Optimal Decision-Making Model for Multi-Process and Multi-Component Production in Enterprises

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Abstract: To optimize the cost structure of enterprises, improve decision-making in the production process, and enhance production and quality control decisions, this study comprehensively considers factors such as the procurement cost of finished products, inspection costs, replacement losses, disassembly, and assembly costs. By employing statistical inference, hypothesis testing, genetic algorithms, dynamic programming, and other methods, along with collected data and materials, a three-stage multi-process, multi-component production optimization decision-making model is established, covering component inspection, finished product inspection, and defective product disassembly. A sampling inspection model was implemented using Python programming, enabling enterprises to effectively control component quality. This model provides the optimal decision-making solution for the production process.

Keywords: Optimal Decision-Making Model; Hypothesis Testing; Dynamic Programming; Sampling Inspection Model; Component Quality Control

1. Introduction

This research problem is derived from Problem B of the 2024 Higher Education Society Cup National Undergraduate Mathematical Modeling Competition[1]. It concerns decision-making in the production process. When an enterprise manufactures a product, various types of components must be purchased separately to ensure the quality of the finished product. During production, the components must undergo rigorous quality inspections to guarantee that the product meets the required standards. In assembling the finished product, the quality criteria for the components are critical; if any single component is defective, the assembled product will inevitably be unqualified. Even if all components are qualified, the finished product may still fail to meet the standards.

For defective finished products, the enterprise has two processing options. Option 1: Scrap the defective product immediately, thereby completely avoiding any subsequent processing costs. Option 2: Disassemble the defective product. Although disassembly does not damage the components, it incurs additional disassembly costs. After disassembly, the components can be re-inspected to determine whether they can be recycled and used in producing qualified products.

Considering that both scrapping and disassembly involve costs, enterprises need to optimize their quality inspection and processing strategies to minimize the overall cost while ensuring product quality. Against this backdrop, it is necessary to design a reasonable sampling inspection scheme and to balance the costs of scrapping versus disassembly to develop an optimal decision-making strategy. Consequently, the following four problems are set up in increasing order of difficulty:

For Problem 1, hypothesis testing is employed to decide whether to accept a batch of components. For large sample sizes, the binomial distribution can be approximated by a normal distribution, allowing the use of normal distribution-based hypothesis testing. Two testing schemes are designed under confidence levels of 95% and 90%: one to reject the components at a 95% confidence level and one to accept the components at a 90% confidence level, respectively determining whether the defect rate exceeds or does not exceed the nominal value. A sample size calculation formula is also provided to ensure that the estimation error remains within acceptable limits, along with specific implementation steps to help enterprises effectively control component quality.

For Problem 2, strategies for component inspection, finished product inspection, and handling defective products are studied. The cost functions for various types of costs are separately formulated, and a genetic algorithm is implemented using Python to search for the optimal combination of decisions that minimizes the total cost.

For Problem 3, a total cost objective function is defined that encompasses multiple costs and fees, and a constraint is established stating that defective products that have not been inspected cannot be disassembled. A dynamic programming method is employed. [2] The production process is divided into three stages: component inspection, semi-finished product inspection and disassembly, and finished product inspection and disassembly. The state of each stage is determined by inspection and disassembly decisions, and the state transition equations are used to calculate the minimum cost.

In Problem 4, the model is constructed by performing sampling inspections to estimate the defect rates of both components and finished products. By approximating the binomial distribution with a normal distribution, the confidence interval for the sample defect rate is calculated. Based on this confidence interval, the estimated defect rate and required sample size are determined. Subsequently, the models for Problems 2 and 3 are adjusted to include factors such as the procurement costs of components and finished products, inspection costs, and the loss incurred from replacing finished products. Finally, the sampling inspection model is implemented using Python, and the cost models for Problems 2 and 3 are re-solved based on the updated defect rates.

2. Construction and Solution of the Optimal Decision-Making Model for Multi-Process and Multi-Component Production

2.1 Establishment and Solution of the Model for Problem 1

This study considers the estimation of the component defect rate $p_0 = 0.10$, assuming that the supplier claims a nominal defect rate of P. Through sampling inspection, the enterprise needs to infer the overall defect rate P based on the number of defective items X in the sample. Therefore, the problem can be described as a hypothesis test:

Null Hypothesis H_0: The defect rate is $p \le p_0$.

Alternative Hypothesis H_1: The defect rate is $p > p_0$.

The results of the sampling inspection follow a binomial distribution $B(n, p)$, where n represents the sample size and X represents the number of defective items in the sample. The sample defect rate $\hat{p} = \frac{X}{n}$ is an unbiased estimator of the overall defect rate. Since the binomial distribution can be approximated by a normal distribution for large sample sizes, a normal distribution is employed for the hypothesis test.

(1) Normal Approximation to the Binomial Distribution

For a large sample size n, the binomial distribution $B(n, p)$ can be approximated by a normal distribution:

$$X \sim N(np, np(1-p)). \tag{1}$$

The distribution of the sample defect rate $\hat{p} = \frac{X}{n}$ is:

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right). \tag{2}$$

The test statistic Z for hypothesis testing can be expressed as:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}. \tag{3}$$

where the Z-value follows the standard normal distribution $N(0,1)$. By calculating the Z-value, we can determine whether the defect rate in the sample differs significantly from the nominal defect rate p_0.

(2) Hypothesis Testing

Based on different confidence levels and decision scenarios, two hypothesis testing schemes are designed:

Scenario 1: Rejecting Components at 95% Confidence Level

At a 95% confidence level, the company aims to determine whether the defect rate exceeds 10%.

This can be achieved using a “one-tailed test (upper-tailed test)”.

“Hypotheses”:

Null hypothesis H_0 : Defect rate $p \le p_0$ (i.e., the defect rate does not exceed the nominal value).

Alternative hypothesis H_1 : Defect rate $p > p_0$ (i.e., the defect rate exceeds the nominal value).

For a 95% confidence level, the critical value from the standard normal distribution is 1.96. Therefore, if the calculated Z-value is “greater than 1.96”, the null hypothesis is rejected, concluding that the defect rate exceeds the nominal value (i.e., $Z > 1.96,$). This can also be translated into a critical threshold for the sample defect rate:

$$\hat{p} > p_0 + 1.96 \times \sqrt{\frac{p_0(1-p_0)}{n}}, \tag{4}$$

When the defect rate $\hat{p}$ in the sample “exceeds this critical value”, the company should reject the batch of components.

Scenario 2: Acceptance of Components at a 90% Confidence Level

At a 90% confidence level, the enterprise aims to determine whether the defect rate does not exceed 10%. This can be addressed using a one-tailed test (lower-tailed test). The hypotheses for the test are as follows:

Null hypothesis H_0 : The defect rate is greater than or equal to 10%.

Alternative hypothesis H_1 : The defect rate is less than 10%.

For a 90% confidence level, the critical value from the standard normal distribution is 1.645. Therefore, if the calculated Z-value is less than 1.645, the null hypothesis is rejected, and it is concluded that the defect rate does not exceed the nominal value. That is: $Z < 1.645$. This can also be expressed in terms of the sample defect rate:

$$\hat{p} < p_0 + 1.645 \times \sqrt{\frac{p_0(1-p_0)}{n}}. \tag{5}$$

If the defect rate in the sample is lower than this critical threshold, the enterprise should accept the batch of components.

The margin of error d refers to the maximum allowable deviation between the sample defect rate and the nominal defect rate as tolerated by the enterprise. The maximum deviation within the confidence interval can be expressed as:

$$d = z_\alpha \cdot \sqrt{\frac{p_0(1-p_0)}{n}}. \tag{6}$$

To ensure that the estimation error does not exceed ddd, the sample size nnn can be derived from the equation above, resulting in the following sample size formula:

$$n = \frac{z_\alpha^2 \cdot p_0(1-p_0)}{d^2}. \tag{7}$$

This formula is used to estimate the required minimum sample size under a given confidence level and allowable margin of error d. Where:

Zα: Critical value from the standard normal distribution corresponding to the chosen confidence level. P: Population (nominal) defect rate. d: Allowable estimation error.

(4) Implementation Plan

The enterprise may implement the inspection procedure according to the following steps:

Determine the Sample Size: Based on the desired confidence level, nominal defect rate, and allowable margin of error, calculate the minimum required sample size using the formula:

$$n = \frac{Z_\alpha^2 \cdot p_0(1-p_0)}{d^2} \tag{8}$$

Conduct Sampling Inspection: Randomly select n components from the supplier’s batch for inspection. Record the number of defective items X, and calculate the sample defect rate:

$$\hat{p} = \frac{X}{n} \tag{9}$$

Perform Hypothesis Testing:

At a 95% confidence level, if $\hat{p} > p_0 + 1.645 \times \sqrt{\frac{p_0(1-p_0)}{n}}$, reject the batch of components.

At a 90% confidence level, if $\hat{p} < p_0 + 1.96 \times \sqrt{\frac{p_0(1-p_0)}{n}}$, accept the batch of components.

2.2 Model Formulation and Solution for Problem Two

2.2.1 Complete cost model

It is now necessary to incorporate the procurement costs C_{b1} and C_{b2} into the model, especially considering that if components are not inspected, defective items may enter the assembly process.

Procurement and inspection costs for components: For component 1 and component 2, the company needs to procure and may choose to inspect them.

Total inspection cost for components:

$$C_{component\ inspection} = C_{component1} + C_{component2} \tag{10}$$

If components are inspected, the company incurs both procurement and inspection costs, and defective products are eliminated;

If components are not inspected, only the procurement cost is incurred, and defective components directly enter the assembly.

Total cost for component 1:

$$C_{component1} = x_1 \cdot N \cdot (C_{b1} + C_{d1}) \cdot (1 - p_1) + (1 - x_1) \cdot N \cdot C_{b1} \tag{11}$$

Total cost for component 2:

$$C_{component2} = x_2 \cdot N \cdot (C_{b2} + C_{d2}) \cdot (1 - p_2) + (1 - x_2) \cdot N \cdot C_{b2} \tag{12}$$

When $x_1 = 1$ or $x_2 = 1$, it indicates the company inspects the components, removing defective ones and keeping only qualified components; the cost includes procurement and inspection expenses.

When $x_1 = 0$ or $x_2 = 0$, components directly enter the assembly stage, and the company only pays the procurement cost, with defective items also entering assembly.

1) Finished Product Inspection Cost

Finished product inspection cost:

$$C_{finished\ product\ inspection} = y \cdot N \cdot C_f . \tag{13}$$

If inspected, defective finished products are removed and do not enter the market. The inspection cost is proportional to the number of finished products inspected.

2) Replacement Loss from Finished Products

If finished products are not inspected, defective items enter the market, leading to return and replacement losses:

$$C_{replacement\ loss} = (1 - y) \cdot N \cdot p_f \cdot C_r . \tag{14}$$

3) Disassembly Cost

If defective finished products are chosen to be disassembled, the disassembly cost is:

$$C_{disassembly\ cost} = z \cdot N \cdot C_t . \tag{15}$$

Disassembly cost is proportional to the number of defective finished products and the disassembly fee.

4) Assembly Cost

Regardless of inspection, a fixed assembly cost is incurred during the assembly stage:

$$C_{assembly\ cost} = N \cdot C_a . \tag{16}$$

Whether or not components and finished products are inspected, assembly costs remain fixed.

5) Recovery Revenue

When defective finished products are disassembled, component of the component costs can be recovered. It is assumed that the recovery price of components is 50% of the purchase price, and the recovery revenue is:

$$C_{recovery\ revenue} = z \cdot N \cdot \sum_{i=1}^{2} 0.5 \cdot C_{bi} \cdot (1 - p_i) . \tag{17}$$

Where z indicates whether the defective finished product is disassembled, and C_{bi} is the purchase price of the component.

Objective Function:

The objective of this study is to minimize costs, i.e.:

$$Minimize Z = C_{component\ inspection} + C_{assembly\ cost} + C_{finished\ product\ inspection} + C_{replacement\ loss} + C_{disassembly\ cost} - C_{recovery\ revenue} \tag{18}$$

Combining all components, the total cost model for the company is:

$$\begin{aligned} Minimize Z = & \left[x_1 \cdot N \cdot (C_{b1} + C_{d1}) \cdot (1 - p_1) + (1 - x_1) \cdot N \cdot C_{b1}\right] + \left[x_2 \cdot N \cdot (C_{b2} + C_{d2}) \cdot (1 - p_1) + (1 - x_1) \cdot N \cdot C_{b1}\right] \\ & + \left[x_2 \cdot N \cdot (C_{b2} + C_{d2}) \cdot (1 - p_2) + (1 - x_2) \cdot N \cdot C_{b2}\right] + y \cdot N \cdot C_f + N \cdot C_a + (1 - y) \cdot N \cdot p_f \cdot C_r + z \cdot N \cdot C_t \\ & - z \cdot N \cdot \sum_{i=1}^{2} 0.5 \cdot C_{bi} \cdot (1 - p_i) \end{aligned} \tag{19}$$

Constraints: Decision variables x_1, x_2, y, and z are binary variables:

$$x_1, x_2, y, z \in \{0,1\}. \tag{20}$$

the production quantity N must be greater than 0:

$$N > 0. \tag{21}$$

2.2.2 Genetic algorithm optimization

To find the optimal decision combination for each scenario, the company adopts a genetic algorithm [3] for solution. The flowchart of the genetic algorithm is shown in Figure 1.

Step 1: Population Initialization – Generate the initial population, with each individual representing a possible decision combination (x_1, x_2, y, z).

Step 2: Fitness Evaluation – Calculate the total cost for each individual. The fitness value is the negative of the total cost; the goal is to maximize fitness (i.e., minimize cost).

Step 3: Selection – Based on fitness values, use a tournament selection strategy to select individuals with higher fitness for the next generation.

Step 4: Crossover – Perform crossover on selected parent individuals to generate offspring.

Step 5: Mutation – Perform mutation on offspring individuals to ensure population diversity.

Step 6: Termination Condition – The algorithm terminates based on the number of iterations or early stopping criteria, and outputs the optimal solution.

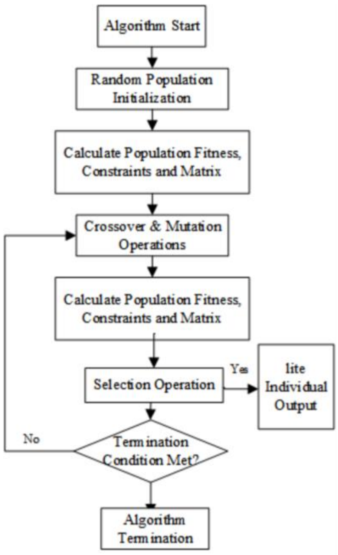

Figure 1. Genetic Algorithm Flow Chart

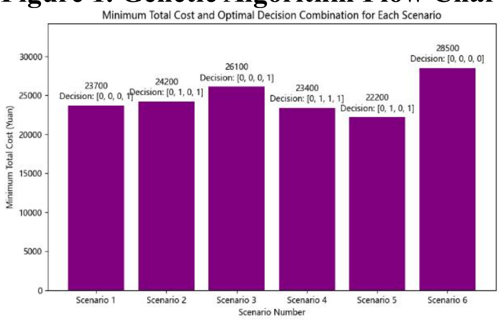

Figure 2. Minimum Total Cost and Optimal Decision Combination Chart for Each Situation

2.2.3 Results and analysis

A genetic algorithm was implemented using Python to optimize the six scenarios involved in the company's production.[4] The optimal plans for the six scenarios were obtained, and the optimal solutions and corresponding costs were visualized, as shown in Figure 2 above.

For each scenario, specific decision plans and corresponding minimum total costs are provided. Table 1 presents the optimal decisions for each scenario (whether to inspect component 1, component 2, and finished products, and whether to disassemble defective finished products), as well as related metric results.

2.2.4 Decision basis and analysis

The company needs to optimize decisions across four stages in the production process:

(1) Whether to inspect component 1 or component 2

Decision basis: If the inspection cost is high and the defect rate is low, the company may choose not to inspect, thereby reducing inspection costs. Conversely, if the defect rate is high and inspection cost relatively low, the company may opt for inspection to reduce defective rate after assembly.

(2) Whether to inspect the assembled finished product

Decision basis: If the defect rate is high, the company may inspect finished products to reduce return loss. If inspection costs are high and the defect rate is low, they may choose not to inspect and send directly to market.

(3) Whether to disassemble defective finished products

Decision basis: If the disassembly cost is high, the company may discard the defective products. If disassembly costs are low, they may opt to disassemble and reassemble.

(4) Whether to replace defective products returned by users

Decision basis: Replacement is the final step in handling defective products. Unconditional replacement causes replacement losses, so defective finished products should be prevented from reaching the market to reduce replacement costs.

Decision Analysis:

Components Inspection Decision: When the defect rate of component 2 is high, inspecting component 2 can effectively reduce defective products entering the market (e.g., scenario 2 and 5). In some cases (e.g., scenario 1 and 3), when the defect rate is low, the company opts not to inspect components to save on inspection cost.

Table 1. Optimal Decisions and Corresponding Metric Results for Each Scenario

Scenario ID	Component 1 Inspection	Component 2 Inspection	Product Inspection	Disassemble Defective Product	Minimum Cost (Yuan)	Decision Basis
1	0	0	0	1	23,700	Low defect rate of components and products; disassembling products yields recovery revenue
2	0	1	0	1	24,200	High defect rate in component 2, low

						product defect rate; disassembly reduces cost
3	0	0	1	1	26,100	High product defect rate; disassembly recovers value and reduces loss
4	0	1	1	1	23,400	High defect rate in component 2 and finished products; inspection reduces loss; disassembly effective
5	0	1	0	1	22,200	High defect rate in component 2, low in finished products; high disassembly return
6	0	0	0	0	28,500	Low defect rate; high cost of inspection/disassembly; not inspecting reduces cost

Finished Product Inspection Decision: When the product defect rate is high, inspection significantly reduces replacement loss (e.g., scenario 4). However, when defect rate is low and inspection cost is high (e.g., scenarios 1 and 6), the company may choose not to inspect and send products directly to market.

Disassembly Decision: In several cases (e.g., scenarios 1, 2, 3, 4, and 5), the company chooses to disassemble defective products because the recovered component value offsets the disassembly cost, reducing overall production cost. In scenario 6, due to low defect rates in both products and components, disassembly is not chosen.

2.3 Problem 3 Model Establishment and Solution

For Problem 3, the total cost needs to be clarified first. It includes the procurement and inspection costs for components, assembly and inspection costs for semi-finished products, inspection costs for finished products, and the replacement losses and disassembly costs due to defective items. Additionally, an important constraint must be considered: only finished and semi-finished products that have been inspected and confirmed as defective can be disassembled; uninspected defective products cannot be disassembled. To solve this problem, dynamic programming[5] needs to be used. This is because the production process can be divided into multiple decision stages, and each stage's decision will impact the subsequent stages' state and cost. Dynamic programming breaks down large problems into smaller ones and uses the results of these smaller problems to construct the final solution, making it ideal for solving multi-stage decision problems.

2.3.1 Total cost

(1) Components Procurement and Inspection Costs

$$C_{component} = \sum_{i=1}^{8} N \cdot \left(x_i \cdot \left(C_{bi} + C_{di}\right) \cdot \left(1 - p_i\right) + \left(1 - x_i\right) \cdot C_{bi}\right) \quad (22)$$

Where x_i indicates whether component i is inspected, p_i is the defective rate, C_{bi} is the procurement cost, and C_{di} is the inspection cost.

(2) Semi-Finished Product Assembly and Inspection Costs

$$C_{semi-finished\ product} = \sum_{j=1}^{3} N \cdot \left(C_{aj} + y_i \cdot C_{fi}\right). \quad (23)$$

Where C_{a_j} is the assembly cost of semi-finished product j, C_{fj} is the inspection cost, and y_j indicates whether it is inspected.

(3) Finished Product Inspection Costs, Replacement Losses, and Disassembly Costs

$$C_{finished\ product} = N \cdot \left(z \cdot C_{f_f} + (1 - z) \cdot C_r + t_f \cdot C_{t_f}\right). \quad (24)$$

Where C_{f_f} is the finished product inspection cost, C_r is the replacement loss, and C_{t_f} is the disassembly cost.

(4) Disassembly Recovery Profit

If disassembling semi-finished products or finished products is selected, component of the cost of the components can be recovered. When disassembling semi-finished products, the recovery cost is 50% of the unit price of components. When disassembling finished products, component of the component cost can be recovered.

$$C_{disassembly\ recovery\ profit} = \sum_{j=1}^{3} t_j \cdot \left(N \cdot C_{t_j} - 0.5 \cdot \sum_{i \in P_j} C_{b_i}\right) + t_f \cdot \left(N \cdot C_{t_f} - 0.5 \cdot \sum_{i=7}^{8} C_{b_i}\right). \quad (25)$$

Where P_j represents the set of components involved in semi-finished product j, and t_j indicates whether disassembly is performed.

Total Cost:

$$C_{total} = C_{component} + C_{semi-finished\ product} + C_{finished\ product} + C_{disassembly\ recovery\ profit} \quad (26)$$

Based on the above, the cost minimization model is:

$$C_{total} = \sum_{i=1}^{8} N \cdot \left(x_i \cdot \left(C_{bi} + C_{di} \right) \cdot \left(1 - p_i\right) + \left(1 - x_i\right) \cdot C_{bi} \right) + \sum_{j=1}^{3} N \cdot \left(C_{aj} + y_i \cdot C_{fi} \right)$$
$$+ N \cdot \left(z \cdot C_{f_f} + \left(1 - z\right) \cdot C_r + t_f \cdot C_{t_f} \right) + \sum_{j=1}^{3} t_j \cdot \left(N \cdot C_{t_j} - 0.5 \cdot \sum_{i \in P_j} C_{b_i} \right) + t_f \cdot \left(N \cdot C_{t_f} - 0.5 \cdot \sum_{i=7}^{8} C_{b_i} \right). \quad (27)$$

Constraints: If defective items are not inspected, they cannot be disassembled. That is:

$$t_j \le y_j \text{'} t_f \le z. \quad (28)$$

Only defective finished products and semi-finished products that have been confirmed after inspection can be disassembled.

2.3.2 Dynamic programming solution

(1) Applicability of Dynamic Programming

Dynamic programming is a solution method suitable for multi-stage decision problems.[6]-[7] The production process of an enterprise can be divided into multiple decision stages, where each stage's decision impacts the state and cost of subsequent stages. Dynamic programming decomposes the problem into sub-problems and uses the results of these sub-problems to construct the final solution, making it well-suited for solving production optimization problems.

(2) Stage Division and State Definition

This problem can be divided into three main stages:

Stage 1: Decide whether to inspect each component, determining the quality of the components entering assembly.

Stage 2: Based on the semi-finished products generated from assembly, decide whether to inspect the semi-finished products and whether to disassemble them, which will affect the quality of the finished products.

Stage 3: Inspect the finished products, and based on the inspection results, decide whether to disassemble them or sell them directly to avoid defective products entering the market and causing losses.

The state of each stage consists of decisions on inspection and disassembly:

$x = \left(x_1, x_2, ..., x_8 \right)$: Decisions on the inspection of 8 components.

$y = \left(y_1, y_2, y_3 \right)$: Decisions on the inspection of 3 semi-finished products.

z_f : Decision on finished product inspection.

$w = \left(w_1, w_2, w_3, w_f \right)$: Decisions on disassembly of semi-finished and finished products.

(3) State Transition Equation

The core of dynamic programming lies in state transitions.[8]-[9] The optimal decision at each stage depends on the state and decisions made at previous stages. For example, in Stage 1, if a component is not inspected, defective components may enter the semi-finished product assembly stage, affecting the subsequent quality and cost of the semi-finished and finished products.

The recursive equation can be expressed as:

$$V\left(t, s\right) = \min\left\{ inspection\ cost + disassembly\ cost + recovery\ revenue \right\}. \quad (29)$$

Where $V\left(t, s\right)$ represents the minimum cost at state s in stage t.

(4) Solution Steps

a. Initialize all states in Stage 1 and calculate the cost of component inspection.

b. Based on the results of component inspections, recursively calculate the cost of semi-finished product inspection and disassembly.

c. Make decisions on finished product inspection and disassembly, and obtain the final total cost and optimal solution.

2.3.3 Results and analysis

Using Python to build the dynamic programming model, and based on the results of dynamic programming, this study obtained the following optimal decision scheme:

Optimal Decision: The optimal combination of decision variables is (0,0,0,0,0,0,0,0,1,1,0,1,1,1,0,0), and the corresponding minimum total cost is 92,000 yuan. Table 2 is an explanation of the decision variables in the combination:

Table 2. Explanation of Decision Variables in the Combination

Decision Stage	Decision Object	Decision Value	Meaning
Component Inspection	Components 1-8	0	No inspection for any components
Semi-Finished Product	Semi-finished Product 1	1	Inspect Semi-finished Product 1

Inspection	Semi-finished Product 2	1	Inspect Semi-finished Product 2
	Semi-finished Product 3	0	Do not inspect Semi-finished Product 3
Finished Product Inspection	Finished Product	1	Inspect Finished Product
Semi-Finished Product Disassembly	Semi-finished Product 1	1	Disassemble Semi-finished Product 1
	Semi-finished Product 2	1	Disassemble Semi-finished Product 2
	Semi-finished Product 3	0	Do not disassemble Semi-finished Product 3
Finished Product Disassembly	Finished Product	0	Do not disassemble Finished Product

Based on this optimal decision variable combination, the study can analyze the meaning of each decision variable in the production process as follows:

Components Inspection Decision: Do not inspect components 1 to 8. This suggests that for this batch of components with a low defect rate, the inspection cost is too high, and the enterprise chooses not to inspect them and directly proceeds with assembly.

Semi-Finished Product Inspection and Disassembly Decision: Inspect semi-finished products 1 and 2, but do not inspect semi-finished product 3. Disassemble semi-finished products 1 and 2, but do not disassemble semi-finished product 3. This indicates that semi-finished products 1 and 2 might have a higher defect rate, and inspection and disassembly are effective methods to reduce the defect rate in subsequent finished products. However, inspecting and disassembling semi-finished product 3 is not economical.

Finished Product Inspection and Disassembly Decision: Inspect finished products and disassemble defective finished products. This decision ensures that defective products do not enter the market, avoiding replacement losses caused by defective products.

Decision Basis and Corresponding Indicators:

Components Inspection: Decision Basis: Defective rate and inspection cost of each component. The defective rate of the components is relatively low (10%), and the inspection cost is high. Therefore, not inspecting them avoids unnecessary costs and proceeds directly to assembly.

Semi-Finished Product Inspection and Disassembly: Decision Basis: The quality and defect rate of semi-finished products after assembly. The inspection and disassembly of semi-finished products 1 and 2 are based on their higher defect rates and the inspection costs after assembly. Inspection and disassembly can effectively reduce the defect rate of finished products. For semi-finished product 3, the cost-benefit ratio of inspection and disassembly is not high, so the decision is to not inspect or disassemble it.

Finished Product Inspection and Disassembly: Decision Basis: Market replacement losses for finished products. Finished product inspection prevents defective products from entering the market and reduces additional losses caused by returns or exchanges. Disassembling finished products recovers component of the component costs, so inspecting and disassembling finished products is the optimal choice.

2.4 Model Construction and Solution for Problem 4

It is assumed that the defective rates of components, semi-finished products, and finished products in Problem 1 and Problem 3 are all estimated through the method of sampling inspection[10]. The enterprise needs to infer the overall defective rate through sampling inspection and re-optimize decisions based on the inspection results. In order to solve the problem based on sampling inspection, this study combines the method used in Problem 1 to recalculate the defective rates in Problem 2 and Problem 3, and then updates the decision model accordingly.

2.4.1 Sampling inspection model

To estimate the defective rates of components and finished products, it is assumed that the enterprise can infer the defective rates by conducting sampling inspections on components, semi-finished products, and finished products, and optimize based on the results. The method of approximating the binomial distribution with the normal distribution is used to infer the confidence interval of the defective rate. [11]The steps for sampling inspection are as follows:

(1) Estimation of sample defective rate

By counting the number of defective items in the sample, the overall defective rate can be inferred. Assuming that the number of defective items in the sample follows a binomial distribution, the confidence interval of the sample defective rate is calculated using the method of approximating the binomial distribution with the normal distribution. According to the given confidence level and error range, the defective rate and sample size can be estimated using the following formula.

Assuming the enterprise requires a 95% confidence interval, 1.96 from the standard normal distribution is used as the critical value of the confidence interval. Let the defective rate be p, and the sample size be n, then the upper and lower bounds of the confidence interval are:

$$p \pm Z \times \sqrt{\frac{p(1-p)}{n}}. \tag{30}$$

Where: $Z = 1.96$ (the standard normal distribution value corresponding to the 95% confidence level), p is the defective rate, and n is the sample size.

(2) Calculation of Sample Size

In order to control the estimation error within the given error range e the sample size n can be calculated using the following formula:

$$n = \frac{Z^2 \cdot p(1-p)}{e^2}. \tag{31}$$

Where: $Z = 1.96$, e is the allowed error range, and p is the initial estimate of the defective rate. The confidence interval is composed of the upper and lower limits p_{lower} and p_{upper} .:

$$p_{lower} = \max\left(0, p - Z \times \sqrt{\frac{p(1-p)}{n}}\right), \tag{32}$$

$$p_{upper} = \min\left(1, p + Z \times \sqrt{\frac{p(1-p)}{n}}\right). \tag{33}$$

The calculated confidence interval represents a 95% probability that the true defective rate is contained within this interval.

Estimated Average Defective Rate

To provide a simplified estimate, the average value of the confidence interval is used as the final defective rate:

$$p_{estimate} = \frac{p_{lower} + p_{upper}}{2}. \tag{34}$$

This estimated average defective rate is used as the input for the defective rate in the cost models and optimization algorithms of Problem 2 and Problem 3, helping the enterprise make reasonable inspection and production decisions.

2.4.2 Modification of problem 2 model

After obtaining the defective rate from the sampling inspection, this study incorporates it into the cost model. Based on the model of Problem 2, the following modifications are required:

(1) Components Procurement and Inspection Costs

For components 1 and 2, the enterprise needs to procure components and may inspect them. If defective components are detected, the non-conforming components are discarded. The cost model is as follows:

$$C_{component1} = x_1 \cdot N \cdot (C_{b1} + C_{d1}) \cdot (1 - p_1) + (1 - x_1) \cdot N \cdot C_{b1}. \tag{35}$$

$$C_{component2} = x_2 \cdot N \cdot (C_{b2} + C_{d2}) \cdot (1 - p_2) + (1 - x_2) \cdot N \cdot C_{b2}. \tag{36}$$

Where p_1 and p_2 are the defective rates of components 1 and 2, x_1 and x_2 indicate whether the components are inspected.

(2) Finished Product Inspection Costs

Finished product inspection can reduce the risk of defective products entering the market. If inspected, defective finished products will be discarded to avoid replacement losses. The finished product inspection cost is as follows:

$$C_{finished\ product\ inspection} = y \cdot N \cdot C_f. \tag{37}$$

Where y indicates whether the finished product is inspected, and C_f is the finished product inspection cost.

(3) Finished Product Replacement Losses

Undetected defective products will enter the market, leading to replacement losses. The replacement loss when not inspected is:

$$C_{\text{replacement losses}} = (1 - y) \cdot N \cdot p_f \cdot C_r. \tag{38}$$

Based on the above, the updated model for Problem 2 is:

$$\begin{aligned} MinimizeZ &= x_1 \cdot N \cdot (C_{b1} + C_{d1}) \cdot (1 - p_1) + (1 - x_1) \cdot N \cdot C_{b1} + x_2 \cdot N \cdot (C_{b2} + C_{d2}) \cdot (1 - p_2) \\ &+ (1 - x_2) \cdot N \cdot C_{b2} + N \cdot C_a + y \cdot N \cdot C_f + (1 - y) \cdot N \cdot p_f \cdot C_r + z \cdot N \cdot C_t - z \cdot N \cdot \sum_{i=1}^{2} 0.5 \cdot C_{bi} \cdot (1 - p_i) \end{aligned} \tag{39}$$

Where p_f is the defective rate of the finished product, and C_r is the cost of each replacement loss.

2.4.3 Reworking the solution for problem 3 model

(1) Components Cost

For each component i , assume the defective rate is p_i (obtained through sampling inspection), the procurement cost is C_{bi} , the inspection cost is C_{d_j} , the assembly quantity is

N , and the decision variable x_i indicates whether the component is inspected:

$$C_{component} = \sum_{i=1}^{8}\left[C_{bi}\cdot N + x_i \cdot C_{d_i}\cdot N + \left(1-x_i\right)\cdot p_i \cdot C_{b_i}\cdot N\right]. \quad (40)$$

(2) Semi-Finished Product Cost

For each semi-finished product j, the defective rate is p_j , the assembly cost is C_{a_j} , the inspection cost is C_{d_j} , the disassembly cost is C_{t_j} , the recovery benefit is R_j ,, and the decision variable y_j indicates whether the semi-finished product is inspected, while t_j indicates whether it is disassembled:

$$C_{semi-finished\ product} = \sum_{j=1}^{3}\left[C_{a_j}\cdot N + y_j \cdot C_{d_j}\cdot N + \left(1-y_i\right)\cdot p_j \cdot C_{a_j}\cdot N + t_j \cdot C_{t_j}\cdot N - t_j \cdot R_j\right]. \quad (41)$$

Where R_j represents the recovery value of components after disassembly, and the calculation formula is:

$$R_j = \sum_{k}\left(0.5\cdot C_{b_k}\cdot N\right). \quad (42)$$

That is, after disassembly, component of the component cost can be recovered.

(3) Finished Product Cost

The finished product inspection cost is C_f , the replacement loss cost is C_r , the disassembly cost is C_t , and the decision variable z indicates whether the finished product is inspected, while t_f indicates whether defective finished products are disassembled:

$$C_{finished\ product} = z\cdot C_f \cdot N + \left(1-z\right)\cdot p_f \cdot C_r \cdot N + t_f \cdot C_t \cdot N - t_f \cdot R_f. \quad (43)$$

Based on the above, the updated model for Problem 3 is:

$$\begin{aligned} C_{component} &= \sum_{i=1}^{8}\left[C_{bi}\cdot N + x_i \cdot C_{d_i}\cdot N + \left(1-x_i\right)\cdot p_i \cdot C_{b_i}\cdot N\right] \\ &+ \sum_{j=1}^{3}\left[C_{a_j}\cdot N + y_j \cdot C_{d_j}\cdot N + \left(1-y_i\right)\cdot p_j \cdot C_{a_j}\cdot N + t_j \cdot C_{t_j}\cdot N - t_j \cdot R_j\right] \\ &+ z\cdot C_f \cdot N + \left(1-z\right)\cdot p_f \cdot C_r \cdot N + t_f \cdot C_t \cdot N - t_f \cdot R_f. \end{aligned} \quad (44)$$

Constraints: If defective products are not inspected, they cannot be disassembled. That is:

$$t_j \le y_j ’ t_f \le z \quad (45)$$

Only defective finished products and semi-finished products confirmed after inspection can be disassembled.

2.5 Solution Steps

① Construct a sampling inspection model through programming (Python).

② Assume an error range of 0.03 and an initial nominal defective rate of 0.10 (which can be set by the enterprise).

③ Substitute the defective rate obtained from the sampling model into the defective rate in the models of Problem 2 and Problem 3, and finally obtain new decision schemes by solving the models.

2.6 Decision Result Analysis

Reconstruct a new model for Problem 2 and Problem 3 using programming (Python), and obtain new decision schemes through execution. For Problem 2, the defective rate is calculated through the sampling inspection model and the results are visualized, as shown in Figure 3:

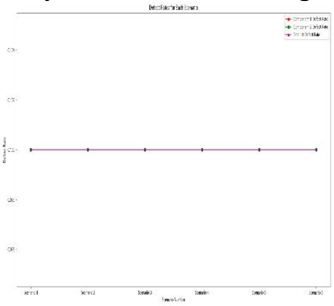

Figure 3. The Defect Rate of Each Situation

It can be seen that the predicted defective rate for each component under various conditions is 0%. By substituting this defective rate into the original model of Problem 2, the latest decision scheme is obtained and visualized using matplotlib in Python, as shown in Figure 4:

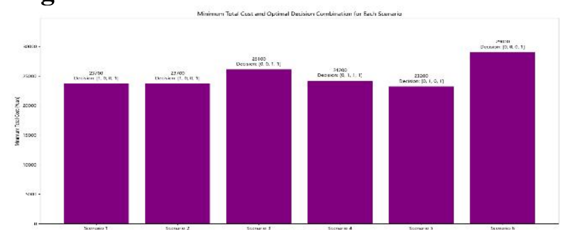

Figure 4. The Lowest Total Cost and Optimal Decision Combination for Each Situation

The decision scheme is shown in Table 3:

Table 3. Decision Plan Table

Case No.	Component 1 Inspection	Component 2 Inspection	Final Product Inspection	Disassembly of Defective Products	Minimum Cost (Yuan)
1	0	0	0	1	23,700
2	0	0	0	1	23,700
3	0	0	1	1	26,100
4	0	1	1	1	24,200
5	0	1	0	1	23,200
6	0	0	0	0	29,000

Components Inspection: When the defective rate is relatively high, components inspection can effectively reduce the number of defective finished products after assembly, thereby lowering the costs of replacement and disassembly. When the inspection cost is high and the defective rate is low, the company chooses not to inspect in order to save inspection expenses.

Finished Product Inspection and Replacement Loss: Finished product inspection can significantly reduce replacement loss when the defective rate is high, while in the case of a low defective rate, not inspecting helps avoid unnecessary inspection costs.

Finished Product Disassembly: Disassembling finished products can recover component of the component cost. When disassembly costs are low and the recovery benefit is considerable, disassembly becomes an effective cost control measure.

For Problem 3, similarly, the latest optimal scheme and the minimum cost are obtained as follows:

Optimal Decision: (0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0), Minimum Cost: 96400.0

Decision Analysis:

Components Inspection ($x1–x8$ = 0, 0, 0, 0, 0, 0, 0, 0):

All components are not inspected, indicating that these components have a relatively low defective rate or the inspection cost is high. The company directly uses these components for assembly without inspection. This strategy avoids unnecessary inspection costs, though it may allow defective components into the assembly line.

This also implies that the defective rate of components is relatively low, or the company has other methods to mitigate the impact of defective components entering the assembly line.

Semi-Finished Product Inspection and Disassembly ($y1–y3$ = 1, 1, 0; $t1–t3$ = 1, 1, 0):

Semi-finished products 1 and 2 are inspected, and those identified as defective are disassembled. This indicates that the defective rate of semi-finished products 1 and 2 is relatively high, and inspection and disassembly can effectively reduce the defective rate of the final products.

Semi-finished product 3 is neither inspected nor disassembled, suggesting that its defective rate is low or that its impact on the final product is minimal, thus inspection is unnecessary.

Finished Product Inspection and Disassembly ($y_f = 0$; $t_f = 0$):

The finished product is not inspected, indicating a low defective rate, high inspection cost, and minimal benefit from inspection. Therefore, the company skips finished product inspection to reduce inspection expenses.

At the same time, finished product disassembly is not selected, suggesting that even if a small number of defects exist in the final product, the potential loss is lower than the cost of inspection and disassembly.

3. Conclusion and Outlook

This model demonstrates strong practicality: it encompasses multiple cost components in the production process, including components procurement, assembly, finished product inspection, and disassembly, making it suitable for real-world production optimization; the sampling inspection method is reasonable: it estimates the defective rate through hypothesis testing and makes acceptance or rejection decisions under different confidence levels; comprehensive cost consideration: the model accounts for the recovery value after disassembly, providing a more complete assessment of the enterprise’s cost; dynamic optimization: it applies dynamic programming to make stage-wise decisions, flexibly responding to changes in the production process.

However, it is worth noting that the model does not consider the impact of external factors on the defective rate and neglects fluctuations in the production environment. The sample size may not be sufficient to accurately represent the defective rate, affecting the reliability of decisions. It does not take into account risk factors such as price fluctuations and logistics in the supply chain. Some costs are difficult to obtain accurately, which affects practical application.

Directions for Model Improvement:

(1) Introduce stochastic factors to enhance the model's ability to cope with uncertainty.

(2) Consider supply chain management to further optimize cost control throughout the production chain.

(3) Improve the method for estimating sample size to ensure a more accurate estimation of the defective rate.

(4) Introduce more sophisticated dynamic programming methods.

Acknowledgments

This paper was supported by the following projects:

Guangdong Province Higher Education Association "14th Five-Year Plan" Research Project for 2024 on the Evaluation of Mathematical Modeling Capabilities of Undergraduates in New Era Applied Universities (No. 24GQN35).

Research Project of the 2024 Annual Project by the Professional Committee on Teaching Quality Management of Private Universities under the Guangdong Higher Education Teaching Management Association: Blended Teaching Quality Evaluation for Mathematics Courses Based on Neural Networks (No. GDZLGL2423)

Guangdong Technology College 2024 "Quality Engineering" Project: Blended Teaching Quality Evaluation for Mathematics Courses Based on Neural Networks (No. JXGG2024150).

Guangdong Technology College 2025 first-class course construction project Economic Mathematics (II) (No.YLKC202521).

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