

Research on Shadow Price in the Valuation of Carbon Emission Rights

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Abstract: Carbon emissions trading, a key component of green finance, serves as an important economic tool to achieve the "dual carbon" goals. Establishing a scientific and efficient carbon trading mechanism and fostering the sustainable development of the carbon market have become focal points in the industry, with carbon emission rights valuation being a fundamental task. This paper explores the application of the shadow price model in valuing carbon emission rights, using Beijing's carbon market as a case study. By analyzing data from 2000 to 2023, it assesses the option characteristics and marginal abatement costs of carbon emission rights. The study first highlights the significance of carbon trading and China's challenges in meeting its "dual carbon" targets. It then examines China's carbon market, particularly Beijing's, and employs principal component analysis and regression modeling to estimate the shadow price based on output value per unit of carbon emissions. The findings suggest that when the shadow price is lower than the market price, it may incentivize emission reductions and improve resource allocation efficiency, as demonstrated by Beijing's carbon trading data.

Keywords: Carbon Credits; Valuation; Shadow Price Modeling

1. Introduction

Accelerated global industrialization and continued technological development have led to a steady rise in carbon dioxide emissions, exacerbating global warming and causing ecological problems such as the frequency of extreme weather events and sea level rise. 2023 data show that global energy-related carbon dioxide emissions have surged to 37.4 billion tons, an increase of 4.1 billion tons, or 11%, compared with 2022. Meanwhile, the

European Union has succeeded in reducing its carbon emissions trading system (ETS) emissions by 1.55 percent from 2022 by vigorously promoting the application of renewable energy technologies and improving the carbon emissions trading system.

In China, CO₂ emissions in 2023 are 56.5 billion tons higher than in 2022, the highest increase in the world, and both total and per capita emissions exceed the average of developed economies by 15 percent. This increase is closely related to the post-epidemic economic recovery and the rapid development of emission-intensive industries. In the face of the increasingly severe emissions reduction situation, China is still under tremendous pressure to reduce emissions. At the national level, the dual-carbon goals of "achieving carbon peak by 2030 and carbon neutrality by 2060" have been clearly established as strategic priorities. Subsequent policy frameworks consistently emphasize the importance of actively and steadily advancing towards these targets. The strategic task of actively and steadily promoting carbon peaking and carbon neutrality was emphasized. The report also proposed to further improve the carbon emission statistics and accounting system, and to improve the carbon emission right market trading mechanism. Against this background, pricing of carbon emission rights has become a key issue. Therefore, how to develop a scientific and effective method to assess the value of carbon emission rights has become an important issue that needs to be solved urgently.

With the development of China's carbon emissions trading market, some problems have emerged one after another. From the macroscopic point of view, there are problems such as insufficient government supervision, difficult monitoring of carbon emission sources, and lack of initial allocation system of gas emission rights; from the microscopic point of view, there are problems such as

relatively backward technological conditions, irrational pricing of carbon emission rights trading, and low sewage charging standards. Among them, unreasonable pricing of carbon emission right trading is the core problem of researching carbon emission right market [1]. Enhancing carbon pricing mechanisms through a practical method for valuing carbon emission rights, while advancing ecological civilization, remains a critical challenge for carbon trading. The practical significance in the field of valuation of carbon emission rights is extensive and far-reaching, which has a significant impact on the behavioral decisions of various stakeholders.

2. Literature Review

2.1 The Concept of Carbon Credits

At present, academics and the industry have basically reached a consensus on the asset attributes of carbon emission rights, but there is still a big controversy over its specific classification. According to Qiao Haisu and Liu Xiaoli, carbon emission rights not only have the characteristics of financial assets, but also have the attributes of financial functions, especially in its "quasi-monetization" characteristics, which is particularly obvious [2]. Ding Ding and Pan Fangfang analyzed carbon emission right from the legal perspective, they think that carbon emission right has the dual attributes of property right and environmental right, and these two attributes are interrelated and dialectical unity [3]. Yuan Zeming and Li Yuanzhen proposed that carbon emission rights should be regarded as intangible assets for accounting measurement from the perspective of accounting [4]. Zhao Yanfeng et al. proposed that the difference in the purpose of holding is the key factor to distinguish the attributes of carbon emission rights [5]. Ren Hongtao analyzes from the perspective of civil code and believes that carbon emission right transaction is essentially a kind of data transaction, so its legal attribute should be defined as "data property right" [6]. Liu Mingming points out that it is inappropriate to interpret carbon emission right as a single private or public right, and argues that carbon emission right is an environmental right with both public and private attributes [7]. Li Junyi suggests that the essence of carbon emission right assets is a

public resource right granted by the state, and this right can be measured by fair value [8]. Feng Tian's study found that there is a strong correlation and long-term cointegration between carbon futures and carbon spot, which further confirms that carbon emission right assets have strong option properties [9].

2.2 Valuation of Carbon Credits

In the field of carbon emission rights valuation, scholars have developed a variety of methods to provide in-depth analysis and empirical research. These methods include the protection cost method [10], which evaluates the replacement cost of carbon emission rights by considering factors such as carbon neutrality, clean energy development, and carbon capture and sequestration; the income method [11], which focuses on the income potential of corporate carbon emission rights and proposes improvements to enhance the accuracy of the valuation [12,13]; the market method, which reduces subjective judgments and improves objectivity through the introduction of fuzzy correlation theories and other key factors such as trading time, industry and region. The market approach, which reduces subjective judgment and increases objectivity by introducing fuzzy correlation theory and other key factors such as trading time, industry, and region [14]; the real options approach, which uses the option mechanism and the binomial pricing model to assess the potential cash flows of carbon emission rights [15]; the Monte Carlo simulation approach, which simulates the future trading price of carbon emission rights by predicting volatility; and the shadow price approach, which uses the transcendental logarithmic production function and the SBM dyadic programming model to compute a shadow price for carbon emission rights, as well as to calculate a shadow price for carbon emission rights. emission rights' shadow price and compares it with the market trading price [16]; and the Putty-Clay Vintage model, which combines the shadow price concept with an improved production function to calculate the cost of carbon emission rights and validates the model with a case study. These valuation techniques provide a scientific basis for pricing carbon emission rights and help the market reflect their value more accurately [17].

2.3 Factors Affecting Carbon Credits Value

Chen and Wang concluded that under the joint influence of policies and institutions, carbon allowances issued by the government are the most critical factor affecting the trading price [18]. Blyth and Bunn considered policy, market and technology factors in a comprehensive manner in a study for the EU's carbon price, and found that when these risks were analyzed jointly, policy uncertainty may significantly increase the risk of carbon pricing. If the emission reduction target is raised from 20% to 30%, the impact of uncertainty caused by policy on the carbon price will be relatively weaker, while market factors will dominate the movement of the carbon price more at this point [19]. Zhao and Hu (2016) found that policy factors are the main influencing factors of carbon trading price.

3. Theoretical Foundation

3.1 The Theory of Externalities (math.)

The theory of externalities is a concept in economics that describes the effects of an economic agent's behavior on other agents not directly involved in the activity that are not compensated or charged for by the market. This theory distinguishes between positive externalities (e.g. knowledge diffusion) and negative externalities (e.g. environmental pollution). In the valuation of carbon credits, the application of the theory of externalities is reflected in the internalization of the negative externality of greenhouse gas emissions through market mechanisms, so that the cost of emissions is reflected in the price. For example, through a carbon tax or emissions trading, polluters must pay a cost for their emissions behavior, thus incentivizing emissions reductions.

3.2 Alternative Theory

Substitution theory in asset valuation emphasizes the effect of the existence of substitutes in the marketplace on the value of an asset [20]. The idea underlying this theory is that a rational consumer making an investment decision would tend to choose a lower-cost alternative asset unless there are certain non-price factors that make the asset being valued uniquely valuable. In the case of carbon credits valuation, this means that valuers need to consider other alternative

mitigation measures or carbon credits available in the marketplace, and the price and availability of these alternatives directly affects the value of a given carbon credit.

3.3 Polluter Pays Principle (PPP)

The polluter pays principle (PPP) is a core principle of environmental management that asserts that the costs of polluting activities should be borne by polluters. This principle is based on the theory of externalities, which requires polluters to bear the costs of reducing pollution emissions and the damage they cause to the environment and health. In carbon credits assessment, this principle is reflected in the fact that emitting companies must pay the corresponding costs for the greenhouse gases they emit, thus incentivizing them to seek more efficient emission reduction strategies.

3.4 Linear Programming and Lagrange Multipliers

The shadow price model has its origins in the study of linear programming problems, which use Lagrange multipliers to represent the marginal value of adding one unit of a resource under conditions of resource scarcity. In the context of carbon credits, the Lagrange multiplier can be interpreted as a shadow price, i.e., the marginal value of adding one unit of carbon credits under aggregate carbon emission controls.

4. Model Building

4.1 Carbon Market Profile

China's carbon market has launched eight pilot carbon trading markets between 2013 and 2016, covering the regions of Beijing, Tianjin, Shanghai, Chongqing, Hubei, Guangdong (Guangzhou), Shenzhen and Fujian, and encompassing a wide range of sectors, including power, industry, construction and transportation. The government allocates carbon allowances to enterprises free of charge based on their environmental protection and energy efficiency standards. These allowances are determined based on a comprehensive assessment of the enterprise's production scale, energy consumption and emission reduction potential. When an enterprise's actual carbon emissions are lower than the carbon allowances it has received, the remaining carbon allowances become a valuable asset,

and the enterprise can choose to sell these remaining carbon allowances in the carbon trading market to realize the realization of carbon assets.

In July 2021, the national carbon emissions trading market started online trading from the power generation industry, and has now included 2,257 key emission units, covering about 5.1 billion tons of carbon dioxide emissions annually, accounting for more than 40% of national carbon dioxide emissions, making it the largest market in the world in terms of the amount of greenhouse gas emissions covered. Through these pilots, China has accumulated valuable experience, laying the foundation for the officially launched national carbon emissions trading market. The layout of the national carbon market focuses mainly on the power sector, which covers a total annual CO₂ emissions of about 4.5 billion tons. Within this market, 2,162 key emission units have been included, thus making it the carbon market with the widest coverage of greenhouse gas emissions globally. Detailed data shows that by the end of 2023, the total trading volume of the national carbon emissions trading market has exceeded 440 million tons, and the corresponding trading volume has reached about RMB 24.9 billion. The market activity has gradually increased with the passage of time. The turnover of the second compliance cycle (2021 to 2022) has increased by 89% compared with the first compliance cycle (2019 to 2020). Enterprises' initiative to participate in transactions showed a significant upward trend, especially in the second fulfillment cycle, the proportion of enterprises actually participating in transactions rose to 82% of the total number of transactions, an increase of nearly 50% compared to the first fulfillment cycle.

In January 2024, the national voluntary greenhouse gas emission reduction trading market was formally launched, encouraging broad participation by the whole society, and interconnecting with the mandatory carbon market through the quota clearing and offsetting mechanism, together constituting the national carbon market system. In addition, on April 24, 2024, the closing price of the national carbon emission rights trading market exceeded 100 yuan per ton for the first time, showing that the green financial attributes of carbon emission rights have gradually gained

market recognition. The development of China's carbon market not only promotes the green transformation and sustainable development of the local economy, but also contributes to the innovation of the global carbon market mechanism.

4.2 Shadow Price

The shadow price model originated from the study of linear programming problems, which is based on the scarcity of resources and comprehensively reflects the external economic effects of scarce assets. Carbon emission right, as a right to utilize the environmental capacity, has obvious scarcity, and with the continuous depletion of environmental resources, this scarcity becomes more significant, which is in line with the application scenario of the shadow price model. Calculating the shadow price of carbon emission rights is actually assessing the price per unit of carbon emission rights to achieve its optimal utilization in the context of total carbon dioxide emission control. This price is not a trading price in the market, but an estimated value based on the contribution of carbon emissions in the production process and environmental impacts, which reflects the true or intrinsic value of carbon emissions when the optimal allocation of resources is achieved [21].

Measuring the value contribution of carbon emissions based on the production function is the basis for constructing the shadow price model of carbon emission rights, and the core idea of the Cobb-Douglas production function is that "the real price of a production factor depends on its marginal output", which is mainly analyzed through the marginal analysis, and the study of the marginal output of the production factor, the marginal rate of substitution, the output elasticity and the substitution elasticity, and so on, and elasticity of substitution. In this paper, carbon emission right is regarded as an important and scarce production factor, which generates income together with fixed assets and labor inputs, and the marginal productivity of production factors is calculated through the traditional Cobb-Douglas production function, and its formula is:

$$\ln Y = \alpha_0 + \alpha_1 \ln K + \alpha_2 \ln L + \alpha_3 \ln E + \varepsilon \quad (1)$$

where Y denotes regional GDP; K denotes fixed asset input; L denotes labor input; E

denotes carbon dioxide emissions; and ϵ denotes the random disturbance term. Since carbon emitting enterprises are mainly concentrated in industrial enterprises (i.e., the secondary industry), this paper uses the regional economic value added of the secondary industry to replace the overall regional GDP.

The super logarithmic function is a square reflective surface model from the structural point of view, which has the advantages of easy estimation and inclusiveness, and can effectively reflect the interactions between the factors and the differences in technological progress generated by the inputs of the factors, so this paper adopts the super logarithmic production function to improve the traditional Cobb-Douglas production function, and obtains the formula:

$$M = \varepsilon + \alpha_E \ln E_t + \alpha_K \ln K_t + \alpha_L \ln L_t + \alpha_{KE} \ln K_t \ln E_t + \alpha_{LE} \ln L_t \ln E_t + \alpha_{KE} \ln K_t \ln E_t + \alpha_{EE} (\ln E_t)^2 + \alpha_{LL} (\ln L_t)^2 + \alpha_{KK} (\ln K_t)^2 \quad (2)$$

Where $\ln Y_t$ is the logarithm of GDP in year t ; $\ln K_t$ is the logarithm of fixed asset inputs in year t ; $\ln L_t$ is the logarithm of labor inputs in year t ; $\ln E_t$ is the logarithm of carbon emissions in year t ; and ϵ is the random disturbance term.

$$\frac{dY/Y}{dE/E} = \frac{d \ln Y_t}{d \ln E_t} = \alpha_E + \alpha_{KE} \ln K_t + \alpha_{LE} \ln L_t + 2\alpha_{EE} \ln E_t \quad (3)$$

By shifting the terms in the above equation, the value of the regional contribution per unit of carbon emission, $P(e)$, can be expressed as follows

$$P_e = \frac{dY}{dE} = \frac{Y_t}{E_t} \times (\alpha_E + \alpha_{KE} \ln K_t + \alpha_{LE} \ln L_t + 2\alpha_{EE} \ln E_t) \quad (4)$$

4.3 Shadow Price Construction

Based on the value contribution of carbon emissions, the shadow price model of carbon emission rights is further constructed using a linear programming function, which needs to satisfy the following two assumptions for the construction of the model:

Assumption 1: The region has perfect carbon emissions trading market conditions, and the regional government based on the carbon emission reduction target to determine the total amount of carbon emissions in the region in the current year Q , in order to meet the scarcity of carbon emission rights, the region has n emission control enterprises.

Assumption 2: The yield of output value of the

region is A_i , the annual output value is B_i , the output value is proportional to the carbon emissions, and the relevant ratio coefficient is set to be r , i.e., $r = \text{carbon emissions/annual output value}$, and r is the intensity of carbon emissions.

Therefore, the model takes the total control of carbon dioxide emissions and profit maximization under paid allocation as the objective function and carbon emissions as the constraints, and the model is constructed as follows according to the assumed conditions:

$$\max V = \sum_{i=1}^n (A_i \times B_i), \sum_{i=1}^n Q_i \leq Q, \quad (5)$$

$$L = \sum_{i=1}^n (A_i \times \frac{Q_i}{r}) + \lambda (Q - \sum_{i=1}^n Q_i),$$

$$\frac{\partial L}{\partial Q} = \sum_{i=1}^n \frac{A_i}{r} - \lambda n = 0, \lambda = \frac{1}{n} \sum_{i=1}^n \frac{A_i}{r} = \frac{\bar{A}}{r}$$

V in the above equation represents the highest output value of the region, and the Lagrange multiplier method is used to take the partial derivation of the constraint function, resulting in the Lagrange multiplier (λ), which is the shadow price of the region. It represents the value of a unit of carbon emission right to realize its optimal use under the total carbon emission right control, when the price of carbon emission right is higher than the shadow price, the cost of using a unit of carbon emission right is lower than its benefit, and the expansion of the production scale is conducive to the improvement of the overall benefit, and vice versa.

The carbon emission right market is divided into primary market and secondary market according to its functional position, corresponding to the two major links of carbon emission right allocation and trading, respectively. The shadow price PE calculated by the shadow price model only reflects the degree of scarcity of the resource and its relationship with the economic efficiency, which is a kind of static allocation value, and it can't reflect the optimal price for the dynamic allocation of the resource in different periods, and it can only reflect the opportunity cost (i.e., carbon emission reduction cost) of environmental management of carbon emission right. Environmental governance opportunity cost (i.e., carbon emission reduction cost). UI believes that the shadow price of carbon emission rights can be used as a basis for the government to price the initial

allocation of carbon emission rights, and also improve the pricing efficiency of the carbon trading secondary market. Therefore, the author believes that the shadow price of carbon emission rights can be used as the basis for the initial carbon allowance pricing in the primary market. As for the secondary market of carbon emission rights, the price of carbon emission rights is intertwined with more complex economic relations, mainly regulated by the market mechanism, and its potential market value should not be ignored.

5. Case Studies

5.1 Background of Beijing Carbon Emission Rights Market

As one of the first pilot cities for carbon emissions trading in China, Beijing, since the official launch of the carbon market in November 2013, has established a market system with the participation of nearly 1,300 key emission units in eight industries, including power generation, petrochemicals and services, covering about 50 percent of Beijing's carbon emissions. The Beijing carbon market has launched a variety of trading products such as Beijing Carbon Allowance (BEA), Beijing Green Travel Emission Reduction (PCER), and Beijing Forestry Carbon Exchange (FCER), etc. As of the end of 2023, the market's total turnover reached 3.93 million tons, with a turnover of RMB 378 million yuan, and an average price of RMB 96.32 yuan per ton, which is the highest in all pilot carbon markets. The Beijing Municipal Bureau of Ecology and Environment is responsible for managing and regulating carbon emissions trading and has issued relevant management measures. The Beijing carbon market has accumulated rich experience in system innovation, market trading rules formulation, carbon financial innovation, etc. It has initially established a regional carbon emission right trading market with a perfect system, standardized market, active trading and strict supervision, and effectively utilized the market mechanism to promote energy conservation and emission reduction. In addition, Beijing has issued the Administrative Measures for Carbon Emission Offsetting in Beijing Carbon Emission Right Trading Market, which further standardizes the work of carbon emission offsetting and

provides policy support and market mechanism for realizing carbon emission reduction targets.

5.2 Data Processing

In this paper, the relevant data from 2000 to 2021 are selected as the basis for analysis and measurement, in which the relevant data of GDP, fixed asset investment and employed population are obtained by consulting the website of Beijing Municipal Bureau of Statistics, and the data of carbon emissions from 2000 to 2023 are obtained by CEADS database. As shown in Figure 1, after curve fitting analysis of the data, it is found that the polynomial function has the optimal fit.

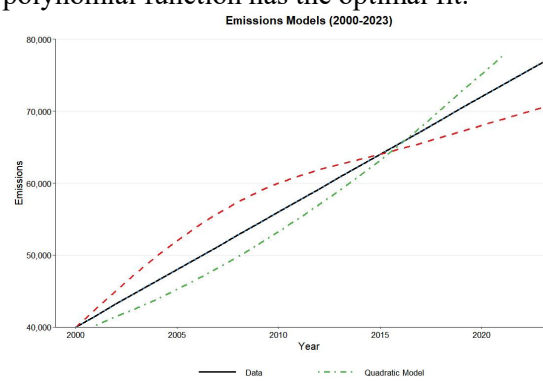


Figure 1. Fitted Trend Plot

Exponential model: with initial loop values in r set to $a = 21402$, $b = 0.035$.

Secondary model:

$$E = 31182938.5 \times \text{Year}^2 - 33004.888 \times \text{Year} + 8.7179 \quad (6)$$

Linear Models:

$$E = -4050440.17 \times \text{Year} + 2044.83 \quad (7)$$

To verify the model's goodness of fit for carbon emissions, we selected data from 2000 to 2021 and made predictions using exponential, quadratic, and linear models. Table 1 presents a comparison of the predicted values from these models with the actual carbon emissions, aiming to identify the model with the best fit, and the results are shown in Table 1.

Based on the fitting results, the polynomial function was found to have the best fit.

To reduce the heteroskedasticity of the data, it is necessary to logarithmize the data, and the results of carbon emission data from 2000 to 2021 are shown in Table 2. The rest of the parameters need to be statistically analyzed and the regression test is significant before substituting into the formula: and the results are shown in Table 2.

Table 1. Comparison of Predicted and Actual Values

	Year	Actual Emissios	Exponential Model	Quadratic Model	Linear Model
1	2021	82116.44	84417.05	82771.62	82161.46
2	2020	81295.20	81632.20	80552.46	80116.63
3	2019	78445.38	78939.21	78350.73	78071.79
4	2018	76633.84	76335.07	76166.43	76026.96
5	2017	74811.04	73816.84	73999.57	73982.13
6	2016	71790.48	71381.68	71850.14	71937.30
7	2015	68943.00	69026.85	69718.14	69892.47
8	2014	67022.84	66749.71	67603.58	67847.64
9	2013	65214.60	64547.69	65506.45	65802.81
10	2012	62936.96	62418.32	63426.75	63757.98
11	2011	60888.34	60359.19	61364.49	61713.15
12	2010	58281.00	58367.98	59319.66	59668.32
13	2009	58004.06	56442.47	57292.26	57623.49
14	2008	55540.24	54580.48	55282.30	55578.66
15	2007	54033.48	52779.91	53289.77	53533.83
16	2006	52581.88	51038.74	51314.67	51489.00
17	2005	49341.00	49355.02	49357.01	49444.17
18	2004	47193.60	47726.83	47416.78	47399.34
19	2003	46248.40	46152.36	45493.98	45354.51
20	2002	43173.24	44629.83	43588.62	43309.68
21	2001	40866.84	43157.53	41700.69	41264.85
22	2000	39834.40	41733.80	39830.19	39220.02

Table 2. Logarization

Year	dq	Emissions	lnEt
2021	Beijing	82116.44	11.31589352
2020	Beijing	81295.2	11.30584225
2019	Beijing	78445.38	11.27015787
2018	Beijing	76633.84	11.24679403
2017	Beijing	74811.04	11.22272075
2016	Beijing	71790.48	11.18150716
2015	Beijing	68943	11.14103536
2014	Beijing	67022.84	11.11278874
2013	Beijing	65214.6	11.08543865
2012	Beijing	62936.96	11.04988887
2011	Beijing	60888.34	11.01679697
2010	Beijing	58281	10.97303142
2009	Beijing	58004.06	10.96826829
2008	Beijing	55540.24	10.92486308
2007	Beijing	54033.48	10.89735913
2006	Beijing	52581.88	10.87012685
2005	Beijing	49341	10.80651066
2004	Beijing	47193.6	10.76201357
2003	Beijing	46248.4	10.74178215
2002	Beijing	43173.24	10.67297614
2001	Beijing	40866.84	10.61807426
2000	Beijing	39834.4	10.59248614

5.3 Shadow Price Assessment

R was used to analyze the principal components of each element, as shown in

Table 3, principal component 1 yielded an eigenvalue of 8.5326, with a variance contribution rate of 94.81%, the model fit was high, and the residuals were autocorrelated,

which indicated that carbon emissions could be used as an important principal component indicator.

To determine the importance of carbon emissions among various indicators, we conducted a principal component analysis on the relevant elements, and the results are shown in Table 3.

Table 3. Analysis of the Results of the Principal Components

Name	Principal Component 1	Score factor	W
characteristic root	8.5326		
variance explained rate	94.81%		
lnEt	0.3263	0.3263	10.88%
lnKt	0.3237	0.3237	10.79%
lnLt	0.3409	0.3409	11.37%
lnLtlnKt	0.3392	0.3392	11.31%
lnLtlnEt	0.3409	0.3409	11.37%
lnKtlnEt	0.3383	0.3383	11.28%
lnEt	0.3254	0.3254	10.85%
lnLt	0.3238	0.3238	10.80%
lnKt	0.3406	0.3406	11.36%

The principal component expression is derived based on the magnitude of principal component contribution in the following equation:

$$M = 0.108 \ln E_t + 0.108 \ln K_t + 0.114 \ln L_t \quad (8)$$

$$+ 0.113 \ln K_t \ln L_t + 0.114 \ln L_t \ln E_t + 0.113 \ln K_t \ln E_t + 0.109 (\ln E_t)^2 + 0.108 (\ln L_t)^2 + 0.114 (\ln K_t)^2 \quad (9)$$

$$\ln Y_t = \varepsilon + \alpha M$$

The analysis of the results of the F-test can be obtained that the significance P-value is 0.000***, presenting significance at the 0.001 level, rejecting the original hypothesis that the regression coefficient is 0, and the model basically meets the requirements. Since the VIF are less than 5, the model has no multicollinearity problem and the model is well constructed. The regression equation is as follows:

$$\ln Y_t = -3.99 + 0.13M \quad (10)$$

Based on the results of the principal component analysis, we constructed a regression model and performed diagnostics. Table 4 lists the estimates, standard errors, statistics, and p-values of the parameters in the regression equation. The purpose of this table is to verify the significance of the model and provide a basis for calculating the contribution of carbon emissions to output, and the results are shown in Table 4. As shown in Figure 2,

the regression diagnostic plot shows that the model fits well, with evenly distributed residuals, which further verifies the reliability of the model.

Table 4. Regression Results

term	estimate	std. error	statistic	p. value
(Intercept)	-3.99008	0.649615	-6.14222	5.31E-06
M	0.13231	0.006342	20.86317	4.81E-15

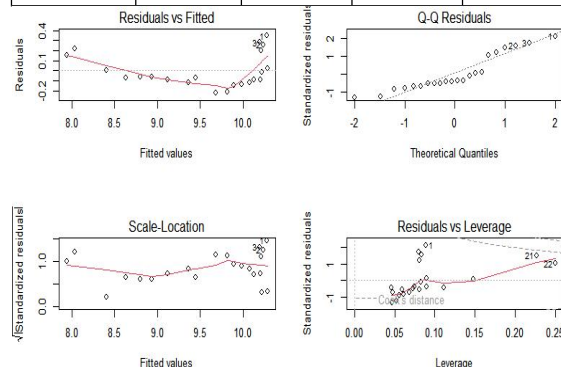


Figure 2. Regression Diagnostic Chart

Substituting M gives:

$$\ln Y_t = -3.99 + 0.13 \times (0.108 \ln E_t + 0.108 \ln K_t + 0.114 \ln L_t + 0.113 \ln K_t \ln L_t + 0.114 \ln L_t \ln E_t + 0.113 \ln K_t \ln E_t + 0.109 (\ln E_t)^2 + 0.108 (\ln L_t)^2 + 0.114 (\ln K_t)^2) \quad (11)$$

Substituting the above Parameters into the above equation, the contribution PE per unit of carbon emission output value from 2000 to 2021 is obtained as follows:

$$P_E = \frac{dY}{dE} = \frac{Y_t}{E_t} \times (-0.01404 + 0.01469 \ln K_t + 0.01482 \ln L_t + 0.02834 \ln E_t) \quad (12)$$

Using the regression results as a foundation, we calculated the annual contribution of carbon emissions to output in Beijing from 2000 to 2021. Table 5 presents the specific values for each year, reflecting the interannual variation trends of carbon emissions' contribution to output. This provides data support for the subsequent calculation of the shadow price, and the results are shown in Table 5.

$$P_E = \frac{dY}{dE} = \frac{Y_t}{E_t} \times (-0.08 - 0.00864 \ln K_t - 0.00912 \ln L_t - 0.0174 \ln E_t) \quad (13)$$

Table 5. Contribution of Output Value

year	dq	PE
2021	Beijing	3530.63078
2020	Beijing	3118.887781
2019	Beijing	3182.075319
2018	Beijing	3041.849632
2017	Beijing	2816.762715
2016	Beijing	2649.583421

2015	Beijing	2525.103857
2014	Beijing	2396.889973
2013	Beijing	2266.789341
2012	Beijing	2108.074606
2011	Beijing	1962.752684
2010	Beijing	1780.03098
2009	Beijing	1536.025348
2008	Beijing	1457.838693
2007	Beijing	1318.552934
2006	Beijing	1081.211556
2005	Beijing	975.9138867
2004	Beijing	887.8763247
2003	Beijing	759.2361205
2002	Beijing	692.9905947
2001	Beijing	616.4202235
2000	Beijing	534.9969805

Comprehensively accounting for the shadow price of specific carbon emission rights, the accounting results are shown in Table 6. By examining the difference between the carbon market trading price and the shadow price in Beijing from 2016-2021, it is found that the shadow price is lower than the current market trading value, and that the shadow price being lower than the market price may promote emission reduction activities, improve the efficiency of resource allocation, and may have a more positive impact on the environment. and the results are shown in Table 6.

Table 6. Shadow Prices

YEAR	r	PE	shadow price
2021	11.2974	3530.63078	27.1422765
2020	14.1652	3118.887781	21.64733818
2019	13.8415	3182.075319	22.1534877
2018	13.9909	3041.849632	21.91691802
2017	14.8158	2816.762715	20.69663293
2016	15.3865	2649.583421	19.92897101

6. Conclusion

The results of the study show that the shadow price model is an effective tool for assessing the value of carbon emission rights and can reflect the marginal abatement cost of carbon emission rights. In the case of Beijing, the shadow price was found to be lower than the market trading price, which may promote the participation of enterprises in emission reduction, improve the efficiency of resource allocation, and have a positive impact on the environment. However, it also implies that there may be a risk of excessive speculation or imbalance between supply and demand in the market, and that appropriate regulation and

adjustments are needed to ensure the healthy development of the market. In addition, the shadow price can be used as a basis for the government to price the initial allocation of carbon emission rights and improve the pricing efficiency of the secondary carbon trading market. The study emphasizes the importance of improving the valuation methodology of carbon emission rights to realize a scientific and effective carbon trading mechanism, which is of great significance in promoting the healthy development of the carbon trading market and realizing the goal of "double carbon".

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