Construction and Implementation of a Three-Dimensional Development Model for the Mathematical Teaching Content Knowledge of Pre-Service Teachers

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Abstract: Exploring effective development strategies for pre-service teachers' MPCK is crucial for cultivating high-quality mathematics teachers. Guided by the knowledge-creation spiral theory, constructivist learning theory, and the theory of collaboration and learning communities, this study constructed and implemented a three-dimensional development model for pre-service teachers' MPCK along with a "two-stage, seven-step" development strategy, followed by an analysis of its effectiveness. The results indicate that through activities such as observing exemplary MPCK cases, engaging in collaborative learning, conducting lesson studies, and developing MPCK cases, pre-service teachers' MPCK showed significant improvement.

Keywords: Pre-Service Teachers; MPCK Three-Dimensional Development Model; MPCK Development Strategy; Development Community

1. Introduction

In the new era, high-quality, professionally trained teachers must substantially enhance their instructional competencies and reform teacher-education programs in order to raise teacher quality at its source. Emphasizing the training and development of pre-service teachers, reforming and strengthening teacher-education institutions, and elevating the caliber of teacher preparation align both with the new era's demands for teacher quality and with the requirements of the national curriculum standards. For mathematics educators in particular, improving their professional knowledge and pedagogical skills hinges on a deep command of Mathematical Pedagogical Content Knowledge (MPCK). Research on the development of mathematics teachers' MPCK has attracted attention from mathematics-education scholars at home and abroad; however, systematic studies of pre-service teachers' MPCK development remain scarce in China. Consequently, it is especially urgent to explore a set of concrete and effective strategies for fostering MPCK among pre-service mathematics teachers.

In China, the majority of primary- and secondary-school teachers are drawn from normal-university programs. The quality of pre-service mathematics-teacher training directly determines these teachers' professional competence and, by extension, influences the quality of mathematics instruction in primary and secondary schools. China mandates a three-level monitoring and accreditation system for teacher-education majors in higher education. Among the evaluation dimensions is "Curriculum and Instruction," which is assessed through four indicators-curriculum design, structure, content, and implementation. These four indicators, to a considerable extent, define the level of professional knowledge and MPCK that normal universities impart to future secondary-school teachers. Therefore, investigating the development of pre-service teachers' MPCK carries significant practical implications for enhancing the accreditation process of China's mathematics-teacher education programs.

The development of pre-service teachers' MPCK is a critical step in the education of mathematics teachers (normal university students) and is one of the current international hotspots in mathematics teacher education research. It is also an important issue faced by higher normal universities in China. A notable characteristic of pre-service mathematics teacher education in China is that its "normality" has not been fully

embodied and its academic rigor is relatively low. Mathematics teachers need to know not only "what to teach" but also "how to teach." For pre-service mathematics teachers, possessing MPCK is essential as it reflects the dual professional nature of mathematics teaching. However, the current development of MPCK in pre-service mathematics teacher education in China is somewhat inadequate; the present MPCK levels of pre-service mathematics teachers do not yet meet the requirements for primary and secondary school mathematics teachers. Therefore, the development of pre-service teachers' MPCK is extremely necessary.

The author conducted a survey on the MPCK status of pre-service teachers (junior and senior undergraduates) at three local normal universities in Province A, preliminarily analyzing the MPCK of 478 pre-service teachers, and conducting an in-depth analysis on the MPCK across five dimensions and three content areas (specifically, middle school functions) for 43 pre-service teachers. Overall, the MPCK levels among pre-service mathematics teachers showed little variance, with the overall scores below passing levels and a majority of the teachers scoring poorly. Significant differences were observed in MPCK across different dimensions based on gender, grade level, and region of origin, and the scoring rates for the five dimensions and three content areas were low, indicating substantial room for improvement.

Based on the internal demands of reforming basic mathematics education, the practical requirements of accreditation in higher normal university mathematics education, and the shortcomings in the development of MPCK in pre-service mathematics teacher education, this paper constructs a development model for pre-service mathematics teachers' MPCK, proposes and implements a corresponding development strategy, and verifies its effectiveness.

2. Construction of the Three-Dimensional MPCK Development Model for Pre-service Teachers

2.1 Clarifying the Core Goal of Development – Promoting the Development of Pre-service Mathematics Teachers' MPCK

"Promoting the development of pre-service mathematics teachers' MPCK" is the core objective of MPCK development activities. This is mainly reflected in three aspects:

First, the "theme of MPCK development" is derived from the challenges in mathematics learning among middle school students and the difficulties in middle school mathematics teaching, which in turn inform the key focus areas of pre-service teachers' MPCK development.

Second, pre-service mathematics teachers are the main agents of MPCK development activities. Their participation is required at every stage-from the determination of the MPCK development theme to the four formal stages of development.

Third, there is an emphasis on cultivating the developmental abilities of pre-service mathematics teachers. Through MPCK development activities, their awareness and ability to autonomously develop MPCK are further nurtured.

2.2 Finding a Solid Theoretical Foundation – Epistemology, Learning Theory, and Sociological Theory

In the 1990s, two Japanese scholars, Hirotaka Takeuchi and Ikujiro Nonaka, proposed the SECI spiral model of knowledge creation (as shown in Figure 1) during their research on knowledge creation[1]. Pre-service mathematics teachers gradually update their "tacit knowledge base" in mathematics education by incorporating their own beliefs, observations, and experiences. However, these beliefs are largely incomplete, untested, and sometimes even unreasonable. Therefore, the MPCK of pre-service teachers is highly personalized and cannot be transmitted in a standardized manner, though this does not imply that their MPCK cannot be developed.

Pre-service mathematics teachers share or create tacit knowledge through socialization-for example, by exchanging ideas during simulated classrooms and teaching competitions. In-depth and effective communication occurs among individual pre-service teachers, between individuals and their peer groups, and between pre-service teachers and mentors or in-service teachers both within and outside their institutions (such as forming a community for MPCK development and engaging in collaborative learning); this process is known as externalization. Next is combination, where pre-service teachers systematically organize and utilize the formal knowledge or information they

have acquired-for instance, through lesson study or by independently developing MPCK cases. Finally, internalization occurs, whereby pre-service teachers absorb the formal knowledge obtained during MPCK development activities, integrate it into their personal tacit knowledge, update their MPCK, and thereby transform their cognitive structures, ultimately promoting the development of their MPCK.

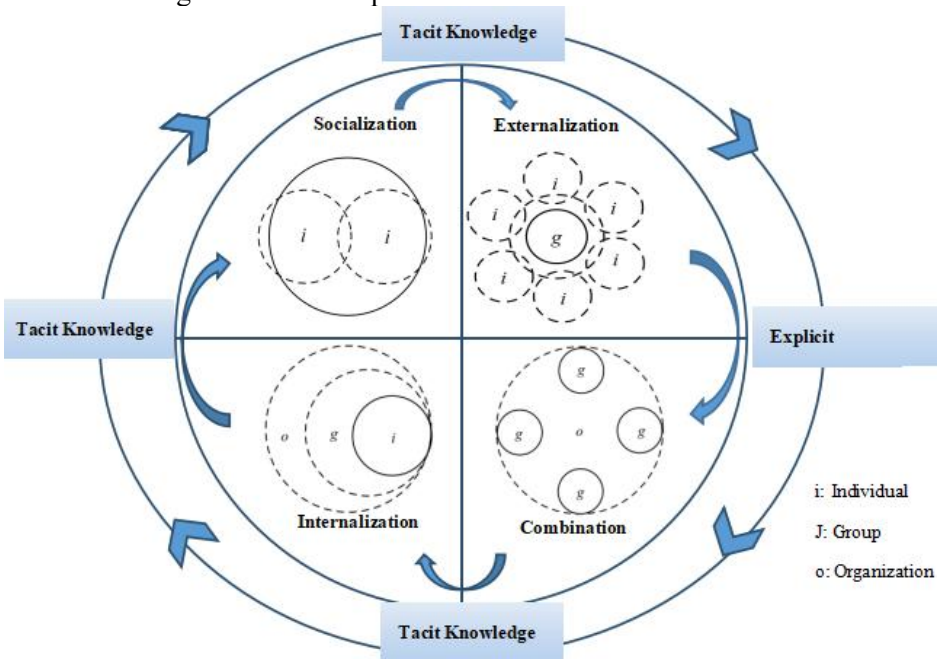

Figure 1. SECI Spiral Model of Knowledge Creation

Jean Piaget's theory emphasizes the dynamic, open, and creative nature of knowledge[2]. The knowledge possessed by pre-service mathematics teachers differs from that of teachers at other levels, stages, and types, and while constructivist learning theory can to some extent explain the learning process of pre-service teachers, the learning experiences of pre-service mathematics teachers differ from those of other university students in various disciplines and from in-service mathematics teachers. Pre-service mathematics teachers have their own unique learning characteristics, meaning that constructivist learning theory cannot be entirely applied to their learning.

Pre-service mathematics teachers are at the first stage of teacher development (the pre-service focus stage). They exhibit the following learning characteristics: They have limited teaching practice experience. They pay little attention to students' learning status and behaviors. Although they value learning, their understanding of the significance of university education is rather utilitarian. They desire more knowledge, yet what they seek is applied knowledge. They have high expectations for the learning environment and conditions, but their learning is not very proactive or in-depth. They wish to achieve good learning outcomes, but they do not know how to learn or what they have learned. Based on the learning needs and characteristics of pre-service teachers[3] and grounded in constructivist learning theory, four traits or factors in the development process of the learning environment or mathematical pedagogical content knowledge for pre-service mathematics teachers have been identified (Table 1).

Table 1. Characteristics of the Learning Environment for Pre-Service Mathematics Teachers

Trait	Notes
"Context"	The learning environment must support the construction of meaning for the content learned.
"Collaboration"	Collaboration should occur throughout the learning process, aiding in material collection, hypothesis formation, and evaluation.
"Conversation"	Group members must converse to plan how to complete the assigned tasks, making the learning process a conversation.
"Meaning Construction"	The nature, patterns, and internal relationships of the content learned.

The development process of pre-service mathematics teachers' MPCK is a learning process based on the theory of collaboration[4]. It exhibits the following learning characteristics: The MPCK development process for pre-service teachers is supported by technology or the learning environment. Pre-service mathematics teachers proactively and spontaneously engage in effective communication and interaction. This includes effective exchanges and mutual learning among individual pre-service teachers, as well as in-depth communication and collaborative learning between pre-service teachers and organizations or groups. Collaborative learning occurs both among individuals and between individuals and organizations or groups, with participants working toward learning objectives, mutually promoting progress, assisting one another, and advancing together. During the collaborative learning process, pre-service mathematics teachers assimilate or accommodate the mathematical pedagogical content knowledge they acquire into their cognitive structures, thereby optimizing or updating their MPCK. Through continuous collaborative interactions among individuals, organizations, or groups, pre-service teachers eventually achieve an internal state of balance. The characteristics of pre-service teachers' collaborative learning are shown in Figure 2, and the model of their collaborative learning is depicted in Figure 3.

In the 1880s, the broad concept of "community" was proposed [5], a concept that belongs to the field of sociology. In the 1950s, Ernest L. Boyer first introduced the concept of the "learning community"[6], which falls within the realm of education. At its core, education is a social activity aimed at cultivating individuals. After carefully considering the concepts of "community" and "learning community," this study defines the learning community of pre-service mathematics teachers as a group composed of learners (pre-service mathematics teachers) and facilitators (including on-campus and off-campus mentors, in-service teachers, and mathematics education experts) who work together towards a common developmental goal (i.e., the development of mathematical pedagogical content knowledge).

In the systematic MPCK development activities for pre-service teachers, individual pre-service mathematics teachers engage in effective communication, exchange, and sharing, while on-campus and off-campus mentors and mathematics education experts provide guidance and support. This collective effort among group members promotes the development of pre-service teachers' MPCK.

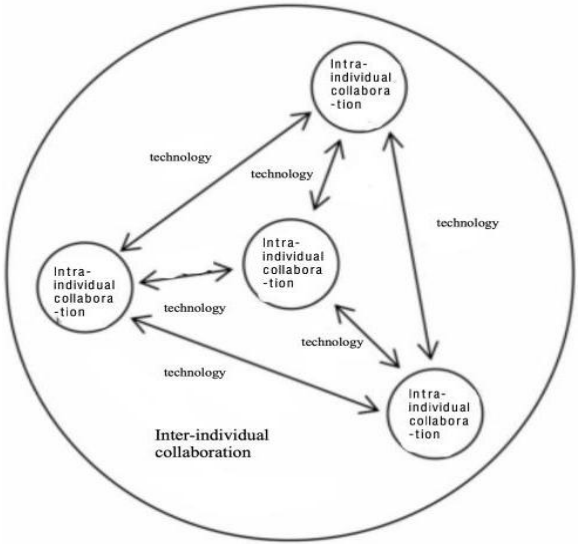

Figure 2. Characteristics of Collaborative Learning among Pre-Service Mathematics Teachers

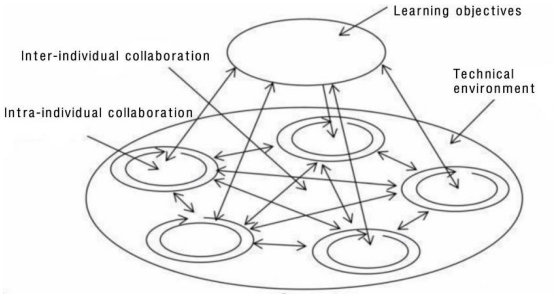

Figure 3. Collaborative Learning Model for Pre-Service Mathematics Teachers

2.3 Establishing an Extensive Development Platform – A Platform for Observation, Research, and Interactive Exchange

This study has established a comprehensive development platform for pre-service mathematics teachers' MPCK. The platform includes a dedicated observation room, an MPCK lesson study platform, and a group interactive exchange platform (or learning community interactive exchange platform). These facilities are primarily used for observing exemplary MPCK cases, conducting MPCK lesson studies, and enabling pre-service mathematics teachers to discuss, exchange ideas, and learn within small groups.

2.4 Facilitating Deep and Effective Interaction – Leveraging the Combined

Efforts of Individuals, Peers, and On-Campus/Off-Campus Mentors

Effective interaction and rational dialogue are crucial to ensuring high-quality learning within a learning community. Only when there is sufficient depth and breadth of interaction among members of the MPCK development community can the synergistic effects of individual reflection, peer assistance, and mentor guidance be fully realized to create a dynamic group atmosphere and achieve high-quality developmental outcomes. For this study, the effective interaction and rational dialogue among individual pre-service mathematics teachers, the pre-service mathematics teacher community, and on-campus/off-campus mentors are key to promoting the development of pre-service teachers' MPCK.

Only through MPCK development activities that meet these conditions can the intrinsic motivation of pre-service mathematics teachers be effectively integrated with external drives (such as peer assistance and mentor guidance), forming a synergistic force. This force continuously promotes the development of their Mathematical Expression Knowledge (MEK), Mathematical Communication Knowledge (MCK), knowledge of how students learn mathematics (HSLM), Mathematical Curriculum Knowledge (MCK1), and Instructional Strategy Knowledge (ISK) throughout the process of "observing exemplary MPCK case repositories – engaging in collaborative learning within MPCK development groups – conducting lesson studies – and independently developing MPCK cases."

2.5 Constructing a Three-Dimensional Development Model – The MPCK Development Model for Pre-Service Mathematics Teachers

Based on an analysis of the core objective of developing pre-service mathematics teachers' MPCK, a solid theoretical foundation, an extensive development platform, deep and effective interaction, and the existing strategies or recommendations for pre-service teachers' MPCK development both domestically and internationally[7]-[13], the author has constructed the "MPCK Development Model for Pre-service Mathematics Teachers" (see Figure 4). This model can be simply described as follows: Based on a solid theoretical foundation drawn from epistemology, learning theory, and sociological theory, and supported by an extensive development environment that includes observation, research, and interactive exchange platforms, the core objective of "promoting the development of pre-service mathematics teachers' MPCK" is pursued. Members of the development community-pre-service mathematics teachers-engage in research-oriented activities such as observing exemplary MPCK cases, conducting case studies, engaging in critical reflection, participating in discussions and seminars, and independently developing MPCK cases. By focusing on the proactive developmental process of the "MPCK development theme" and continuously exploring the principles of education and teaching, the collective development of MPCK among the members of the pre-service mathematics teacher community is achieved.

3. Proposal and Implementation of the MPCK Development Strategy for Pre-service Teachers

Guided by the MPCK development model for pre-service mathematics teachers-and in conjunction with the theoretical foundations, the professional training curriculum for pre-service teachers, and the current state of mathematics teachers' MPCK-a development strategy for pre-service teachers' MPCK has been constructed (see Figure 5). This MPCK development strategy for pre-service teachers consists of "two stages and seven steps," namely the development preparatory stage and the formal development stage.

3.1 Development Preparatory Stage

Step 1: Forming the MPCK Development Community

The members of the MPCK development community primarily consist of pre-service mathematics teachers. Considering factors such as gender, grade, region of origin, type of high school attended, academic performance, and willingness to pursue a career in mathematics teaching, 10 students were randomly selected as research subjects to participate in this study's MPCK development activities. (See Table 2.)

Step 2: Determining the MPCK Development Theme

In this study, the MPCK development theme uses middle school function knowledge as an example. Based on the analysis of the current status of the five dimensions and three content areas of pre-service mathematics teachers'

MPCK, the MPCK development theme for pre-service mathematics teachers was determined (see Table 3).

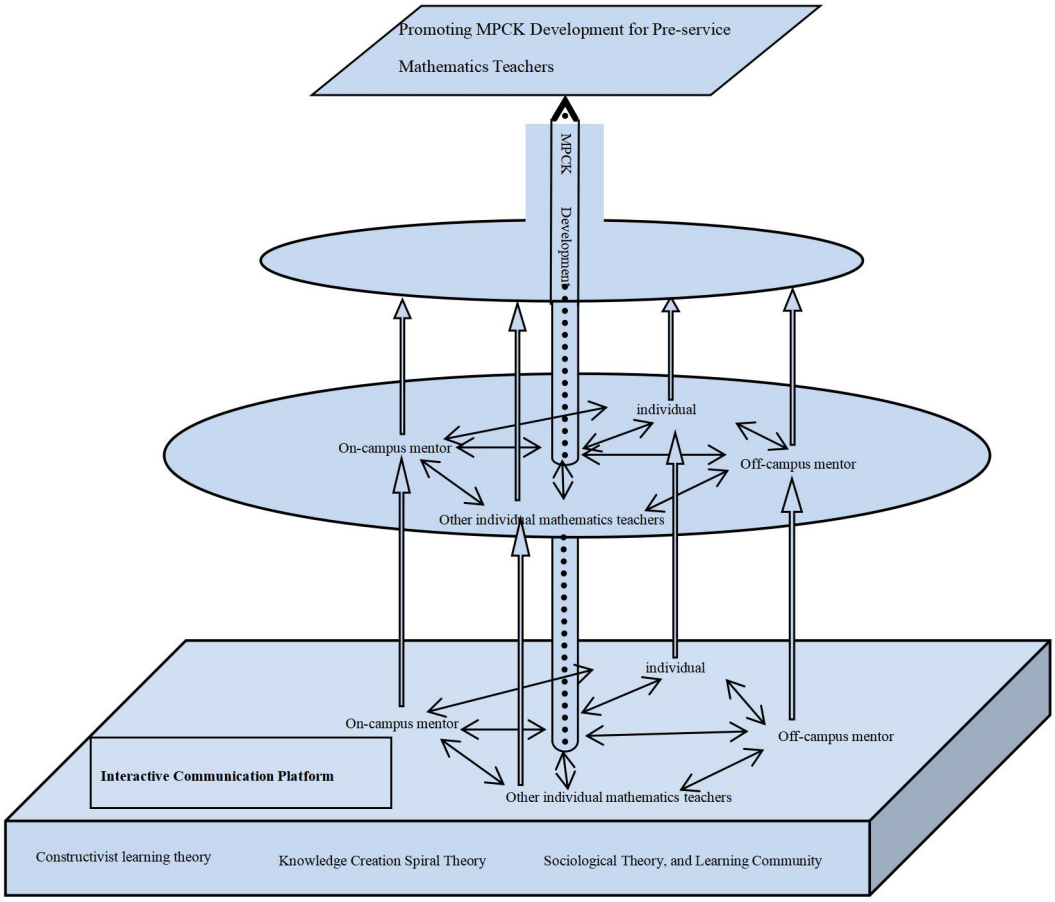

Figure 4. Three-Dimensional MPCK Development Model for Pre-Service Mathematics Teachers

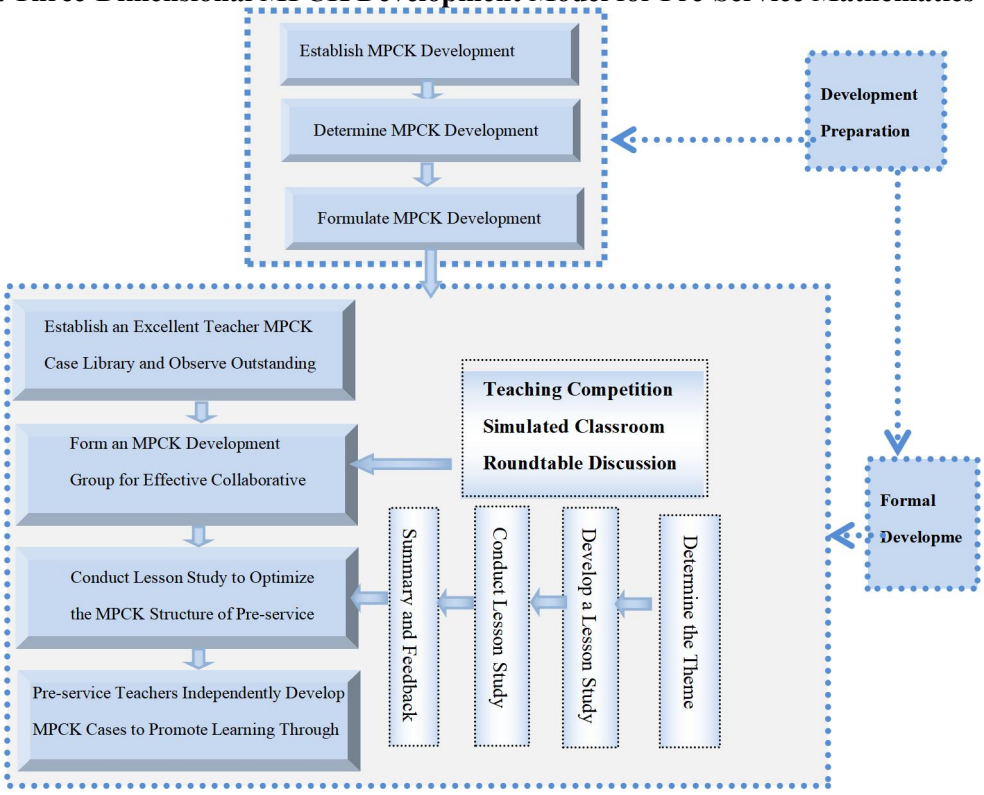

Figure 5. MPCK Development Strategy for Pre-Service Teachers

Table 2. Composition of the MPCK Development Community

Category	Pre-service Mathematics Teachers	Persons in Charge of MPCK Development Activities	On-campus and Off-campus Mentors	Support Staff
Number	10	3	2	2
Remarks	Female:Male=7:3 Third-year:Fourth-year=6:4 Rural:Urban = 7:3 Ordinary High School: County (District) Key: City Key: Provincial = 3:3:2:2 Academic Performance (Average:Good:Satisfactory)= 5:3:2	1 Mathematics Education specialist; 1 class monitor from each grade	2 mentors for theoretical and practical guidance	1 for daily liaison and management of the development platform

Table 3. MPCK Development Themes for Pre-Service Mathematics Teachers

MPCK Development Theme	Sub-theme Content
Functions and Their Graphs	Variables and constants; the concept of function; function values and the range of independent variables; methods of representing functions; graphing.
Linear Functions	The concept, graph, properties, and applications of linear functions.
Quadratic Functions	The concept, graph, properties, and applications of quadratic functions.
Inverse Proportional Functions	The concept, graph, properties, and applications of inverse proportional functions.

Based on the analysis of the current state of pre-service teachers' MPCK and its structure, the MPCK development theme should also focus on the dimension index system for developing pre-service teachers' MPCK (see Table 4).

Step 3: Formulating the MPCK Development Plan

Based on the teaching schedule of a certain normal university in HB City, A Province, and the curriculum for the Mathematics and Applied Mathematics (Teacher Education) program, an MPCK development plan was formulated (see Table 5).

Table 4. MPCK Development Dimension Index System for Pre-Service Mathematics Teachers

MPCK Development Dimension	Description
Mathematical Expression Knowledge (MEK)	Knowledge for expressing mathematical content- Knowledge for expressing mathematical affective experiences- Knowledge for expressing mathematical problems- Knowledge for expressing real-world problems
Mathematical Communication Knowledge (MCK)	Knowledge for exchanging mathematical knowledge and skills- Knowledge for exchanging affect and experiences- Knowledge for exchanging problem-solving insights- Knowledge for exchanging mathematical ideas, concepts, and methods
Knowledge of How Students Learn Mathematics (HSLM)	Experiential knowledge- Cognitive knowledge
Mathematics Curriculum Knowledge (MCK1)	Knowledge about the background of the mathematics curriculum- Mathematics curriculum standards- Knowledge about mathematics textbooks- Knowledge of mathematics curriculum resources
Instructional Strategy Knowledge (ISK)	Knowledge of subject-specific teaching strategies- Knowledge of thematic teaching strategies

Table 5. MPCK Development Plan for Pre-Service Mathematics Teachers

Time	September	October	November	December
MPCK Development Content	Observe "Functions and Their Graphs" in the exemplary MPCK case repository- MPCK development group collaborative	Observe "Linear Functions" in the exemplary MPCK case repository- MPCK development group collaborative	Observe "Quadratic Functions" in the exemplary MPCK case repository- MPCK development group collaborative	Observe "Inverse Proportional Functions" in the exemplary MPCK case repository- MPCK development group

	learning- Conduct lesson studies- Develop MPCK cases	learning- Conduct lesson studies- Develop MPCK cases	learning- Conduct lesson studies- Develop MPCK cases	collaborative learning- Conduct lesson studies- Develop MPCK cases
Person in Charge	Hu XX, Li XX	Hu XX, Li XX	Hu XX, Li XX	Hu XX, Li XX
On-campus and Off-campus Mentors	Li XX, Zhang X	Li XX, Zhang X	Li XX, Zhang X	Li XX, Zhang X

This comprehensive strategy integrates the formation of a collaborative community, the clear definition of development themes, and a detailed, time-bound plan to systematically enhance pre-service mathematics teachers' MPCK.

3.2 Formal Development Stage

Step 4: Establish an Exemplary Teachers' MPCK Case Repository and Observe Exemplary MPCK Cases

With the support of outstanding frontline teachers and the author's mentor, an exemplary teachers' MPCK case repository was established during this stage. This repository mainly includes videos of outstanding teacher lectures, videos from municipal-level outstanding teacher teaching competitions, lesson rehearsals, expert evaluation videos, and demonstration teaching by outstanding teachers. Partial displays of the exemplary MPCK case repository are shown in Figures 6, 7, 8, and 9.

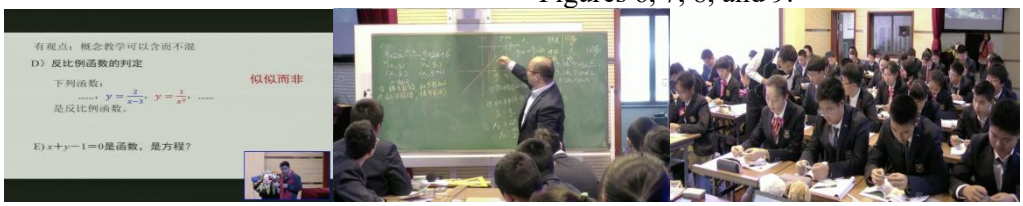

Figure 6. Outstanding Teacher's Lecture

Figure 7. Outstanding Teacher's Demonstration Lesson and Discussion

Figure 8. Municipal-Level Outstanding Teacher Teaching Quality Lesson Competition

Figure 9. Expert Lesson Evaluation

The coordinator of the MPCK development activities organized and arranged for 10 pre-service mathematics teachers to observe and study the exemplary teachers' MPCK case repository according to the plan. In this process, pre-service mathematics teachers learned about exemplary teachers' instructional design, teaching methods, textbook handling, student interaction, and classroom arrangements. They exchanged ideas and discussed the demonstration lessons among peers, learned innovative teaching ideas from high-quality lesson competitions, and observed expert lesson evaluations to gain new perspectives on mathematics instructional design and classroom teaching, thereby gradually updating their own "practical knowledge base."

Step 5: Establish MPCK Development Groups

for Effective Collaborative Learning

Professor Manabu Sato of Japan advocates the construction of a "learning community" to leverage collective strengths, thereby avoiding the limitations of individual teaching experiences and understandings, and promoting cooperative learning and mutual assistance[14]. Consequently, establishing MPCK development groups to organize pre-service mathematics teachers for effective collaborative learning has a positive impact on the development of their MPCK.

During the early, middle, and later stages of observing the exemplary MPCK case repository, the MPCK development groups were formed by pairing teachers into groups of two, resulting in a total of five groups. The group compositions were rotated weekly, and effective collaborative learning was conducted using both online and offline methods. Offline sessions were primarily held in the teacher development center, while online sessions were conducted via DingTalk meetings and WeChat voice calls. In this process, each pre-service teacher's intellectual contributions were shared with the other nine teachers, aiming to achieve a common development goal-namely, the enhancement of the five dimensions of MPCK.

(1) Teaching Competitions

Based on the specific development theme, teaching competitions focusing on mathematics instructional design, mathematics courseware production, and classroom teaching were organized. These competitions promoted learning through competition and, in turn, updated and developed the pre-service teachers' MPCK.

(2) Simulated Classrooms

From an educational philosophy perspective, one of the best strategies for fostering cognition is "dialogue in conjunction with practice." Therefore, simulated classroom activities for pre-service mathematics teachers are essential. Under the guidance of both on-campus and off-campus mentors, simulated classroom activities were conducted in accordance with the MPCK development plan. These activities included instructional design, simulated lessons without students, parallel class constructions, post-lesson reflections, discussions, and group members recording and evaluating each other's lessons. In addition, invited teaching experts and renowned educators provided online evaluations, identifying shortcomings and prompting continuous improvements.

(3) Round Table Meetings

Combining the MPCK development theme with the learning needs, characteristics, and core objectives of pre-service teachers, round table meetings were held. Based on repeated reflections on classic teaching videos of outstanding teachers, participants discussed topics such as macro and micro interpretations of textbooks, reading renowned mathematics education papers and classics, typical teaching cases, mathematics instructional design, and the integration of information technology with mathematics teaching. Guided by mentors, participants engaged in brainstorming, discussions, exchanges, and shared outcomes.

Step 6: Conduct Lesson Study to Optimize the Structure of Pre-service Teachers' MPCK

American scholar Posner posits that "growth = experience + reflection." Generally, experience is derived from practice. MPCK lesson study serves as an opportunity for pre-service mathematics teachers to understand mathematical knowledge and theories of mathematics learning from their own practical perspectives. Therefore, lesson study can, to a certain extent, promote the growth of pre-service teachers' MPCK, optimize its structure, and help them develop the ability to reflect on practice as well as the skills to research and solve problems. Consequently, lesson study is integrated into the MPCK development process for pre-service mathematics teachers.

The pre-service mathematics teachers participating in the MPCK development activities were divided into two groups, with each group focusing on the theme of middle school functions. They engaged in multiple rounds of lesson rehearsals and discussions on various topics, followed by teaching practice in their internship schools, continuously enriching their MPCK through a cycle of "practice-reflection-practice."

The basic steps of MPCK lesson study can be succinctly summarized as: problem → design → practice → reflection → summation. The key to lesson study lies in determining the research theme, with the core being the process of active inquiry during the lesson study. Due to space limitations, the specific processes of these inquiry activities are not provided.

4. Effectiveness of MPCK Development for Pre-service Teachers

Due to space limitations, a brief introduction is provided regarding the development of pre-service teachers' MPCK. A comprehensive evaluation-based on questionnaires, statistical analysis, text analysis, and interviews-indicated that after a semester of development activities, all 10 pre-service mathematics teachers demonstrated significant improvement in their MPCK (see Table 6).

Table 6. Development of MPCK among Pre-Service Teachers

Component	Pre-test	Post-test	Significance
Mathematical Expression Knowledge	3.47	3.92	.000
Mathematical Communication Knowledge	3.57	3.95	.000
Knowledge of How Students Learn Mathematics	3.45	3.77	.000
Mathematics Curriculum Knowledge	3.36	4.01	.000
Instructional Strategy Knowledge	3.59	3.93	.000

In the development of Mathematical Expression Knowledge, the largest increase was observed in the knowledge for expressing mathematical affective experiences (0.69), followed by the knowledge for expressing mathematical problems (0.43), real-world problem expression (0.35), and mathematical content expression (0.33).

In the development of Mathematical Communication Knowledge, the largest increase was in the knowledge for exchanging problem-solving insights (0.47), followed by exchanging affect and experiences (0.38), exchanging mathematical ideas, concepts, and methods (0.37), and exchanging mathematical content and skills (0.32).

For the development of Knowledge of How Students Learn Mathematics, the largest increase was in experiential knowledge (0.34), followed by cognitive knowledge (0.29).

In the development of Mathematics Curriculum Knowledge, the largest increase was in the knowledge of mathematics textbooks (0.85), followed by mathematics curriculum resources (0.80), knowledge about the background of the mathematics curriculum (0.52), and mathematics curriculum standards (0.42).

In the development of Instructional Strategy Knowledge, the largest increase was in the knowledge of specific thematic teaching strategies (ISSSK) (0.47), followed by subject-specific teaching strategies (ISSTK) (0.21).

Practice has shown that this development model can effectively enhance the MPCK of pre-service mathematics teachers. It is important to note that the mathematics teaching classes of pre-service teachers participating in MPCK development activities should be recorded and subjected to video analysis to deeply explore the dynamic, complex, and evolving changes in the knowledge that pre-service mathematics teachers exhibit during the teaching process.

5. Conclusion

The study demonstrates that, under the combined guidance of the Knowledge Creation Spiral Theory, Constructivist Learning Theory, and the Theory of Collaboration and Learning Communities, the two-stage, seven-phase, three-dimensional MPCK development model for pre-service teachers exhibits significant efficacy. Specifically, through activities such as observing exemplary MPCK cases, collaborative discussion and learning, lesson-study and reflective practice, and the autonomous development of MPCK cases, pre-service teachers achieved marked improvements in integrating content knowledge with pedagogical knowledge, constructing teaching contexts, and engaging in reflective practice. Empirical analysis indicates that this development strategy not only systematically fosters multidimensional growth in pre-service teacher MPCK but also offers a replicable framework for the design and evaluation of future teacher-education programs, thereby robustly supporting the goal of cultivating high-quality mathematics educators.

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