# **Exploration of Calculation Methods for a Class of Sparse Determinants**

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**Abstract: Claw-shaped determinants, as a type of sparse determinant, have extensive applications in various fields such as matrix theory, numerical computation, physics, engineering, economics, and computer science. To efficiently compute the values of claw-shaped determinants, this paper first categorizes them into four types. Next, using the properties** of **determinants, the**  $j = 2, 3, \dots, n$ . Determinants of order n: **calculation formulas for these four types of claw-shaped determinants are discussed. Finally, numerical examples demonstrate the rationality of these formulas.**

**Keywords: Determinant; Claw-Shaped; Sparsity; Laplace's Theorem**

### **1. Introduction**

In many practical applications, such as image processing, signal processing, scientific computing, and social networks, the data being processed often takes the form of sparse matrices (matrices where the number of nonzero elements is much smaller than the total number of elements), this has made the theoretical study of sparse matrices one of the research hotspots [1-5].

The problem of calculating the determinant of sparse matrices is significant for improving the theoretical foundation of sparse matrices. For example, the computation of tridiagonal determinants plays an important role in the discretization of differential equations, linear systems of equations, and physical problems. Claw-shaped determinants are another type of determinant with a sparse structure. This sparsity not only significantly reduces computational effort when calculating their values but also minimizes storage costs. Previous researchers have studied the computation of claw-shaped determinants [6-10], but no systematic exploration of their calculation methods has been conducted.

Therefore, this paper will comprehensively discuss the computation formulas for different types of claw-shaped determinants.

To facilitate the discussion, we first define four types of claw-shaped determinants as follows.

Definition 1.1 Let  $a_i$ ,  $b_j$ ,  $c_j$  be numbers not ce Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024<br> **hods for a Class of Sparse**<br> **ants**<br>
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gaing College of International Business and<br> *Author*<br>
Therefore, this paper will comprehensively<br>
discuss the compu all equal to zero, where  $i = 1, 2, \dots, n$ ; **j** Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024<br> **nods for a Class of Sparse**<br> **ints**<br> **an, Jing Zhou**<br> **and Space of International Business and**<br> *Author*<br> *Cherefore, this paper will comprehensively*<br> *Space of claw*re, this paper will comprehensively<br>the computation formulas for different<br>claw-shaped determinants.<br>litate the discussion, we first define<br>pes of claw-shaped determinants as<br>on 1.1 Let  $a_i$ ,  $b_j$ ,  $c_j$  be numbers not<br>al re, this paper will comprehensively<br>the computation formulas for different<br>claw-shaped determinants.<br>litate the discussion, we first define<br>pes of claw-shaped determinants as<br>on 1.1 Let  $a_i$ ,  $b_j$ ,  $c_j$  be numbers not<br>al **Class of Sparse**<br> **Class of Sparse**<br> **Class of Sparse**<br> **Depited Alternational Business and**<br> **paper will comprehensively**<br> **utation formulas for different**<br> **educes of different**<br> **class of different**<br> **class of differe** Il comprehensively<br>rmulas for different<br>ninants.<br>on, we first define<br>ed determinants as<br> $e^{c_j}$  be numbers not<br>re  $i = 1, 2, \dots, n$ ;<br>ts of order n:<br> $a_n$ <br>0 0 0<br>0 0<br>0 0<br>: **a for a Class of Sparse**<br> **for a Class of Sparse**<br> **ng Zhou**<br> **ng Zhou**<br> **ng**, China<br> **ng**<br> **ng**<br> **ng**<br> **ng**<br> **ng**<br> **ng**<br> **ng**<br>  $\begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \end{vmatrix}$  $\cdots$  0 0

1 3 3 1 1 0 0 0 0 0 0 0 0 0 *n n n c b c b D c b c b* , <sup>1</sup> 3 2 1 2 2 2 3 3 1 1 0 0 0 0 0 0 <sup>0</sup> 0 0 0 0 0 *n n n n n <sup>n</sup> <sup>n</sup> a a a a a b c b c D b c b c* , 1 1 3 3 3 2 2 1 2 3 1 0 0 0 0 0 0 0 0 0 0 0 0 *n n n n n n n c b c b D c b c b a a a a a* , 1 1 4 3 3 2 2 <sup>1</sup> 3 2 1 0 0 0 <sup>0</sup> 0 0 0 0 0 0 0 0 *n n n n n n n <sup>b</sup> <sup>c</sup> b c D b c b c a a a a a* .

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These are referred to as the first, second, third, the  $k$   $(k = 1, 2, 3, \dots, n-1)$  column of  $D_n^2$  to and fourth types of claw-shaped determinants, respectively.

For the first type of claw-shaped determinants, extensive research results already exist. Below, we list some of the known conclusions.

**Lemma** 1.1 Let  $D_n^1$  be a determinant of  $D_n^2 =$ order n of the first type of claw-shaped determinant. Then:

e are referred to as the first, second, third,  
fourth types of claw-shaped determinants,  
activity. the first type of claw-shaped determinants,  
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st some of the known conclusions.  
**ma 1.1** Let 
$$
D_n^1
$$
 be a determinant of  
in an **1.1** Let  $D_n^1$  be a determinant of  
minant. Then:  
(i) When  $\prod_{j=2}^n b_j \neq 0$ ,  
 $D_n^1 = \left(a_1 - \sum_{j=2}^n \frac{a_j c_j}{b_j}\right) \prod_{j=2}^n b_j$ .  
(ii) When there is exactly one zero  
ing  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$ , without loss of  
relity, let  $b_i = 0$  ( $i \in \{2, 3, \cdots, n\}$ ), then:  
 $D_n^1 = -a_i c_i \prod_{j=1, j=2}^n b_j$ .  
(iii) When at least two elements among  
 $b_3$ ,  $\cdots$ ,  $b_n$  are zero,

among  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$ , without loss of without *j i j*

$$
D_n^1 = -a_i c_i \prod_{j \neq i, j=2}^n b_j.
$$

(iii) When at least two elements among  $b_2, b_3, \dots, b_n$  are zero,

$$
D_n^1=0
$$

Expanding the formula for case (i) in Lemma 1.1 reveals that the result also satisfies the conclusions in (ii) and (iii). Thus, the computation formula for the first type of claw-shaped determinant can be unified as follows: at least two elements among<br>  $\begin{vmatrix}\n\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\n\end{vmatrix}$  at least two elements among<br>  $\begin{vmatrix}\nP_n^1 = 0 & \frac{1}{2} & -(-1)^{i+n}c_i \\
0 & \frac{1}{2} & \frac{1}{2} & -(-1)^{i+n}c_i \\
0 & \frac{1}{2$ *D<sub>n</sub>* = -*a<sub>i</sub>c<sub>i</sub>* **j**  $\sum_{j\neq i,j=2}^{n}$ <br>
i) When at least two elements among<br>  $\cdots$ ,  $b_n$  are zero,<br>  $D_n^1 = 0$ .<br>  $D_n^2 = (-1)^{i+n}$ <br>
ding the formula for case (i) in Lemma<br>
reals that the result also satisfies the<br>
sions *j*  $\neq i, j=2$ <br>
an at least two elements among<br>
are zero,<br>  $D_n^1 = 0$ .<br>  $D_n^2 = (-1)^{i+n} c_i$ <br>  $\vdots$ <br>
at the result also satisfies the<br>
in (ii) and (iii). Thus, the<br>
formula for the first type of<br>
determinant can be unified as<br> When  $\prod_{j=2}^{n} b_j \neq 0$ ,<br>  $D_n^1 = \left(a_1 - \sum_{j=2}^{n} \frac{a_j c_j}{b_j}\right) \prod_{j=2}^{n} b_j$ .<br>  $D_n^1 = \left(a_1 - \sum_{j=2}^{n} \frac{a_j c_j}{b_j}\right) \prod_{j=2}^{n} b_j$ .<br>
When there is exactly one zero<br>
When the signal one sero<br>  $D_n^1 = -a_i c_i \prod_{j \neq i, j=2}^{n} b_j$ .<br>
W

$$
D_n^1 = a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k \neq j, k=2}^n b_k \right).
$$

Given the above conclusions for the first type of claw-shaped determinant, do similar conclusions exist for the other three types of claw-shaped determinants? The following sections will continue to explore these and provide computation formulas for other types of claw-shaped determinants. above conclusions for the first type<br>
shaped determinant, do similar<br>
sexist for the other three types of<br>
dd determinants? The following<br>
ill continue to explore these and<br>
mputation formulas for other types<br>
aped determ claw-shaped determinant, do similar  $D_n^2 = (-1)^{i+n} c_i (-1)^{i+n}$ <br>nelusions exist for the other three types of<br>aw-shaped determinants? The following<br>tions will continue to explore these and<br>ovide computation formulas for other  $D_n^1 = 0$ .<br>
anding the formula for case (i) in Lemma<br>
reveals that the result talso satisfies the<br>
lutsions in (ii) and (iii). Thus, the<br>
putation formula for the first type of<br>  $D_n^1 = a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k=j,k$ 

#### **2. Main Conclusions**

 $D_n^2$  is a second-type claw determinant, then where  $i \in \{2, \ldots, n\}$ 

$$
D_n^2 = (-1)^{\frac{n(n-1)}{2}} \left[ a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k \neq j, k=2}^n b_k \right) \right].
$$
all their  
\n**Proof** When  $\prod_{j=2}^n b_j \neq 0$ , adding  $-\frac{c_{n+1-k}}{b_{n+1-k}}$  times  
\nNow we  
\n $b_i (i = 2,$ 

to the last column yields:

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\nThese are referred to as the first, second, third,  
\nand fourth types of claw-shaped determinants,  
\nthe next column yields:  
\nthe last column yields:  
\nthe first type of claw-shaped determinants,  
\nwe list some of the known conclusions.  
\norder n of the first type of claw-shaped  
\ndeterminant. Then:  
\n(i) When 
$$
\prod_{j=2}^{n} b_j \neq 0
$$
,  
\n
$$
D_s^1 = \begin{pmatrix} a_i - \sum_{j=2}^{n} \frac{a_j c_j}{b_j} \end{pmatrix} \begin{pmatrix} a_i \\ \frac{a_i}{c_i} \end{pmatrix} \begin{pmatrix} a_i \\ \frac{a
$$

When one element in  $b_2, b_3, \dots, b_n$  is zero, loss of generality, suppose  $b_i = 0 (i \in \{2, 3, \dots, n\}),$ . Expanding  $D_n^2$  along the i-th row gives:

≠ <sup>1</sup> 0 *D<sup>n</sup>* . *D a b a c b* 1 1 2 2 2 1 0 0 0 0 *i n n i i i n b b b* 1 0 0 0 

then expanding the determinant in the formula along the column containing  $a_i$ :

$$
D_n = \begin{pmatrix} 1 & -2 & a_1 - 2 & a_2 - 2 \\ a_1 - \frac{1}{f-2} & b_1 \end{pmatrix} \int_{y=1}^{2} b_j
$$
\n(ii) When there is exactly one zero  
\n(ii) When there is exactly one zero  
\n $b_1, b_2, b_3, \cdots, b_n$ , without loss of  
\n $b_1 = 0 \text{ (} i \in \{2, 3, \cdots, n\})$ , then:  
\n
$$
D_n = -a_i c_j \int_{p=1}^{a} b_j
$$
\n(iii) When at least two elements among  
\n
$$
D_n^1 = -a_i c_j \int_{p=1, b}^{a} b_j
$$
\n
$$
= a_i c_j \int_{p=1, b}^{a} b_j
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= a_i c_j \int_{p=1, b}^{a} b_j
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= a_i c_j \int_{p=1, b}^{a} b_j
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$$
= a_j c_j \int_{p=1, b}^{a} b_j
$$
\n $$ 

**2. Main Conclusions**<br> **Theorem 2.1** If the n-order determinant<br>  $b_2, b_3, \cdots, b_n$  are zero, suppose  $b_i = b_j = 0$ ,<br>
where  $i \in \{2, 3, \cdots, n\};$   $j \in \{2, 3, \cdots, n\}$ : <sup>2</sup> <sup>2</sup> , 2 determinant, do similar  $b_n - (1)^{n-1}$ <br>
or the other three types of<br>
minants? The following<br>
inue to explore these and<br>
on formulas for other types<br>
erminants.<br> **j**  $b_1$ ,  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  are zet<br>
the n-order det . all their second-order minors equal zero. By  $\prod_{j=2}^{n} b_j \neq 0$ , adding  $-\frac{c_{n+1-k}}{b_{n+1-k}}$  times Now we prove that regardless of the values of  $b_{n+1-k}$   $h_i (i = 2, 3, \cdots, n),$ following<br>these and<br>ther types<br> $= (-1)^{\frac{(n-2)(n-2)}{2}}$ <br>When at least<br> $b_2, b_3, \dots, b_n$  are zerc<br>eterminant<br>mant, then<br> $i \neq j$ . selecting the<br> $\prod_{j=1}^{n} b_k$ <br> $\begin{bmatrix} \vdots \\ k \end{bmatrix}$ . selecting the<br>all their second-order<br>Laplace's th these and<br>
other types<br>  $= (-1)^{\frac{(n-2)(n-2)}{2}}$ <br>
When at least<br>
eterminant<br>
mant, then  $i \neq j$ . selecting the<br>  $\prod_{\neq j,k=2}^{n} b_k$ <br>  $\prod_{n+1-k}^{n} b_k$ <br>  $\text{all their second-order Laplace's theorem, it follows we prove that reg}$ <br>  $b_i (i = 2, 3, \dots, n)$ ,<br>
hthere is the second of the seco e types of<br>
following<br>
these and<br>
ther types<br>  $= (-$ <br>
When at<br>  $b_2, b_3, \dots, b$ <br>
terminant<br>
nant, then<br>  $i \neq j$ . sele<br>  $\left[\prod_{j,k=2}^n b_k\right]$ .<br>
all their seconds if  $i \neq j$ .<br>
Laplace's the<br>
Now we prov<br>  $b_i (i = 2, 3, \dots)$ the following<br>
or other types<br>  $= (-1)^{\frac{(n-2)(n-3)}{2}}$ <br>
When at least t<br>  $b_2, b_3, \dots, b_n$  are zero,<br>
when  $i \in \{2, 3, \dots, n\}$ ;<br>
when  $i \in \{2, 3, \dots, n$ When at least two elements in Laplace's theorem, it follows that  $D_n^2 = 0$ .

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\n
$$
D_n^2 = (-1)^{\frac{n(n-1)}{2}} \left[ a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k \neq j, k=2}^n b_k \right) \right].
$$
\nThe computation formulas provided in this paper can then be used to calculate their values.  
\nBelow are specific examples.  
\nWhen  $\prod_{j=2}^n b_j \neq 0$  and at least two  
\nof  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  are zero, expand each  
\nformula to prove it. When  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  has  
\n
$$
b_n
$$
 has

 $j \sim 0$  and

2 *j* of  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  are zero, expand each formula to prove it. When  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  has only one zero, without loss of generality, let 48 Journal of Natural Science Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024<br>  $D_s^2 = (-1)^{\frac{n(s-1)}{2}} \left[a, \prod_{j=1}^n b_j - \sum_{j=2}^n \left(a_j c_j \prod_{j=1}^n b_k\right)\right]$ . The computation formulas provided in this<br>
paper can then the used to calc formulas and comparing the results, it can be observed that the two formulas differ by at most a sign. Thus, it suffices to prove that  $(n-2)(n-3)$ *n*<sub>n</sub> =  $(-1)^{-2}$   $\begin{bmatrix} a_1 \end{bmatrix} \begin{bmatrix} b_j - \sum_{j=2} \end{bmatrix} \begin{bmatrix} a_j c_j \end{bmatrix} \begin{bmatrix} b_k \end{bmatrix}$ . paper<br>
When  $\prod_{j=2}^{n} b_j \neq 0$  and at least two Exam<br>
f  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  are zero, expand each<br>
ormula to prove it. When  $\begin{aligned}\n\mathcal{D}_j &= \sum_{j=2} \left[ a_j c_j \prod_{k \neq j, k=2} b_k \right] \cdot \text{ paper can then} \\
\text{Below are speed} & \text{Example 1 3.} \\
\text{and at least two} & \text{R} \text{ component} \text{ element} \\
\text{are zero, expand each} & n\text{-order determinant} \\
\text{Then } b_2, b_3, \dots, b_n \text{ has } \\ \text{at loss of generality, let} & b_2 \text{ is } \\ 1 \text{ is } \text{by } b_1 \text{ is } \\ \text{by the results, it can be performed by the result, it can be performed by the result,$  $\left[\sum_{j=2}^{n} \left( a_j c_j \prod_{k \neq j, k=2}^{n} b_k \right) \right]$ . The comput<br>paper can the paper can the Below are sp<br>and at least two **Example1**:<br><br><br><br>**n**  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  has<br>oss of generality, let  $D_n = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n - 1 \\ b$ Journal of Natural Science Education (ISSN: 3005-5792) Vol. 1 No. 5,<br>  $\int_{r=2}^{n(x+1)} \left[a_1 \prod_{j=2}^{n} b_j - \sum_{j=2}^{n} \left(a_j c_j \prod_{k=j,k=2}^{n} b_k\right)\right]$ . The computation formulas provided in<br>
paper can then be used to calculate their v 48<br>  $D_s^2 = (-1)^{\frac{n(s-1)}{2}} \left[ a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k=j,k=2}^n b_k \right) \right]$ . The computation (ISS<br>
Delow are specified than the number of the set of the Journal of Natural Science Education (ISSN: 3005-5792<br>  $-1$ )<sup> $\frac{n(n-1)}{2}$ </sup>  $\left[a, \prod_{j=2}^{n} b_j - \sum_{j=2}^{n} \left(a_j c_j \prod_{k \neq j, k-2}^{n} b_k\right)\right]$ . The computation formulas<br>
paper can then be used to cal<br>
EXample1 3.1 Compute the sump 48<br>  $D_n^2 = (-1)^{\frac{n(n-1)}{2}} \bigg[ a_1 \prod_{j=1}^n b_j - \sum_{j=1}^n \bigg( a_j c_j \prod_{k=1}^n b_k \bigg) \bigg]$ .<br>
The computation formulas provided in this<br>
paper can then be used to calculate their values.<br>
When  $\prod_{j=2}^n b_j \neq 0$  and at least two<br>  $\int$ 48 Journal of Natural Science Education (ISS<br>  $D_x^2 = (-1)^{\frac{n(n-1)}{2}} \left[ a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k=1, k=2}^n b_k \right) \right]$ . The computation<br>
when  $\prod_{j=2}^n b_j \neq 0$  and at least two **Example1 3.1**<br>
When  $\prod_{j=2}^n b_j \neq 0$   $\begin{array}{ll} \displaystyle \int_{2}^{\frac{a(x+1)}{2}} \left[a_1 \prod_{j=1}^s b_j - \sum_{j=1}^s \left(a_j c_j \prod_{k=1}^s b_k\right)\right] \cdot \qquad & \text{The computation formulas provided in this paper can then be used to calculate their values.} \\ \displaystyle \prod_{j=2}^n b_j \neq 0 \qquad \text{and at least two.} \\ \displaystyle \int_{3}^{\infty} b_j \neq 0 \qquad \text{and at least two.} \\ \displaystyle \int_{3}^{\infty} b_j \neq 0 \qquad \text{and at least two.} \\ \displaystyle \int_{3}^{\infty} b_j \neq 0$  $D_v^2 = (-1)^{\frac{n(n-1)}{2}} \bigg[ a_j \frac{1}{r_j} b_j - \sum_{r=2}^{\infty} \bigg[ a_r c_j \frac{1}{r_j} b_r \bigg]$ . The computation formulas provided in this spectra then be used to calculate their values.<br>
When  $\int_{r=0}^{\infty} b_j b_j$ , ...,  $b_v$  are zero, expand each Journal of Natural Science Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024<br>  $(-1)^{\frac{n(n-1)}{2}}\left[a_1\prod_{j=2}^{n}b_j\neq 0\right]$  The computation formulas provided in this<br>
paper can then be used to calculate their values<br>  $\prod_{j=2}^{n}b$ When  $\prod_{j=2}^{1/2}$ ,  $\neq 0$  and at least two *a* and  $\pi$  or  $\pi$  b  $\pi$ ,  $\pi$ , L  $f^{-2}$   $h^{-1}$  Below are specific exampled 1.1 Complete.<br>
L  $f^{-2}$   $h^{-1}$  and at least two **Exampled 1.1**<br>  $h_2$ ,  $h_3$ ,  $\cdots$ ,  $h_n$  are zero, expand each<br>
nula to prove it. When  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  has<br>  $\cdots$  one

$$
(-1)^{\frac{n(n-1)}{2}} = (-1)^{\frac{(n-2)(n-3)}{2}+1} \qquad \text{Let} \qquad \qquad 0
$$

Observe that

\n
$$
(-1)^{\frac{n(n-1)}{2}} = (-1)^{\frac{(n-2)(n-3)}{2}+1} \cdot \text{ Let}
$$
\n
$$
F(n) = \frac{n(n-1)}{2} - \left[\frac{(n-2)(n+3)}{2} + 1\right], \text{ it}
$$
\nmust be proven that for any

\n
$$
F(n) = \frac{n(n-1)}{2} - \left[\frac{(n-2)(n+3)}{2} + 1\right], \text{ it}
$$
\n
$$
D_n = (-1)^{n+2} \begin{cases} \frac{1}{2} & \text{if } n \neq 2^+ \\ \frac{1}{2} & \text{if } n = 2^+ \\ \frac{1}{2} & \text{if } n = 2^+ \end{cases}
$$
\nThus, the proof is complete.

\nThus, the proof is complete.

\n
$$
D_n^2 = (-1)^{\frac{n(n-1)}{2}} \left[a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left(a_j c_j \prod_{k \neq j, k=2}^n b_k\right)\right].
$$
\nSimilarly, the calculation formulas for third-type and fourth-type claw determinants

\n
$$
D_n = (-1)^{-(n-2)} \begin{pmatrix} 0 & 0 \\
$$

$$
F(n)
$$
 is always an even number. Because

$$
F(n) = \frac{n(n-1)}{2} - \left[ \frac{(n-2)(n-3)}{2} + 1 \right] = 2n - 2 \in 2Z.
$$

Thus, the proof is complete.

$$
D_n^2 = (-1)^{\frac{n(n-1)}{2}} \left[ a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k \neq j, k=2}^n b_k \right) \right].
$$

Similarly, the calculation formulas for third-type and fourth-type claw determinants can be derived, and their proofs are similar to the proof of Theorem 2.1, so they are omitted. **Theorem 2.2** If the n-order determinant  $\left[\frac{(n-1)}{2} - \left[\frac{(n-2)(n-3)}{2} + 1\right] = 2n - 2 \in 2Z.$  Multiply the<br>
e proof is complete. -1 and add i<br>  $1\right)^{\frac{n(n-1)}{2}}\left[a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left(a_j c_j \prod_{k=j,k=2}^n b_k\right)\right].$ <br>  $\left[\begin{array}{c} \text{where } a_1 \neq b_1, b_2 \neq b_2 \neq b_k \end{array}\right]$ .<br>  $\left$  $f(n) = \frac{1}{2}$   $\left[\frac{n(n-1)}{2} + 1\right] = 2n - 2 \in 2Z$ .<br> **nus, the proof is complete.**<br>  $\left[\begin{array}{c} \frac{n(n-1)}{2} \\ \frac{n(n-1)}{2} \end{array}\right]$   $\left[\begin{array}{c} a_1 \\ a_2 \end{array}\right]$   $\left[\begin{array}{c} a_2 \\ a_3 \end{array}\right]$   $\left[\begin{array}{c} a_3 \\ a_4 \end{array}\right]$   $\left[\begin{array}{c} a_1 \\ a_2 \end{array}\right$ busts and comparing the results, it can be<br>
a sign. Thus, it suffices to prove that<br>  $n = \frac{n(n-1)}{2} = \left[\frac{(n-2)(n+3)}{2} + 1\right]$ , it<br>  $n = \frac{n(n-1)}{2} - \left[\frac{(n-2)(n+3)}{2} + 1\right]$ , it<br>
the proven that for any  $n \in \mathbb{Z}^*$ ,<br>  $n$ ) is al fourth-type claw determinants<br>
and their proofs are similar to<br>
sorem 2.1, so they are omitted.<br>
If the n-order determinant<br>  $\begin{bmatrix}\na_1 \bigr|_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k=j,k=2}^n b_k \right)\n\end{bmatrix}$ .<br>  $\begin{bmatrix}\na_1 \bigr|_{j=2}^n b_j - \sum_{j=$ *n*  $p_s$  in exactuation formulas for<br>
pe and fourth-type claw determinants<br>
derived, and their proofs are smillar to<br>
of of Theorem 2.1, so they are omitted.<br> **m** 2.2 If the n-order determinant<br>
s a third-type claw determ *z* **j** *k* is the set of the set  $\frac{n(n-1)}{2} - \left[ \frac{(n-2)(n+3)}{2} + 1 \right]$ , it<br>
proven that for any  $n \in \mathbb{Z}^*$ ,<br>
always an even number. Because<br>  $\frac{(n-1)}{2} - \left[ \frac{(n-2)(n-3)}{2} + 1 \right] = 2n - 2 \in 2\mathbb{Z}$ .<br>
Multiply the last  $\left\{ a_n + b_n \right\}$ <br>
proof is complete.<br>

 $D_n^3$  is a third-type claw determinant, then

$$
D_n^3 = (-1)^{\frac{n(n-1)}{2}} \left[ a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k \neq j, k=2}^n b_k \right) \right].
$$

**Theorem 2.3** If the n-order determinant  $D_n^4$  is a fourth-type claw determinant, then

$$
D_n^4 = a_1 \prod_{j=2}^n b_j - \sum_{j=2}^n \left( a_j c_j \prod_{k \neq j, k=2}^n b_k \right).
$$

#### **3. Numerical Examples**

If an n-order determinant belongs to one of the types of claw-shaped determinants, its value can be directly obtained using the corresponding computation formula. In general, the structure of an n-order determinant may not be a claw-shaped determinants, their properties can be utilized to transform them into claw-shaped determinants. The computation formulas provided in this paper can then be used to calculate their values. Below are specific examples.

 $\prod_{j=2}^{n} b_j \neq 0$  and at least two **Example1 3.1** Compute the value of the *n*-order determinant: *n*-order determinant:

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\n
$$
\left[a_1 \prod_{j=2}^{n} b_j - \sum_{j=2}^{n} \left(a_j c_j \prod_{k=j, k=2}^{n} b_k\right)\right]
$$
\nThe computation formulas provided in this paper can then be used to calculate their values.  
\n $\neq 0$  and at least two  
\nReic in Wannel 3.1 Compute the value of the  
\n*n*-order determinant:  
\n $b_n$  are zero, expand each  
\nwe it. When  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  has  
\nwithout loss of generality, let  
\n $b_1$ ,  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  has  
\nwithout loss of generality, let  
\n $b_{n-1}$  and  $b_1$  and  $b_2$ ,  $b_3$ ,  $\cdots$ ,  $b_n$  and  $b_n$ ,  $b_n$ ,  $b_n$ ,  $b_n$   
\n $b_{n-1}$  and  $a_{n-1}+b_{n-1}$  and  $b_{n-1}$  and  $b_{n-1}$  and  $b_{n-1}$   
\n $b_{n-1}$  and  $a_{n-1}+b_{n-1}$  and  $b_{n-1}$  and  $b_{n-1}$   
\nThus, it suffices to prove that  
\n
$$
\left(-1\right)^{\frac{(n-2)(n-3)}{2}+1}
$$
 Let  
\n
$$
\left(-1\right)^{\frac{(n-2)(n+3)}{2}+1}
$$
 Let  
\n
$$
b_n
$$
  $\left[\begin{array}{cccccc} 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ b_1 & b_1 & \cdots & b_n & b_n & b_n & b_n \\ b_n & b_n & \cdots & b_n & b_n & b_n \\ b_n & b_n & \cdots & b_n & b_n & b_n \\ b_n & b_n & \cdots & b_n & b_n & b_n \\ b_n & b_n & \cdots & b_n & b_n & b_n \\ b_n & b_n & \cdots & b_n & b_n & b_n \\ b_n & b_n & \cdots & b_n & b_n & b_n \\ b_n & b_n & \cdots & b_n & b_n & b_n \\ b$ 

**Solution** Using the edge-addition method [11], we have:

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\n
$$
\vec{a} = (-1)^{\frac{n(n-1)}{2}} \begin{bmatrix} a_1 \frac{1}{1}b_2 \end{bmatrix} b_2 - \sum_{j=2}^{n} \begin{bmatrix} a_j c_j \frac{1}{1}b_{j+1} \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\text{The computation formulas provided in this paper can then be used to calculate their values.}\n\text{Bolow are specified example.}\n\end{bmatrix}
$$
\nthen\n
$$
\begin{bmatrix}\n\frac{1}{2}b_j \neq 0 \\
\frac{1}{2}b_j \neq 0\n\end{bmatrix} \text{ and at least two linearly independent:}\n\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{j=1}^{n} \begin{bmatrix} a_j \frac{1}{2}b_1 \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{j=1}^{n} \begin{bmatrix} a_j \frac{1}{2}b_1 \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{j=1}^{n} \begin{bmatrix} a_j \end{bmatrix} b_1 \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{j=1}^{n} \begin{bmatrix} a_j \end{bmatrix} b_1 \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{j=1}^{n} \begin{bmatrix} a_j \end{bmatrix} b_2 \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{j=1}^{n} \begin{bmatrix} a_j \end{bmatrix} b_1 \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{j=1}^{n} \begin{bmatrix} a_j \end{bmatrix} b_2 \end{bmatrix}.
$$
\n
$$
\begin{bmatrix}\n\frac{1}{2}b_1 \end{bmatrix} b_2 + \sum_{
$$

 $\left[\begin{array}{ccc} 2 & -1 \end{array}\right]$  =  $2n-2 \in 22$ . Multiply the last column of the determinant by -1 and add it to each of the preceding columns:



Using the computation formula for the second type of claw-shaped determinant:

$$
\text{ determinant} \qquad D_n = (-1)^{\left(\frac{n^2-n-4}{2}\right)} \left[ \prod_{j=1}^n a_j + \sum_{j=1}^n \left( b_j \prod_{k \neq j, k=1}^n a_k \right) \right].
$$
\nnant, then

**Example 3.2** Compute the value of the n-order determinant:

$\mathbf{L} \mathbf{L} \mathbf{v}_k$	$b$	$b$	$\cdots$	$b$	$a$
g(s) to one of the laants, its value using the formula. In an n-order	$D_n = \begin{vmatrix} b & b & \cdots & b & a & b \\ b & b & \cdots & a & b & b \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ b & a & \cdots & b & b & b \\ a & b & \cdots & b & b & b \end{vmatrix}$				

determinant. However, for certain<br>determinants, their properties can be utilized to determinant  $D_n$  by -1 and add it to each of the **Solution** Multiply the last row of the preceding rows:

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\n
$$
b-a
$$
\n
$$
b-a
$$
\n
$$
0 \cdots
$$
\n
$$
0 = a - b
$$
\n

Using the computation formula for the third type of claw-shaped determinant:

$$
D_n = (-1)^{\frac{n(n-1)}{2}} (a-b)^{n-1} [a+(n-1)b].
$$

**Example 3.3** Compute the value of the n-order determinant:

$$
D_n = \begin{vmatrix} a & b & b & \cdots & b & b \\ b & a & b & \cdots & b & b \\ b & b & a & \cdots & b & b \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ b & b & b & \cdots & a & b \\ b & b & b & \cdots & b & a \end{vmatrix}.
$$

**Solution** Multiply the last row of the determinant  $D_n$  by -1 and add it to each of [5] Johann Walter the preceding rows:



Using the computation formula for the fourth type of claw-shaped determinant:

$$
D_n = (a-b)^{n-1} \left[ a + (n-1)b \right].
$$

#### **4. Conclusion**

Claw-shaped determinants are a special type of determinant characterized by sparsity. This paper leverages their structural characteristics to not only provide a rigorous classification of different types of claw-shaped determinants but also derive computation formulas for each type. Numerical examples demonstrate that the computation formulas presented in this paper facilitate more convenient and accurate Calculation calculation of the values of various types of claw-shaped determinants.

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- ral Science Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024<br>
0  $\cdots$  0  $a-b$ <br>
0  $\cdots$  0  $\cdots$ <br>
0  $\cdots$  0  $\cdots$ <br>
0  $\cdots$  0  $\cdots$ <br>
0 ral Science Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024<br>
0  $\cdots$  0  $a-b$ <br>
0  $\cdots$  0  $\cdots$ <br>
0  $\cdots$  0  $\cdots$ <br>
0  $\cdots$  0  $\cdots$ <br>
0 Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024<br>
0 0  $a-b$  of International Business and Economics (Nos.<br>  $\begin{array}{ccc}\n 0 & a-b & 0 \\
 0 & a-b & 0 \\
 \end{array}$ <br>  $\begin{array}{ccc}\n 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\$  Applied Crystallography, 1991, 24(4): [1] Jarmila Jancarik, Sung-Hou Kim. Sparse Matrix Sampling: A Screening Method for Crystallization of Proteins. Journal of 409-411.
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- 1 atural Science Education (ISSN: 3005-5792) Vol. 1 No. 5, 2024 49<br>
0 ... 0 0  $a-b$  of International Business and Economics (Nos.<br>
0 ... 0  $a-b$  o <br>
1 ... at a b 0 ... a b 0 ... a b b cystallization of Proteins. Journal of  $\begin{array}{ccc} \cdots & b & b \end{array}$  Matrices and Improved Normalized Cuts.  $\begin{bmatrix} b & b \\ 18(11): 3027-3040. \end{bmatrix}$ [3] Mingrui Yang, Shibing Zhou, Qian Wang, et al. Fast Multi-view Clustering of Sparse Computer Science and Exploration, 2024,
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- ...  $a-b = 0$  o a Beferences<br>  $b \t\t\t\t\ldots a b = 0$  o a Matrix Sampling: A Screening Method for<br>  $b \t\t\t\t\ldots b = b$  o b b crystallization of Proteins. Journal of<br>
ungard determinant:<br>  $\frac{n-1}{2}(a-b)^{n-1}[a+(n-1)b]$ . Antimetic Based on H <sup>0</sup> 0 0 *n*  $\begin{vmatrix} b-a & 0 & \cdots & a-b & 0 & 0 \\ b & \vdots & \vdots & \vdots & \vdots & \vdots \\ a & b & b & b & b \end{vmatrix}$  **References**<br>  $\begin{vmatrix} 1 & 1 \arcsin b & 1 \end{vmatrix}$  *Ammala Jancarik*, Sung-Hou Kim. Sparse<br>  $\begin{vmatrix} a & b & b & b \end{vmatrix}$  *b* **c** *rystalization* of Proteins. Journal of *a b b a* 0 0 0<br> *A b b a b* Crystallization of Proteins. Journal of the b b Crystallization of Proteins. Journal of Applied Crystallography, 1991, 24(4):<br>
ion formula for the third<br>
determinant:<br>  $(a-b)^{n-1}[a+(n-1)b]$ , Athentic Based **a** b the third  $409-411$ .<br>
[2] Hackbusch Wolfgang. A Sparse Matrix<br>  $\cdot$   $\cdot$   $(n-1)b$ ]. Arithmetic Based on H-Matrices. Part I:<br>  $\cdot$  Introduction to H-Matrices. Computing,<br>  $\cdot$  of the n-order  $1999$ , 62(2): 89-108.<br>
[3] 23) Hackbowshi Wollgaug, A Spare Matrices. Part 1:<br>
(-1)<sup>2</sup> (*a* -*b*)<sup>2</sup> [*a* + (*n* -1)*b*].<br> **b** 3 Conpute the value of the n-order<br> **b** b  $\cdots$  b b a<br> **b**  $\cdots$  b b a<br>  $\cdots$  b b a<br>  $\cdots$  b b a<br>
Matrices and Exploratio  $-b$  0  $\cdots$  0  $b-a$  Converter. APEC. Seventeenth Annual  $-b$   $\cdots$  0  $b-a$  Conference and Exposition, 2002, 2: b b cystalization of Proteins. Journal of the third<br>
main:<br>  ${}^{4}[a+(n-1)b]$  Hotkbusch Wolfgang. A Sparse Matrix<br>
inant:<br>  ${}^{4}[a+(n-1)b]$  Arithmetic Based on H-Matrices. Part I:<br>
lindroduction to H-Matrices. Computing,<br>
value e conputation of the computer Science and Exposition (1)-11.<br>  $\left(-1\right)^{\frac{d(n-1)}{2}}(a-b)^{n-1}\left[a+(n-1)b\right].$ <br>
Arithmetic Based on H-Matrices. Part 1:<br>  $\left(-1\right)^{\frac{d(n-1)}{2}}(a-b)^{n-1}\left[a+(n-1)b\right].$ <br>
Arithmetic Based on H-Matrices. Comp 16.33 Compute the value of the n-order<br>
1999,  $62(2): 89-108$ .<br>
16.33 Compute the value of the n-order<br>
1999,  $62(2): 89-108$ .<br>  $\begin{array}{cccccc} a & b & \cdots & b & b \\ b & a & \cdots & b & b \\ b & a & \cdots & b & b \\ b & b & \cdots & b & b \\ b & b & \cdots & a & b \\ b & b & \cdots & a & b \\ b & b & \cdots & b & d \\ \end$ [5] Johann Walter Kolar, Martin Baumann, Frank Schafmeister, et al. Novel Three-phase AC-DC-AC Sparse Matrix IEEE Applied Power Electronics 777-791.
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