Research on Visualizationization of University Mathematics Curriculum in Applied Undergraduate Institutions under the Background of New Liberal Arts

Qijian Li
Ningbo University of Finance and Economics, Ningbo, Zhejiang 315375, China

Abstract: This paper analyzes the characteristics of students and university mathematics courses in applied undergraduate institutions. It proposes that by using powerful numerical, symbolic, and graphical capabilities of mathematical software such as Matlab and the Geometer's Sketchpad, mathematical concepts, theorems, and calculations in university mathematics courses can be visualizationization. This visualizationization process can transform abstract, complex, and tedious mathematical knowledge and methods into more concrete, vivid, and intuitive forms, presenting them dynamically and instantaneously. This approach not only enhances the students' sense of the reality of knowledge but also helps them discover the beauty of mathematics. It effectively reduces the difficulty of understanding key concepts and increases students' interest in learning mathematics, thereby further improving the quality of university mathematics teaching. Additionally, it helps students develop skills in using mathematical software, which significantly promotes their overall mathematical literacy. This study also provides a reference for applied undergraduate institutions in building new teaching concepts and models.

Keywords: University Mathematics; Matlab; Geometer's Sketchpad; Visualizationization

1. Introduction

The concepts and theorems in university mathematics courses are characterized by their abstract nature, high difficulty, and complex derivation processes.

Students in applied undergraduate institutions generally perform poorly in mathematics [1]. Through a questionnaire survey, the author found that approximately 7% of students scored above 110 in the college entrance examination (Gaokao) in mathematics, about 15% scored between 100 and 110, and around 60% scored between 80 and 100, indicating a relatively weak foundation in mathematics. As a result, these students face significant challenges when learning university mathematics, often feeling intimidated and hesitant to engage with the subject. In light of the inherent characteristics of university mathematics courses and the specific difficulties faced by students in applied undergraduate institutions, how to break through these barriers and make the teaching of university mathematics more accessible to students has always been a key issue for university mathematics teachers in their pedagogical research.

The domain of science and technology is witnessing a surge of growth, a wide variety of teaching software and multimedia technologies have emerged. In the teaching process of university mathematics courses, it is an effective solution to use these rich teaching software, the Internet, the Internet of Things, multimedia technologies and other tools and technologies to concretize, intuitivize and visualizationization [2] abstract concepts and theorems in mathematics courses, thereby improving the traditional teaching method of chalk and blackboard.

2. Visualization Design of Relevant Concepts in University Mathematics Courses

2.1 Visualization Design of Limit Concept Teaching Combined with Function Image and Data

The first major challenge in learning university mathematics courses is the concept of limits [3]. In applied undergraduate institutions, teachers generally explain the concepts of sequence limits, function limits, and double limits in a descriptive way, aiming to reduce the difficulty of understanding the concept of limits and make it easier for students to comprehend. As for the rigorous and accurate " $\varepsilon-N,\varepsilon-\delta$ " language definition of limits, most teachers only briefly mention it without requiring students to deeply understand and master it. When explaining the descriptive definition of limits, the expression "infinitely close" mentioned in the definition can only be understood by students through their own imagination, lacking intuitive and vivid images and data to assist in understanding. This results in a considerable number of students having a shallow understanding of the concept of limits, and some even fail to understand it, leading to unsatisfactory learning and teaching outcomes [4]. If in the teaching process, mathematical software such as Matlab and The Geometer's Sketchpad is used to add visualization design of data and images to the meaning of "infinitely close", it can not only concretize and intuitivize the abstract and difficult concept of limits, reduce the difficulty of the knowledge itself, but also attract students' attention and interest in learning, and improve their enthusiasm for learning, thus achieving twice the result with half the effort.

Visualization design of series limit, function limit and double limit concepts are given below

Example 1: Visualizationization of the limit of the series $\left\{\frac{n}{1+n}\right\}$ as n→∞:

Step 1: Write M file:

n=1:1:100; an=n./(1+n); plot(n,an,'*r'), xlabel('n'), ylabel('an')

Step 2: Observe the generated scatter plot (as shown in Figure 1)

Step 3: Make a list of tables (e.g., Table 1) to observe the distance between $a_n=\frac{n}{1+n}$ and 1 to visualizationization the mathematical meaning of infinite convergence.

Table 1. Distance of the Series $\left\{\frac{n}{1+n}\right\}$ from 1

n	an	|1-an|
10	0.909090909	0.090909091
100	0.99009901	0.00990099
1000	0.999000999	0.000999001
10000	0.99990001	9.999E-05
100000	0.99999	9.9999E-06
……	……	……

Example 2: Visualizationization of the limit of the function $y=(1+\frac{1}{x})^x$ when $x\to+\infty$

Step 1: Write M file:

syms x,y='(1+1/x)^x'; ezplot(y,[1,10000]), xlabel('x'), ylabel('f(x)=(1+1/x)^x')

Step 2: Observe the generated scatter plot (as shown in Figure 2)

Step 3: Make a list of tables (e.g., Table 2) to observe the trends in the values of the f(x) function

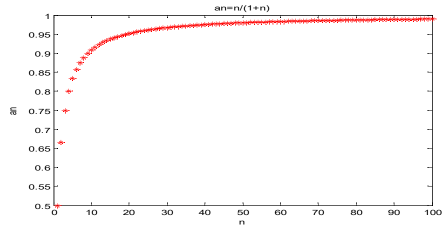


Figure 1. Image of the Series $\left\{\frac{n}{1+n}\right\}$

Table 2. $y=(1+\frac{1}{x})^x$ Function Values

x	f(x)
10	2.59374246
100	2.704813829
1000	2.716923932
10000	2.718145927
100000	2.718268237
……	……

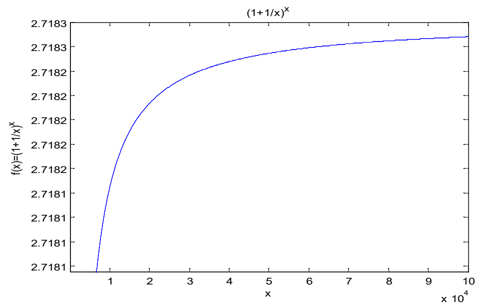


Figure 2. Image of Function $y=(1+\frac{1}{x})^x$

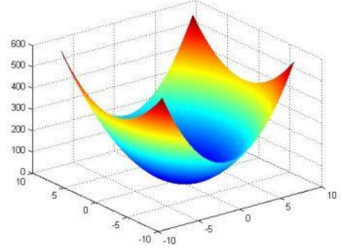
Example 3: Combining the teaching of the concept of double limits with the visualizationization design for finding $\lim\limits_{(x,y)\to(00)} 6x^2+3y^2$

Step 1: Write M file:

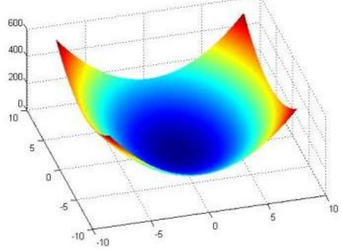
[x,y]=meshgrid(-8:0.05:8);z=6*x.^2+3*y.^2;mesh(x,y,z)

Step 2: Observe the generated 3D surface plot (as shown in Figure 3), and rotate the surface to obtain images of the surface from different angles (as shown in Figures 3a, 3b, and 3c). This helps to intuitively perceive the mathematical concept of function values approaching infinitely close from various perspectives.

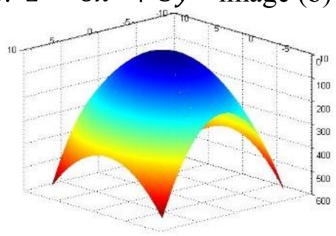
When teaching the definition of double limits, the concept of an arbitrary point $p(x,y)$ in the domain approaching a fixed point $p_0(x_0,y_0)$ in an arbitrary manner is involved. This kind of arbitrary approach is highly abstract, and it is particularly challenging for students in applied undergraduate institutions to understand this arbitrariness of the approach [5]. If we incorporate the graph of a bivariate function into the teaching and rotate the spatial surface in an all-around, no-dead-angle manner, students can intuitively, concretely, and vividly perceive that when the point $p(x, y)$ approaches the fixed point $p_0(x_0, y_0)$ in any way, the corresponding function value also approaches a certain definite constant. Not only does this enhance students' spatial imagination, but it also helps them better understand and master the knowledge points. Moreover, the three-dimensional spatial images, especially the dynamic visualization impact generated by rotating three-dimensional spatial images, can instantly ignite students' enthusiasm for learning and increase the charm of mathematical knowledge [6].



a. $z = 6x^2 + 3y^2$ image (a)



b. $z = 6x^2 + 3y^2$ image (b)



c. $z = 6x^2 + 3y^2$ image (c)

Figure 3. Image of Binary Function $z = 6x^2 + 3y^2$

2.2 Visualization Design of Function Derivative Teaching

Students have some difficulty in understanding the concept of derivatives in university mathematics courses. The main reason is that the definition of a function's derivative as the limit of the ratio of the increment of the function value to the increment of the independent variable cannot be visually demonstrated as easily as the function itself [7]. Although students have a relatively good grasp of the knowledge points and calculation of the first-order derivative of a single-variable function, their understanding of the conceptual knowledge points of higher-order derivatives is very vague, and their understanding of the partial derivatives of multivariable functions is even more difficult. Students only know how to calculate by applying basic formulas, but they are unclear about the mathematical meaning they represent, and thus cannot master them well. When dealing with the derivative calculation of some more complex functions, they often make mistakes. The reason is that students have not truly mastered derivatives and lack an intuitive sense of them [8]. When explaining the concept of derivatives, if the images of derivatives of different orders and their corresponding geometric meanings can be presented to students in a way that they can "see and touch", students will have a sense of reality when learning the knowledge points of derivatives, which will facilitate their understanding and mastery. The following are examples of the visualization design of the derivatives of single-variable functions, higher-order derivatives, and partial derivatives of two-variable functions.

Example 4: Visualizationization of finding the first, second, and third order derivatives of the function y=x^3-2x^2+5x

Step 1: Write M file:

```
symsx,
y='x^3-2*x^2+5*x';y1=diff(y,x,1);y2=diff(y,x,2)
;y3=diff(y,x,3);y1=char(y1);y2=char(y2);
y3=char(y3);subplot(2,2,1),fplot(y,[-10,10]),titl
e('y=x^3-2*x^2+58*x'),subplot(2,2,2),
fplot (y1, [-10, 10]), title ('y=x'3-2*x'2+58*x
first derivative'), subplot (2, 2, 3), fplot (y2,
[-10,10])
title ('y=x'3-2*x'2+58*x second derivative'),
subplot (2, 2, 4), fplot (y3, [-10, 10]),
title ('y=x'3-2*x'2+58*x third derivative')
```

Step 2: Observe the first-order derivative function, second-order derivative function, and third-order derivative function images of the function y = x^3 - 2x^2 + 5x (Figure 4).

Some function expressions are relatively complex. Students can only understand the first-order derivative of such a function at the calculation level, and cannot know the mathematical meaning of the calculated derivative. For higher-order derivatives, students only know how to find the derivative after the derivation, but they have no sense of the mathematical connotation contained in the higher-order derivatives and cannot further understand the mathematical ideas of higher-order derivatives. During the teaching process, according to the geometric meaning of the derivative of a one-variable function, the teacher compares the function image and the first-order, second-order, and third-order derivative images in real time, so that students can combine the geometric display of the derivative when learning the derivative, and their understanding of the derivative is no longer It is an ethereal feeling, and it is easier to understand the mathematical connotation of each derivative.

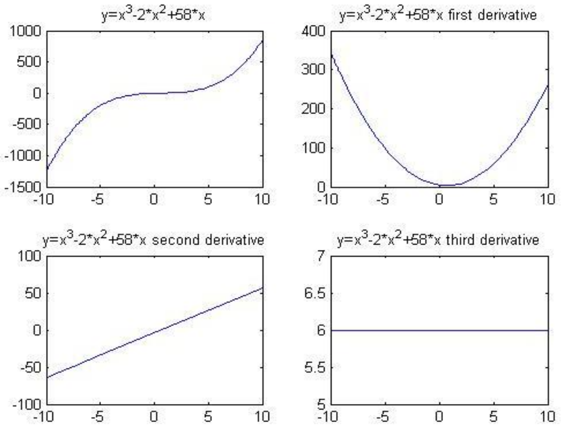


Figure 4. Images of the First, Second, and Third Order Derivatives of the Function y=x^3-2x^2+5x

Example 5: Find $z = x^2 + y^2$ in Visualization Design of Partial Derivative

Step 1: Write a partial derivative geometric meaning image program in the Geometer's Sketchpad:

$f: z = x^2 + y^2$

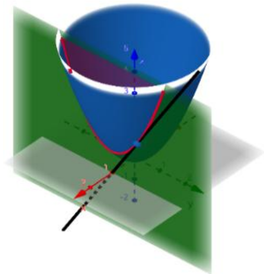
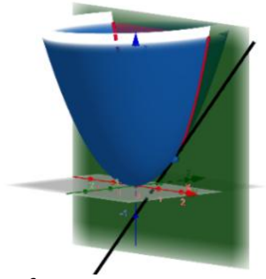
$g: y = 1$ or $g: x = 1$

c: Intersecting paths (f,g)=x=(0,1,1)+ (0.5t,0, 0.25t^2)

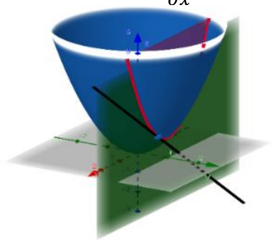
A=stroke point(c)=(.1,2)

Tangent(A,c)=h:x=(0,1,0)+2(-0.5.0.-1)

Step 2: Observe the generated 3D surface plot (as shown in Figure 5), and rotate the curved surface to obtain curve tangent lines with different angles (as shown in Figure 5a, Figure 5b and Figure 5c).

a. $\frac{\partial z}{\partial x}$= 2x tangent (a) b. $\frac{\partial z}{\partial x}$ = 2x tangent (b)



c. $\frac{\partial z}{\partial y}$ = 2y tangent (c)

Figure 5. Image of Geometric Meaning of Partial Derivative of Binary Function

$$z = x^2 + y^2$$

When teaching partial derivatives of bivariate functions, for students who have never learned or seen the graphs of bivariate functions before, the visualization display of the spatial surface images of bivariate functions in class can quickly catch their eyes, stimulate their enthusiasm for learning and enhance their interest. The visualization treatment of the geometric meaning of partial derivatives, especially the dynamic display that can make partial derivatives "come alive" and "move", greatly increases students' attention in class and helps them better understand and grasp the concept of partial derivatives.

2.3 Visualization Design of Definite Integral Teaching

Definite integrals originate from the calculation of the areas of irregular plane figures. The concept of definite integrals is one of the most lengthy and difficult to understand in single-variable calculus [9]. After learning the concept of definite integrals, students often have a blank mind about the arbitrary division of the integration interval and the arbitrary choice of point ε_i in each subinterval, leading to very unsatisfactory learning outcomes. When teachers explain the concept of definite integrals, they can use visualizationization to combine the four steps of "partitioning, approximating, summing, and taking limits" with graphs. Especially by using the dynamic changes of graphs to approximate the real area of curvilinear trapezoids, it can not only arouse students' interest in learning the knowledge points but also transform the abstract and obscure expressions in the concept of definite integrals into intuitive, concrete, and vivid graphical displays. This makes it easier for students to learn and more convenient for teachers to teach. Below is an introduction to the visualizationization design of teaching the concept of definite integrals using The Geometer's Sketchpad software.

Example 6: Combining the teaching of the concept of definite integrals with the visualizationization design for finding $\int_0^{10} \frac{x^2}{4} dx$

Step 1: Write the dynamic area approximation program of the Geometer's Sketchpad image as follows:

A=Intersection (f, x-axis, 1)=(0, 0)

B=stroke point (x-axis)=(10, 0)

a=integral(f,x(A),x(B))

b=1: 1: 100% slider b, number of segments after segmentation

c=lowersum(f,x(A),x(B),b)

d=uppersum(f,x(A),x(B),b)

Step 2: Display the dynamic demonstration that the irregular plane graph is segmented through the change of the numerical value of the slide bar, and intercept 3 illustrations (as shown in Figure 6a, Figure 6b and Figure 6c).

Example 7: Combine the design of a visualizationization to find the volume of a rotating body formed by the graphs enclosed by the curves $y = x^2$ and $y = \sqrt{x}$ around the x-axis and y-axis, respectively:

Step 1: Write the program for displaying the rotating body of space on the Geometer's Sketchpad as follows:

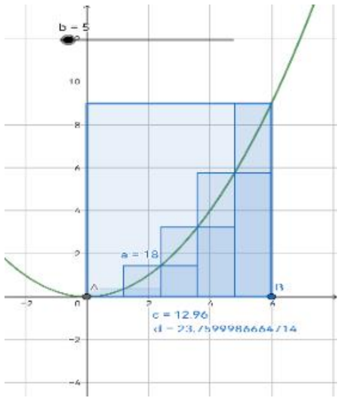
a=curve((t,t^2,,0),t,0,1)=(t,t^2,0),(0≤t≤1),b=curve ((t^2,t,,0),t,0,1)=(t^2,t,0),(0≤t≤1),

$$c=surface(a,2\pi,xAxis)=\begin{pmatrix} u \\ u^2\cos(v) \\ u^2\sin(v) \end{pmatrix},$$

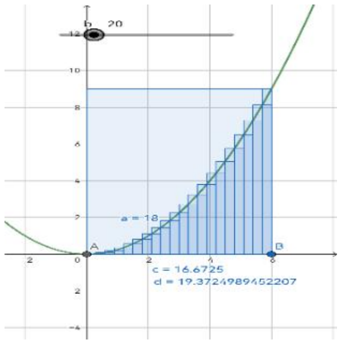
$$d=surface(b,2\pi,xAxis)=\begin{pmatrix} u^2 \\ u\cos(v) \\ u\sin(v) \end{pmatrix}$$

$$e=surface(a,2\pi,yAxis)=\begin{pmatrix} u\cos(v) \\ u^2 \\ u(-\sin(v)) \end{pmatrix},$$

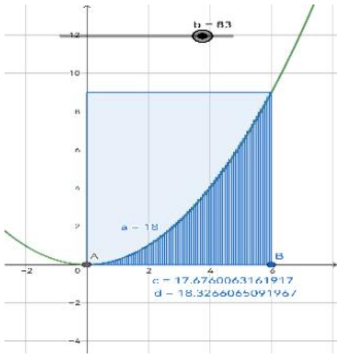
$$f=surface(b,2\pi,yAxis)=\begin{pmatrix} u^2\cos(v) \\ u \\ u^2(-\sin(v)) \end{pmatrix}$$



a. Slide bar b=5



b. Slide bar b=20

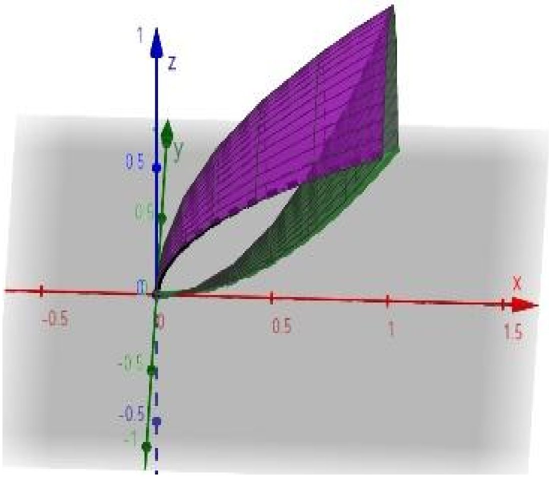


c. Slide bar b=83

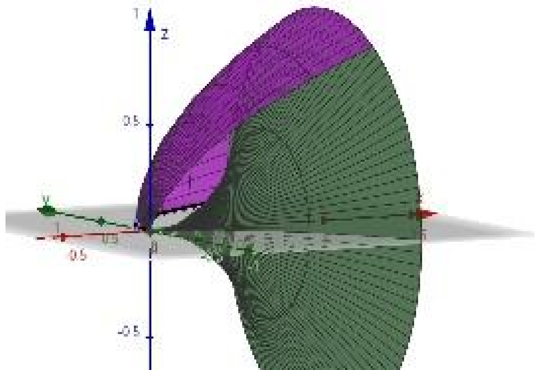
Figure 6. Demonstration of Definite Integral Dynamic Approximation

Step 2: Observe the dynamic diagram of the space rotating body generated by rotating the enclosed plane figure around the x-axis and y-axis respectively (as in Figure 7), which rotates around the x-axis (as in Figures 7a, 7b, and 7c), and rotates around the y-axis (as in Figures 7d, 7e, and 7f).

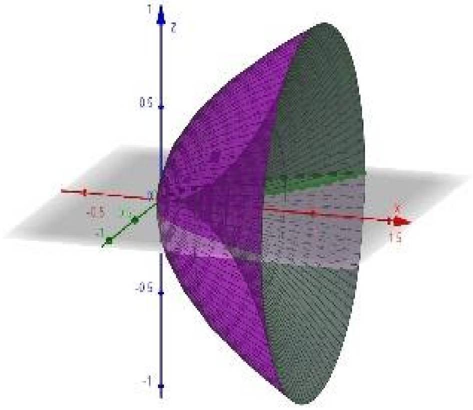
When explaining the geometric application of definite integrals in finding the volume of solids of revolution of irregular shapes, it is quite challenging for teachers to manually draw spatial solids of revolution on the blackboard with chalk, and students also lack the ability to imagine these spatial solids. After learning, students usually only remember to apply formulas to calculate the volume of solids of revolution, but do not have a deep understanding of the volume calculation formulas themselves. If teachers combine 3D animation demonstrations of solids of revolution in the teaching process, the colorful spatial solids of revolution are full of mathematical beauty [10]. The animation during the rotation greatly increases students' interest in learning. Encountering such beautiful "scenery" in the process of learning to calculate the volume of spatial solids of revolution can broaden students' horizons. By allowing students to discover the beauty of mathematics through mathematical graphs, it can not only reduce the difficulty of understanding knowledge points but also increase students' fondness for learning mathematics. It can transform students from being intimidated and unwilling to learn to being willing to face and enjoy learning, achieving the desired teaching effect.



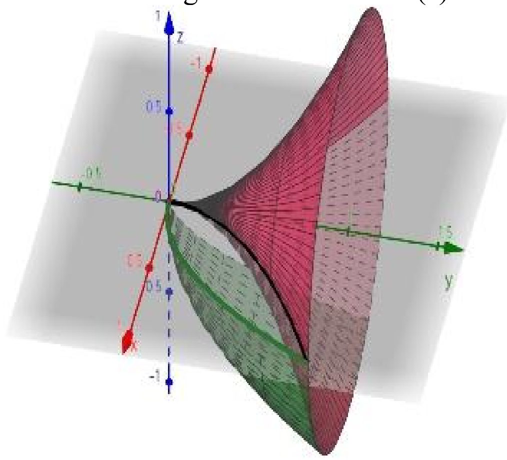
a. Rotating around the x-axis (a)



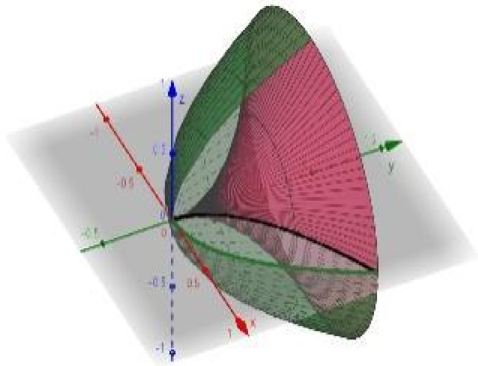
b. Rotating around the x-axis (b)



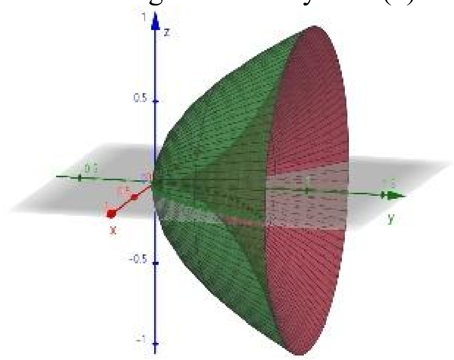
c. Rotating around the x-axis (c)



d. Rotating around the y-axis (a)



e. Rotating around the y-axis (b)



f. Rotating around the y-axis (c)

Figure 7. Dynamic Demonstration of Rotating Body

3. Conclusion

Students in applied undergraduate institutions have relatively weak mathematics foundations and struggle with learning University Mathematics courses, resulting in less than ideal learning outcomes. Teachers can use the help of multimedia devices such as computers and tablets, and appropriately use mathematical software like Matlab and The Geometer's Sketchpad to draw plane and spatial figures corresponding to knowledge points. They can vividly and intuitively present highly abstract, obscure, and complex mathematical concepts, theorems, and calculations through static images or animations, making knowledge more concrete, vivid, and real. Compared with traditional single teaching models, effective visualization teaching can achieve the effect of knowledge visualizationization, allowing students to feel the authenticity of knowledge and greatly reduce the difficulty of learning University Mathematics courses, achieving the desired teaching effect. It can also help students exercise their ability to use mathematical software and is conducive to cultivating students' practical hands-on ability to apply knowledge to solve practical problems. It has an obvious promoting effect on improving students' comprehensive mathematical quality and is well worth promoting and carrying out visualization teaching in applied undergraduate institutions.

Acknowledgement

This paper is supported by Education and Teaching Reform Project of Ningbo University of Finance and Economics: Research on Visualizationization of University Mathematics Curriculum in Applied Undergraduate Institutions under the Background of New Liberal Arts (Project No.: JY2022003801)

References

[1] Xu Dinghua, Liu Dan, Liu Keji. Discussion on Visualization Teaching Design of Analytical Mathematics Course. University Mathematics, 2022(8): 44-51.
[2] Kong Xiangqiang, Liu Xiuying. Application of Dynamic Visualizationization Function of Maple Software in University Mathematics Teaching. Journal of Jianghan University, 2018(4): 130-135.
[3] Wang Qiang, Shen Yongmei, Tu Qingwei. Exploration and Practice of University Mathematics Teaching Reform and Application Ability Cultivation. Journal of Higher Education, 2015(21): 138-139.
[4] Sun Xin, Feng Yan, Wu Zhidan, Wang Tao. Application of Visualization Effect of MATLAB Software in University Mathematics. Journal of Shenyang Normal University, 2014(4): 549-552.
[5] Liu Yanqin. Application of Visualization Teaching in Research Teaching of University Mathematics. Journal of Yulin College, 2012(3): 55-57.
[6] Li Muhua, Jia Liping. Exploration and Practice of Visualization Teaching of Mathematics in University of Arts. Journal of Jiamusi Education Institute, 2012(11): 6-7.
[7] Huang Liqiong, Wang Yuanyuan, Yang Yan. Research on University Mathematics Teaching under the Idea of Wisdom Education. Heilongjiang Science, 2019, 10(3): 16-17.
[8] Zhu Hanmin. Application of Matlab in Higher Mathematics Teaching. Computer Aided Engineering, 2004(2).
[9] Ping Yi. Application of Matlab in University Mathematics Teachin. Journal of Hubei Radio and Television University, 2008, 28(4): 135-136.
[10] Li Kun, Zhao Gang. Application of Mathematical Experiment in Higher Mathematics Teaching. Science and Technology Information, 2011, (23): 452-453.