistency and reliability of decision-making results. Through case analysis, it can be seen that this model can customize accurate decision-making schemes for enterprises in various scenarios and significantly improve economic benefits. In summary, this study provides crucial theoretical support and practical guidance for the improvement of enterprise management level and industrial upgrading, demonstrating significant value in practical applications.

Keyword: Bayesian Model; Enterprise Decision-making; Dynamic Programming Algorithm; statistical hypothesis; defective rate fluctuations

1. Introduction

1.1 Research Background

Suppose a company produces an electronic product that is very popular in the market, and the manufacturing process of this product involves two key components: component 1 and component 2. These two components need to be assembled together to form the final electronic product. In the production process, there are strict requirements for the quality of spare parts and finished products. Specifically, if any part used in the assembly fails, the final product will also be judged to be unqualified. Even if both parts are qualified, the finished product may not be qualified due to possible assembly errors or technical defects.

In the face of unqualified finished products, enterprises have two ways to deal with them: one is to scrap the unqualified finished products directly, which means the waste of raw materials and potential economic losses; the other is to disassemble the unqualified finished products and take out the spare parts for reuse. The disassembly process is designed to be safe enough to ensure that no damage is caused to the parts, so that the disassembled parts can be used again in the production line. However, the dismantling operation is not free, and enterprises need to bear a certain cost of dismantling.

Under this production mode, enterprises are faced with such problems as how to efficiently manage spare parts procurement, quality control, finished product inspection and disposal of unqualified products. The solutions to these problems will directly affect the economic benefits and market competitiveness of enterprises [1].

1.2 Literature Review

In the field of enterprise decision optimization, the existing research provides scientific decision support for the production process through mathematical models and algorithms.

Peng deeply discussed the decision-making problems in the production process of electronic components, and optimized the key links such as spare parts procurement, quality control, finished product inspection and unqualified product disposal by establishing mathematical models. In terms of defective rate detection of spare parts, this paper designs a sampling detection scheme based on statistical hypothesis testing, which echoes the relationship between binomial distribution, Poisson distribution and normal distribution discussed in Yu research, and provides a theoretical basis for sampling detection. By accurately calculating the minimum sample size, the defective rate of spare parts can be accurately evaluated with the least number of tests, and the cost of testing can be effectively reduced. The application of dynamic programming algorithm ensures that the decision-making of each stage is based on global optimization, effectively balances the detection cost and potential loss, and achieves the goal of minimizing the cost and maximizing the profit. He et al. research on dynamic programming algorithm for optimal solution provides a reference for algorithm selection and optimization. In addition, Li et al. although the distribution network real-time coordinated voltage regulation strategy based on approximate dynamic programming is applied in different fields, its methodology is enlightening for the construction of dynamic programming model in this study.

The introduction of the Bayesian updating model considers the uncertainty of the estimation of the defective rate, and improves the robustness of the decision-making scheme by combining the prior information with the new sample data. The method of Wang in the research and application of weighted Bayesian network classification algorithm provides technical support for the Bayesian updating model of this study. Wang et al. Research on landing distance of civil aircraft based on Bayesian network, although the field is different, the application of Bayesian network in dealing with uncertainty problems provides valuable experience for this study. Although the existing research has achieved remarkable results in decision optimization, there is still room for improvement. Future research can further relax the assumptions of the model and introduce more factors affecting decision-making, such as equipment depreciation, labor costs, and changes in market demand, so as to improve the universality and dynamic adaptability of the model. In addition, it is also an important direction for future research to verify and optimize the model through experiments or simulations to ensure the effectiveness and robustness of the model in different scenarios.

2. Research and Analysis

2.1 Specific Issues

2.1.1 Question 1

Design a sampling inspection plan to evaluate whether the defective rate of parts supplied by suppliers exceeds the nominal value (the nominal value assumed in this paper is 10%). According to the test results, we will decide whether to accept this batch of spare parts. Reject when the defective rate exceeds the nominal value at 95% confidence level, and accept when the defective rate does not exceed the nominal value at 90% confidence level [2].

2.1.2 Question 2

In view of the specific situations listed in Table 1, this paper needs to determine whether to test or not according to the unqualified rate of spare parts and finished products. O as to ensure the quality of market products. Whether the unqualified finished products are disassembled, and the spare parts are tested and utilized. Unqualified products are exchanged unconditionally by users, and returned products are disassembled and reused. Decision logic and indicator results: Optimize the inspection and reuse process based on cost-benefit analysis. Improve the qualified rate of finished products and reduce consumer complaints. Through the reuse of spare parts, the production cost is reduced. Exchange policy to maintain customer satisfaction and protect corporate reputation. Finally, according to the six situations, the decision schemes of minimizing the cost and maximizing the profit are put forward respectively.

2.1.3 Question 3

For the manufacturing process involving m processing steps and n parts, it is assumed that the defective rate of each part, semi-finished product to the final product, the purchase unit price, the cost of inspection and assembly, and the cost of disassembly have been given. Now, referring to the problem 2 discussed before, the decision-making strategy of production activities is designed in this case. This paper assumes a production scenario with 2 processing stages and 8 spare parts, and the specific data are detailed in Table 2. Finally, it is still a scheme that can minimize the cost and maximize the benefit.

2.1.4 Question 4

Recalculate the defective rate of parts, semi-finished products, and finished products in questions 2 and 3 based on the sample inspection results in question 1. The sampling inspection method in Problem 1 can be applied to the evaluation of the defective rate at each stage, and the inspection and production decisions at each stage can be readjusted.

2.2 Concrete Analysis

2.2.1 Analysis of question 1

It is assumed that the supplier guarantees that the failure rate of the part (which may be part 1 or part 2) will not be higher than a specified value. In order to decide whether to accept the goods, it is planned to adopt a sampling inspection strategy, and the inspection cost will be borne by the enterprise. Parts will be rejected when a 95% confidence level determines that the part's failure rate is greater than specified 10%. Parts will be accepted with a 90% confidence level that the failure rate is not greater than specified 10%.

In this paper, the method of statistical hypothesis testing is used, as follows:

Determine the sample size: Use an accurate statistical formula, such as the binomial distribution or normal approximation, to calculate the minimum sample size that will detect a significant difference from the nominal value at a given level of reliability. A certain number of samples shall be taken at random from the lot of parts. These samples are tested and the number of nonconforming products is recorded.

It is assumed that the failure rate of parts is equal to or lower than 10% (null hypothesis). Binomial test and Z-test were used to test the null hypothesis according to the test results.

If the test statistic falls in the rejection region, i.e., the failure rate is significantly higher than 10%, the lot is rejected at 95% confidence (Scenario 1). If the test statistic does not fall within the rejection region, i.e., there is insufficient evidence that the failure rate is higher than 10%, the lot is accepted at 90% confidence (Scenario 2). Specific to the value, it needs to be calculated according to the sampling theory to ensure that the required level of reliability is achieved.

2.2.2 Analysis of problem 2

For the six specific cases listed in Table 1, it is necessary to discuss and analyze them in stages:

(1) Whether the spare parts 1 and 2 are tested;

(2) Whether the finished product is tested;

(3) Whether the returned unqualified finished products are disassembled and reused;

(4) Whether the returned unqualified finished products are exchanged or discarded.

The dynamic programming algorithm can be used to solve the problem, and the decision logic and index results can be calculated to optimize the detection and reuse process according to the cost-benefit analysis. Through the various stages to make decisions, the final six cases were given to minimize the cost, profit maximization program.

2.2.3 Analysis of problem 3

The third problem is based on a variant of problem 2, which reduces the number of basic parts from six to one, but the number of basic parts increases to eight, and an additional semi-finished stage is added, but the essence is not much different from problem 2, and it can still be solved by dynamic programming algorithm to find out whether each stage needs to be tested and the minimum number of times of testing. And then combining the optimal solution of each stage to obtain the optimal decision scheme. The assembly process is shown in Figure 1.

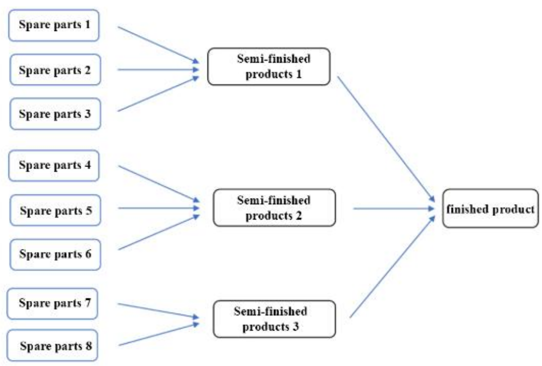

Figure 1. Assembly Flow Chart

2.2.4 Analysis of question 4

This question is a summary of the first three questions. It is assumed that in questions 2 and 3, the defective rate of spare parts, semi-finished products and finished products is derived from sampling inspection. This method is consistent with that used in question 1 of this paper. Based on this premise, questions 2 and 3 need to be revisited and answered to ensure that the inherent uncertainty of sample testing and its impact on the estimate of the defective rate are taken into account. This means that this paper should not only make decisions based on the defective rate obtained by sampling, but also take into account the factors such as sampling error, confidence level and sample size to evaluate the robustness and reliability of decisions.

3. Model Establishment

3.1 Symbol Explanation

See Table 1 above, clearly define the meanings of each symbol, such as the minimum number of samples, error rate, market price, inspection cost and disassembly cost of parts or finished products, etc. to provide clear variable definitions for subsequent model construction.

Table 1. Symbol Explanation

Symbol	Symbol Description
n	Minimum Sample Size
Z	Standard Normal Distribution Critical Value Corresponding to Confidence Level
E	Error Rate
$P0$	Nominal Value (assumed to be 10% in this paper)
$pc[i]$	Cost of Component i
$icp[i]$	Inspection Cost of Component i
$s1\_i$	Inspection Decision for Component i (0 or 1)
$sdu[j]$	Updated Defective Rate of Semi-finished Product j
$sd[j]$	Initial Defective Rate of Semi-finished Product j
$pd[i]$	Defective Rate of Component i
s1[i]	Inspection Decision for Component i (0 or 1) (Note: This is the same as s1_i, ensure consistency in notation)
ics[j]	Inspection Cost of Semi-finished Product j
dcs[j]	Disassembly Cost of Semi-finished Product j
R_p[j]	Potential Revenue from Disassembling Semi-finished Product into Components
s2_j	Inspection Decision for Semi-finished Product j (0 or 1)
d2_j	Disassembly Decision for Semi-finished Product j (0 or 1)
p_f'	Revised Defective Rate of Finished Product
rl	Replacement Loss
mp	Market Price
acp	Assembly Cost of Finished Product
icp	Inspection Cost of Finished Product (Note: This is the same as icp[i] in the context of components, ensure context clarity)
dcp	Disassembly Cost of Finished Product
R_h	Potential Revenue from Disassembling Semi-finished Product into Components
Γ	Gamma Function

3.2 Model Assumptions

(1) Sample representativeness assumption: It is assumed that the sample of spare parts taken from the supplier can represent the quality level of the whole batch of spare parts, that is, the defective rate of the sample can accurately reflect the defective rate of the whole batch of spare parts.

(2) Large sample hypothesis: It is assumed that the number of each batch of spare parts is large enough to meet the conditions of the central limit theorem, so that the distribution of the defective rate can be approximated as a normal distribution, which is convenient for statistical inference.

(3) Defective rate independence assumption: It is assumed that the defective rate of spare parts is independent among batches and spare parts, that is, the defective rate of one spare part will not be affected by the defective rate of other spare parts.

(4) Cost fixity assumption: It is assumed that costs such as purchase unit price, testing cost, assembly cost, market selling price, exchange loss and disassembly cost remain unchanged

during the decision-making period, without considering price fluctuation and cost change factors.

(5) Dynamic programming optimality assumption: It is assumed that the solution obtained by the dynamic programming algorithm is the global optimal solution, that is, there is no other better decision sequence that can produce higher total revenue or lower total cost.

4. Establishment and Solution of the Model

4.1 Question 1

4.1.1 Question 1: model establishment idea

For each batch of spare parts, this paper assumes that the defective rate is *p*. The supplier claims that the defective rate will not exceed a certain nominal value P_0 (e.g. 10% given in the title). Therefore, the task of this paper is to find the minimum sample size under the constraints of case 1 and case 2 in the actual case where P_0 has been given through sampling inspection.

4.1.2 Establishment of model

In this paper, we need to carry out one-tailed test for the two cases in the title:

For the case (1), under the 95% reliability, it is determined that the defective rate of spare parts exceeds the nominal value P_0, and the hypothesis is established in this paper:

The original hypothesis H_0: $p \leq P_0$ and the alternative hypothesis H_1: $p > P_0$. Therefore, this paper needs to test whether the original hypothesis H_0 can be rejected by sampling.

For case (2), under 90% reliability, it is determined that the defective rate of spare parts exceeds the nominal value P_0, and two assumptions are established in this paper:

The original hypothesis H_0: $p \geq P_0$ and the alternative hypothesis H_1: $p < P_0$. Therefore, this paper needs to test whether the original hypothesis H_0 can be rejected by sampling.

Since the defective rate is a probability event, the binomial distribution is used for simulation. Suppose the sample size is n, which is subject to the binomial distribution. However, considering the actual situation of the subject [3], the sample size is relatively large.[4] According to the famous De Moivre-Laplace central limit theorem in probability theory, the theorem shows that when n is sufficiently large, the binomial distribution can be approximated by the normal distribution, that is, the normal approximation of the binomial distribution. So as to establish a decision-making model:

As shown in formula (1), let random variables, then for any X we have:

$$X_n \sim b(n,p)\left(0 < p < 1, n = 1,2,\ldots\right)$$

$$\lim_{n\to\infty}\left\{\frac{X_n - np}{\sqrt{np(1-p)}} \leq x\right\} = \int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt = \Phi(x) \quad (1)$$

Because this paper uses a continuous distribution to approximate the discrete distribution, in practical applications, in order to reduce the approximation error, we often use:

$$P\{a \leq X_n \leq b\} \approx \Phi\left(\frac{b+0.5-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{a-0.5-np}{\sqrt{np(1-p)}}\right) \quad (2)$$

Instead of Formula (2) [1].

The process of normal approximation to the binomial distribution is shown in Figure 2.

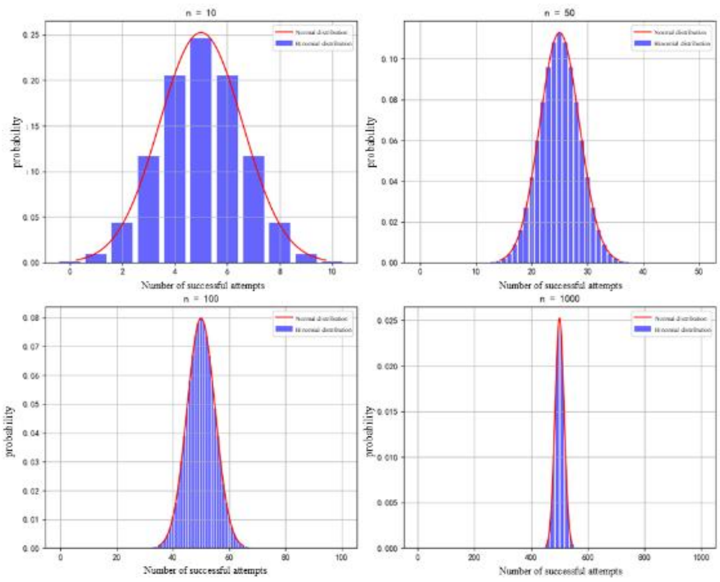

Figure2. Schematic Diagram of Normal Approximation of Binomial Distribution

However, in the verification of binomial distribution, there are two kinds of errors:
The first type of error: the actual defective rate is 10% (that is, the original hypothesis H_0 is true), but the test result mistakenly believes that the defective rate exceeds 10% and rejects the original hypothesis. The first type of error was made at this time.
The second type of error: the actual defective rate exceeds 10% (that is, the alternative hypothesis H_1 is true), but the test result mistakenly believes that the defective rate is equal to or less than 10%, failing to reject the original hypothesis. The second type of error was made at this time.
In the case of H_0, assuming the defective rate is P_0, this paper wants to reject H_0 at the significance level α; in the case of H_1, assuming the actual defective rate is $P1$, this paper wants to reject H_0 correctly with the power $1\text{-}\beta$. The actual defective rate $\hat{p}$ is obtained by sampling, and the test statistic is calculated based on it. For the large sample size assumed in the hypothesis, the test statistic is usually the standardized Z value as shown in formula (3) and (4), which is used to control the two types of errors mentioned above:

$$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{3}$$

And

$$Z = \frac{p_1-p_0}{\sqrt{\frac{p_1(1-p_1)}{n}}} \tag{4}$$

In order to ensure the accuracy of detection and limit the error, this paper needs to determine the required sample size according to the confidence level and the adjusted error rate.
The error rate is assumed to be 0.05, and the sample size can be calculated according to the standard sample size formula (5):

$$n = \frac{Z^2 \cdot p_0 \cdot (1-p_0)}{E^2} \tag{5}$$

Set the error rate interval at $(0.04 \le E \le 0.06)$, and the quantity of parts to be sampled and tested in the two cases can be determined by calculation, as shown in the Table 2:

Table 2. Required Sample Size for Different Error Rates

Error Rate (E)	Minimum Sample Size (n) at 95% Confidence Level	Minimum Sample Size (n) at 90% Confidence Level
0.04	153	93
0.042	139	84
0.044	126	77
0.046	116	70
0.048	106	65
0.05	98	60
0.052	91	55
0.054	84	51
0.056	78	48
0.058	73	44
0.06	68	42

The minimum number of samples of the two kinds of reliability within the specified error interval is obtained by calculation. When the error rate is 0.05, it is relatively gentle and moderate. Therefore, the error rate of the hypothesis E = 0.05 is determined.
4.1.3 Solution of the model
For case (1), if the Z value exceeds the corresponding critical value, the null hypothesis is rejected and the defective rate is considered to exceed the nominal value, while for case (2), if the Z value is less than corresponding critical value, the null hypothesis is rejected and the defective rate is considered not to exceed the nominal value. And the sample results are analyzed to make an acceptance or rejection decision. The sample size calculation of the confidence Z values in the two cases is as follows:
The rest of the parameters are set by consulting and assuming the Table 3:

Table 3. Parameters for Sample Size Calculation

Symbol	Set Value
E	0.05
Z_{95}	1.96
Z_{90}	1.64

Substitute Z_{95}, Z_{90} and E into the formula (6) and (7) to calculate:
The sample size at 95% confidence is:

$$n_{95} = \frac{(1.96)^2 \cdot 0.1 \cdot (1-0.1)}{0.05^2} \approx 98 \tag{6}$$

The sample size at 90% confidence is:

$$n_{90} = \frac{(1.64)^2 \cdot 0.1 \cdot (1-0.1)}{0.05^2} \approx 60 \tag{7}$$

4.1.4 Conclusion:
According to the calculation results, it can be concluded that under the 95% confidence level, 98 parts can meet the requirements of rejecting this batch of parts, while under the 90% confidence level, 60 parts can meet the requirements of receiving this batch of parts.

4.2 Question 2

4.2.1 Question 2: model establishment

In question 2, it is required to make multiple decisions on the production process of the enterprise, including whether to test spare parts, whether to test finished products, whether to disassemble unqualified finished products, etc. It is necessary to establish a decision-making model that includes costs and benefits, and the goal is to maximize the profits of enterprises or minimize losses. Because the six cases given in the table are subdivided into several links, the dynamic programming algorithm is used to split the whole process into spare parts, finished products, unqualified finished products and returning and discarding unqualified finished products to the third stage for calculation. The specific process is shown in the Figure 3.

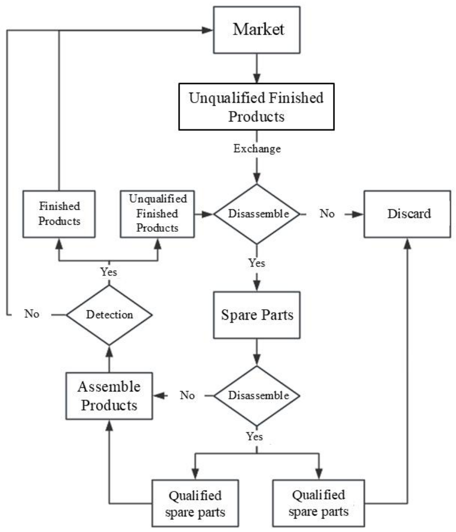

Figure 3. Decision flow chart

4.2.2 Establishment of model

The problem is structured as a multi-stage decision-making process that focuses on parts procurement and testing, assembly and testing of finished products, and management of nonconforming finished products. Specifically, it is divided into three key stages: procurement and testing of spare parts (stage 1), assembly and testing of finished products (stage 2), and disassembly and replacement of unqualified finished products (stage 3).

The state is represented in the form of a tuple, recording each stage and the decision made (whether to detect or not). Decision variables cover the selection of various operations, such as inspection or disassembly. The recursive relationship and the state transition equation ensure that the decision is based on the results of the previous stages, and evaluate the expected cost or benefit of different decisions. For example, the detection of spare parts in stage 1 directly affects the cost of stage 2, and then affects the processing strategy of stage 3.The bottom-up calculation strategy is used to evaluate the disassembly cost from stage 3 to stage 1step by step to ensure that each decision is based on the optimal solution of the subsequent stages. Finally, through the comparison of the global state, the optimal total income and the corresponding path are determined, and the optimal decision sequence running through the three stages is obtained by backtracking.

The basic idea of dynamic programming algorithm is to divide the problem to be solved into several interrelated sub-problems, and to calculate the optimal value by solving the sub-problems in a bottom-up way. The dynamic programming algorithm reduces the time complexity by increasing the space complexity of the program. It can be seen that the problem solved by dynamic programming algorithm needs to have two properties, namely, the optimal substructure property and the overlapping subproblem property [5]. Figure 4 is a visualization of the principle and process of dynamic programming.

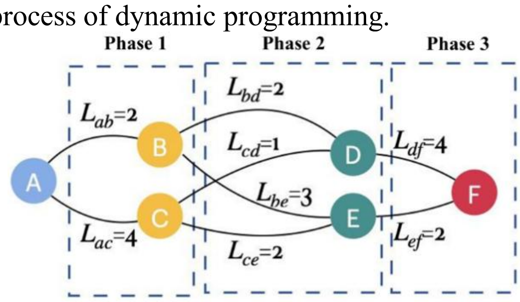

Figure 4. Schematic Diagram of Dynamic Programming

4.2.3 Corresponding schemes under 6 scenarios

In discussing the optimization of decision-making in the production process of enterprises, we adopted the dynamic programming algorithm to address complex multi-stage decision problems. The core idea of the dynamic programming algorithm is to decompose the problem into a series of interrelated sub-problems, solving from the last stage upward to find the global optimal solution. This method is particularly suitable for our case, as it allows us to formulate optimal strategies for each stage while considering the optimal decisions of subsequent stages.

Assuming the finished product quantity is

10,000, we applied the dynamic programming algorithm to six different production scenarios in order to determine the optimal decision-making plans regarding component inspection, finished product inspection, and handling of defective finished products. These scenarios reflect different defect rates, costs, and revenues that may be encountered during the production process. Through this method, we were able to maximize the economic benefits of the enterprise while ensuring product quality.

The following are the optimal decision plans and corresponding optimal total revenues derived from the dynamic programming algorithm for the six scenarios, summarized in detail in Tables 4 and 5:

Table 4. Hypothetical Scenarios Encountered in the Production Process of the Enterprise

Phase Scenario	Component 1	Component 12	Finished Product	Defective Finished Product	Optimal Total Revenue (Yuan)
1	Not Inspected	Not Inspected	Not Inspected	Not Disassembled	467000
2	Not Inspected	Not Inspected	Not Inspected	Not Disassembled	467000
3	Inspected	Inspected	Not Inspected	Not Disassembled	443000
4	Inspected	Inspected	Inspected	Not Disassembled	378000
5	Not Inspected	Inspected	Not Inspected	Not Disassembled	464000
6	Not Inspected	Not Inspected	Not Inspected	Not Disassembled	511500

Table 5. Optimal Decision-Making Schemes and Optimal Total Revenue Results for Six Scenarios

	Part1 (Component)			Part2 (Component)			Finished Product			Defective Finished Product		
	Defective Rate	Purchase Unit Price	Finished Product Inspection	Defective Rate	Purchase Unit Price	Finished Product Inspection	Defective Rate	Assembly Cost	Inspection Cost	Market Price	Replacement Loss	Disassembly Cost
11	10%	4	2	10%	18	3	10%	6	3	56	6	5
22	20%	4	2	20%	18	3	20%	6	3	56	6	5
33	10%	4	2	10%	18	3	10%	6	3	56	30	5
44	20%	4	1	20%	18	1	20%	6	2	56	30	5
55	10%	4	8	10%	18	1	10%	6	2	56	10	5
66	5%	4	2	5%	18	3	5%	6	3	56	10	40

4.2.4 Conclusion

It can be seen from Table 5 that the optimal total revenue is the same when the detection decisions in cases 1 and 2 are the same; different detection decisions corresponding to different situations have different degrees of impact on the total revenue, which needs to be considered comprehensively according to the defective rate, exchange loss and disassembly cost of different situations.

4.3 Question 3

4.3.1 Question 3: Model establishment ideas

The problem is structured as a multi-stage decision-making process, focusing on parts procurement and testing, assembly and testing of semi-finished products, assembly and testing of finished products, and management of non-conforming finished products. On the basis of problem 2, it is divided into four key stages: the procurement and inspection of spare parts (stage 1), the assembly and inspection of semi-finished products (stage 2), the assembly and inspection of finished products (stage 3), and the disassembly and exchange of unqualified finished products (stage 4); the decision-making basis and the corresponding indicators are the optimal total revenue.

The model algorithm used in this problem is the same as that used in problem 2, which can find the optimal solution through dynamic programming. The dynamic programming method transforms the multi-stage optimization problem into a series of single-stage decision-making problems, and then uses the transfer and constraint relationship between the stages to solve the single-stage optimization problem one by one, and gives the dynamic decision according to the real-time state of the system, so as to simplify the complex problem. In the application of dynamic programming theory, it is necessary to traverse all the state space and decision space to obtain the value function of environment, the system in a certain state in the current stage may transfer to infinite different States in the next stage, resulting in the problem of "dimension explosion" [6], which has been explained more concretely in the literature [7]. Therefore, the part of model establishment will not be elaborated as shown in Figure 5.

4.3.2 Solution of model

When applying the dynamic programming algorithm to solve multi-stage decision-making problems, Table 6. provides a key dataset that details various hypothetical situations that enterprises may encounter in the production process. These situations include the defect rates of components, purchase unit prices, inspection costs, and the related costs of semi-finished and finished products. These parameters are crucial for constructing the dynamic programming model because they directly affect the decisions at each stage and the ultimate economic benefits.

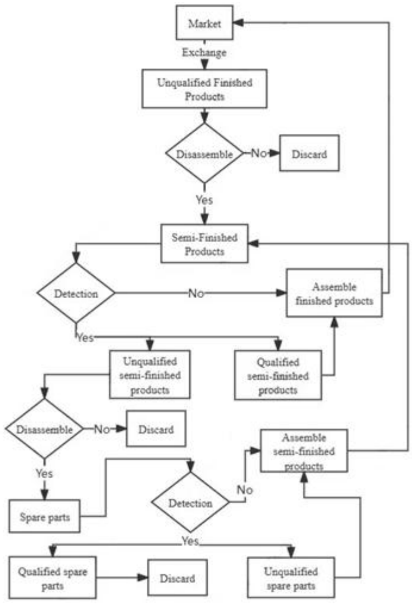

Figure 5. Scheme Flow Chart

Table 6. Hypothetical Scenarios Encountered in the Production of the Enterprise

Spare Part(s)	Defective-Rate	Purchase Unit Price	Inspection Cost	Semi-finished Product	Defective Rate (for-Semi-finished Product)	Assembly Cost	Inspection Cost	Disassembly Cost
1	10%	2	1	1	10%	8	4	6
2	10%	8	1	2	10%	8	4	6
3	10%	12	2	3	10%	8	4	6
4	10%	2	1					
5	10%	8	1	Finished Product	10%	8	6	10
6	10%	12	2					
7	10%	8	1		Market Selling Price		Replacement Loss	
8	10%	12	2	Finished Product	200		40	

Calculated from the data given in the table:

The first stage:

Part inspection cost as shown in formula (8):

$$cp = \sum_{i=1}^{8} [s1\_i * icp[i] + pc[i]] \quad (8)$$

Part Stage Benefits as shown in formula (9):

$$dp[1, s1\_1, s1\_2, ..., s1\_8] = cp + \max \quad (9)$$

In the aforementioned formula, max represents the earnings at the relevant semi-finished stage.

The second stage as shown in formula (10-12):

Defect rate update for sub-assembly J:

$$sdu[j] = sd[j] + \sum_{i \in RelatedPartset} pd[j] * (1 - s1[j]) \quad (10)$$

Cost of semi-finished product inspection and disassembly decision:

$$cs = \sum_{j=1}^{3} [s2\_j * ics[j] + d2\_j * sdu[j] * (dcs[j] - R\_p[j])] \quad (11)$$

Semi-finished goods stage income:

$$dp[(2, s2\_1, d2\_1, s2\_2, d2\_2, s2\_3, d2\_3)] = -cs + \sum_{j=1}^{3} (1 - sd[j]) * \max \quad (12)$$

The third stage as shown in formula (13-15):

Benefits of not testing the finished product and not-disassembling=

$$p\_f * rl + (1 - p\_f') * mp - acp \quad (13)$$

Benefits of testing the finished product without-disassembly=

$$-icp + (1 - p\_f) * mp - acp \quad (14)$$

Benefits from inspection and disassembly of-finished-products=

$$-icp - p\_f * (dcp - R\_h) + (1 - p\_f) * mp - acp \quad (15)$$

The optimal decision scheme obtained through three stages of calculation is shown in the Table 7.

4.3.3 Conclusion:

When enterprises exchange the unqualified products purchased by users, the best strategy is to test and disassemble the finished products after exchange, while not to test and disassemble the semi-finished products. As a result, no additional spare parts are created in the process, so there is no need to test them.

Table 7. Solution Results for Question 3

Spare Part	Inspect or Not	Semi-finished Product	Inspect or Not	Disassemble or Not	Finished Product	Inspect or Not	Disassemble or Not
1	Not Inspected	1	Not Inspected	Not Disassembled	1	Inspect	Disassemble
2	Not Inspected	2	Not Inspected	Not Disassembled			
3	Not Inspected	3	Not Inspected	Not Disassembled			
4	Not Inspected						
5	Not Inspected						
6	Not Inspected						
7	Not Inspected						
8	Not Inspected					Optimal Total Revenue (Yuan)	183.14

4.4 Question 4

4.4.1 Question 4: Model establishment ideas

Because it is assumed that the defective rates of spare parts, semi-finished products and finished products are obtained by sampling inspection, their defective rates are uncertain, so this paper needs to recalculate an appropriate defective rate, complete the optimization of the model and ensure the robustness, so that the solutions of problem 2 and problem 3 are more reasonable.

Therefore, this paper introduces the binomial distribution-beta distribution model based on the Bayesian update of the binomial distribution. The model combines the prior information with the new sample data, and updates the defective rate through Bayesian inference. The prior information is expressed by beta distribution, the new sample data is obtained by binomial distribution, and the updated defective rate is also expressed by beta distribution, whose parameters are determined by the prior distribution parameters and the sample data. The unique advantage of Bayes theorem is that it can skillfully transform the calculation of complex event probability into the calculation of multiple independent simple event probabilities. This shift not only makes computing more efficient, but also makes dealing with the relationship between different events more flexible, so as to understand and deal with the complexity of probability analysis more comprehensively [8]. The optimization comparison results are shown as in Figure 6.

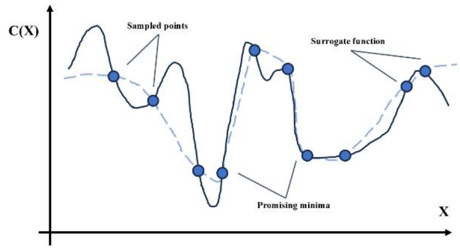

Figure 6. Bayesian Optimization Curve (Dotted Line)

4.4.2 Beta distribution

Beta Distribution as shown in formula (16) is a continuous probability distribution, which is usually used to describe random variables taking values in the interval (0, 1). The beta function as shown in Figure 7 is defined as [9]:

$$B(\alpha,\beta)=\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad (16)$$

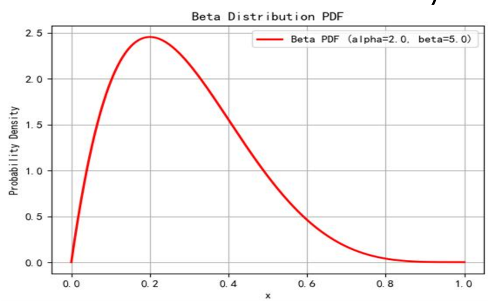

Figure 7. Probability Density Plot of Beta Function

Bayesian parameter learning

First, Bayesian estimation is selected for parameter learning of defective rate, and the mathematical description of Bayesian estimation is shown in formula (17) [10].

$$\pi(\theta|x)=\frac{f(x|\theta)\pi(\theta)}{m(x)}=\frac{f(x|\theta)\pi(\theta)}{\int f(x|\theta)\pi(\theta)d(\theta)} \quad (17)$$

Where: *π(θ)* is the prior distribution of parameter *θ*; *π(θ)* is the posterior distribution of parameter *θ*.

Therefore, Bayesian estimation can be regarded as correcting the prior distribution according to the sample information to obtain the posterior distribution on the premise that *θ* obeys the prior distribution of *π (θ)*.

Usually, the expectation of the posterior distribution is taken as the estimated value of the parameter. The expectation of the posterior distribution is shown in formula (18):

$$\hat{\theta}_{be}=E\pi(\theta|x) \quad (18)$$

4.4.3 Solution of optimization model

we have implemented a Bayesian optimization strategy, synergistically integrated with binomial and Beta distribution models, to recalibrate the defect rates associated with Component 1, Component 2, and the final products. This recalibration is imperative for augmenting the accuracy of the dynamic programming model and bolstering the robustness of the decision-making framework. By amalgamating prior knowledge with empirical data, we have refined the estimation of defect rates, culminating in the formulation of optimal decision schemes predicated upon these refined rates. Tables 8, 9, 10, and 11 delineate the refined defect rates in conjunction with their corresponding optimal decision schemes and the resultant optimal total revenues. These outcomes not only substantiate the efficacy of our model but also bestow upon enterprises a data-centric decision-making apparatus, enabling the crafting of more judicious production decisions amidst conditions of uncertainty.

Table 8. Optimized Defective Rates (%) of Components and Finished Products for Question 2

Phase Scenario	Component 1	Component 2	Finished Product
1	9.7%	5.9%	12.9%
2	6.9%	5.9%	14.4%
3	8.3%	8.8%	14.9%
4	8.3%	8.8%	14.9%
5	9.7%	10.8%	12.4%
6	9.7%	6.9%	10.9%

Note: The result of the calculation is an infinite repeating decimal, so rounding is to one decimal place

Table 9. Solution Results for Question 2 After Optimizing the Model

Phase Scenario	Component 1	Component 2	Finished Product	Defective-Finished-Product	Optimal-Total-Revenue (Yuan)
1	Not Inspected	Not Inspected	Not Inspected	Not Disassembled	430833.3
2	Not Inspected	Not Inspected	Not Inspected	Disassembled	456666.7
3	Inspected	Not Inspected	Not Inspected	Disassembled	436666.7
4	Inspected	Inspected	Inspected	Not Disassembled	396666.7
5	Not Inspected	Inspected	Inspected	Disassembled	443333.3
6	Not Inspected	Inspected	Not Inspected	Not Inspected	369166.7

Table 10. Optimized Defective Rates (%) of Components, Semi-Finished Products, and Finished Products for Question 3

Component	Defective Rate	Semi-finished-Product	Defective Rate	Finished Product	Defective Rate
1	7.8%	1	4.8%	1	13.9%
2	9.8%	2	9.6%		
3	11.8%	3	11.3%		

4	8.8%				
5	3.9%				
6	14.7%				
7	10.8%				
8	7.8%				

Note: The result of the calculation is an infinite repeating decimal, so rounding is to one decimal place

Table 11. Solution Results for Question 3 After Optimizing the Model

Spare Part	Inspect or Not	Semi-finished Product	Inspect or Not	Disassemble or Not	Finished Product	Inspect or Not	Disassemble or Not
1	Not Inspected	1	Not Inspected	Not Disassembled	1	Inspect	Disassemble
2	Not Inspected	2	Not Inspected	Not Disassembled			
3	Not Inspected	3	Not Inspected	Not Disassembled			
4	Not Inspected						
5	Not Inspected						
6	Not Inspected						
7	Not Inspected						
8	Not Inspected					Optimal Total Revenue (Yuan)	183.14

4.4.4 Conclusion:
From the column of optimal total revenue, compared with the optimal total revenue obtained from the original problem 2, Table 9 is more stable as a whole, and the decision scheme of whether to detect and dismantle at each stage is more reasonable; compared with the optimal total revenue obtained from the problem 3, Table 11 increases by 125.12%, and the optimization effect is good.

5. Model Evaluation and Promotion

5.1 Advantages of the Model

(1) High efficiency: The sampling detection scheme designed by statistical hypothesis testing can minimize the number of detections and reduce the cost of detection while ensuring the accuracy of detection.
(2) Economy: The application of dynamic programming algorithm ensures that the decision-making in each stage is based on global optimization, effectively balances the detection cost and potential loss, and achieves the goal of cost minimization and profit maximization.
(3) Robustness: The Bayesian updating model considers the uncertainty of the estimation of the defective rate, optimizes the estimation of the defective rate by introducing the combination of prior information and sample data, and improves the robustness of the decision-making scheme.
(4) Comprehensiveness: The model not only considers the inspection decision of spare parts and finished products, but also covers the disassembly and reuse of unqualified finished products, as well as the replacement of unqualified products by users, forming a comprehensive production process decision-making system.

5.2 Disadvantages of the Model

(1) Strong data dependence: The accuracy of the model is highly dependent on the defective rate data of spare parts, semi-finished products and finished products, and the accuracy of the data directly affects the decision-making effect.
(2) Long-term impact is not considered: The model mainly focuses on the decision-making optimization of a single production process, and does not fully consider the impact of equipment wear, technology upgrading and other factors on decision-making in long-term production.

5.3 Areas to Be Improved in the Model

Relax the assumptions: Further relax the assumptions in the model to improve the universality of the model in practical applications. For example, one may consider a sampling detection scheme for the case of a small sample size.

Introduce more influencing factors: introduce more factors that affect the decision-making in the model, such as equipment depreciation, labor costs, changes in market demand, etc., so that the decision-making scheme is closer to the actual production situation.

Enhance dynamic adaptability: By integrating the prediction model, dynamically adjust the cost, selling price and other parameters to improve the adaptability of the model to market changes. At the same time, machine learning methods can be introduced to learn historical data and optimize decision-making strategies.

Verification and optimization: verify and optimize the model through experiment or simulation to ensure the effectiveness and robustness of the model in different situations.

5.4 Model Outlook

Cross-industry application: The model proposed in this paper is not only applicable to the field of electronic product production, but also can be extended to other manufacturing fields such as machinery manufacturing, food processing and so on, providing production decision support for different industries.

Supply chain collaboration: In supply chain collaboration management, the model can be used to evaluate the quality of spare parts of different suppliers, optimize the allocation of supply chain resources, and improve the overall operational efficiency.

Policy-making reference: When formulating relevant industrial policies, government departments can refer to this model to supervise and optimize the production process of enterprises in the industry and promote the healthy development of the industry.

Academic research expansion: The research ideas and methods of this paper provide new perspectives and tools for academic research in the field of production decision-making, and can be further expanded to more complex production systems and decision-making environments

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References

[1] LI Jia-rong, MEI Hua-ping, Luo Dan-yan, et al. Ordering and transportation of raw materials in manufacturing enterprises based on mathematical programming model. Heilongjiang Science, 2024, 15 (16): 77-81.

[2] CuiRumai, SiShoukui. Mathematical model of data visualization processing. Journal of Naval Aeronautical Engineering College, 2010,25 (01): 105-108 + 112.

[3] Jiang Qiyuan, Xie Jinxing, YeJun, Mathematical Model. 5th Edition. Beijing: Higher Education Press, 2018.

[4] Yang. Analysis of the relationship among binomial distribution, Poisson distribution and normal distribution. Enterprise Science and Technology and Development, 2008, (20):108-110.

[5] He Siyao, Shen Yue, Xin Yanyu. Dynamic programming algorithm for optimal solution problem. Electroacoustic Technology,2019,43(03): 42-44. DOI: 10.16311/J.Audioe.2019.03.011.

[6] Li Ruijie, CuiShichang, ZhangYihan, et al. Real-time coordinated voltage regulation strategy of distribution network based on approximate dynamic programming. Power Grid Technology,1-12[2024-09-08]. Https://doi.org/10.13335/j.1000-3673.pst.2023.1996.

[7] Powell W.B. Approximate dynamic programming: Solving the curses of dimensionality. Hoboken John Wiley & Sons,2007.

[8] Wang Di. Research and Application of Weighted Bayesian Network Classification Algorithm. Hubei Normal University, 2024. DOI:10.27796/d.CNKI. Ghbs f. 2024. 000228.

[9] Zhang Sichen, Fang Huan. Emergency decision model for hazardous chemicals leakage based on dynamic Bayes. Journal of Lanzhou Institute of Technology, 2024, 31 (04): 94-99.

[10]Wang Ke, Shao Jingwen, Yang Jun. Research on landing distance of acivilaircraft based on Bayesian network. Progress in Aeronautical Engineering,1-8[2024-09-08]. http://kns.cnki.net/kcms/detail/61.1479.V.20240719.1249.004.html.

