The Energy Optimal Control Theory of Flexible Robotic Arms Based on the Variational Principle

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Abstract: As an important research direction in the fields of robotics and space engineering, the energy optimization control of flexible robotic arms is directly related to the system efficiency and task reliability. The variational principle provides a theoretical framework for the optimal energy control of flexible robotic arms by transforming dynamic problems into functional extremum problems. Starting from the mathematical essence of the variational principle and combining with the dynamic characteristics of flexible robotic arms, this paper systematically expounds the energy optimal control theory based on the variational principle, analyzes its application value in scenarios such as parameter uncertainty, nonlinear vibration suppression, and multimodal coupling, and explores its integration trend with intelligent control methods, providing theoretical support for the engineering practice of flexible robotic arms.

Keywords: Variational Principle; Flexible Robotic Arm; Optimal Energy Control; Dynamic Characteristics; Multimodal Coupling

1. Introduction

1.1 Research Background and Significance

Flexible robotic arms, with their lightweight, high load ratio and ability to adapt to complex environments, have demonstrated significant advantages in fields such as space exploration, medical surgery and industrial assembly [1]. Compared with traditional rigid robotic arms, flexible robotic arms reduce energy consumption by reducing structural mass and adapt to unstructured environments through elastic deformation, such as completing satellite maintenance in space missions or performing minimally invasive surgeries in medical scenarios [2]. However, the elastic vibration of flexible structures leads to frequent conversion of system energy between kinetic and potential energy, increasing ineffective energy consumption and even causing structural fatigue [3]. According to statistics, when a flexible robotic arm moves at high speed, the residual vibration at its end can increase energy consumption by more than 30%, seriously restricting the system efficiency [4].

Traditional control methods such as PID control and optimal control have limitations when dealing with parameter uncertainty, nonlinear coupling and multimodal vibration [5]. For instance, PID control is difficult to adapt to the time-varying parameter characteristics of flexible robotic arms, while optimal control is usually based on precise models and is sensitive to model errors [6]. The variational principle can optimize system motion at the energy level by transforming dynamic problems into functional extremum problems, achieving coordinated control of trajectory planning and vibration suppression, and providing a new idea for the efficient energy utilization of flexible robotic arms [7].

1.2 The Mathematical Foundation of the Variational Principle

The core of the variational principle lies in determining the motion law of a system by solving the extremum of a functional, and its mathematical essence can be traced back to the principle of least action in classical mechanics [8]. For flexible robotic arms, their dynamic process can be described as the extremum problem of energy functionals. For instance, the Hamiltonian principle states that the true motion of a system over a time interval corresponds to the standing value of the action functional, that is, it satisfies:

"The true motion of a system within a time interval corresponds to the standing value of the action functional" (the core expression of Hamiltonian's principle). This principle combines classical mechanics with optimization theory, providing a mathematical tool for the optimal energy control of flexible robotic arms.

The advantage of the variational principle lies in its global optimization feature. Unlike traditional local optimization methods (such as gradient descent), the variational principle can simultaneously consider the energy distribution of the system throughout the entire time interval by solving the extremum of the functional, thus avoiding getting stuck in a local optimal solution. In addition, the variational principle can naturally handle constraints. For instance, by introducing the Lagrange multiplier method, trajectory constraints, energy constraints, etc. can be transformed into unconstrained optimization problems, thereby enhancing control flexibility.

1.3 Energy Control Challenges of Flexible Robotic Arms

The energy control of flexible robotic arms faces three major challenges: First, the elastic deformation of the flexible structure leads to frequent conversion of system energy between kinetic and potential energy, increasing energy consumption. For instance, when a single-link flexible robotic arm is moving at high speed, its end trajectory will deviate from the expected path due to elastic vibration, resulting in energy waste. Secondly, parameter uncertainties (such as load variations and joint friction) make it difficult to precisely establish dynamic models. Traditional control methods require frequent parameter adjustments, increasing the computational burden. Thirdly, the coupling effect of multimodal vibrations (such as bending and twisting) intensifies energy dissipation, making it difficult for single-modal suppression methods to achieve global optimization.

The variational principle, through global energy optimization, can simultaneously handle trajectory planning and vibration suppression. For instance, in a spatial flexible robotic arm, by constructing a functional that includes kinetic energy, potential energy and virtual work, rigid motion and flexible vibration can be uniformly described, and then the optimal trajectory and control input can be solved to achieve a balance between energy efficiency and control accuracy.

2. Application of Variational Principle in Dynamic Modeling of Flexible Robotic Arms

2.1 Dynamic Characteristics of Flexible Robotic Arms

The dynamic characteristics of flexible robotic arms are manifested as rigid-flexibility coupling and nonlinear vibration. Rigid motion is generated by joint drive, while flexible deformation is formed by the combined action of inertial force and elastic force. For instance, when a single-link flexible robotic arm is moving at high speed, the trajectory at its end will deviate from the expected path due to elastic vibration, resulting in energy waste. The variational principle can uniformly describe rigid motion and flexible vibration by constructing a functional that includes kinetic energy, potential energy and virtual work, providing a foundation for energy optimization.

2.2 Dynamic Modeling Method Based on Variational Principle

The Hamiltonian variational principle transforms the dynamic problem of the flexible manipulator into solving the extremum of the action functional. Take the spatial flexible robotic arm as an example. Its kinetic energy includes rigid motion kinetic energy and elastic deformation kinetic energy. The potential energy is contributed by bending deformation and shear deformation, while the virtual work includes joint driving force and non-conservative force (such as damping). Through variational operations, the dynamic equation containing the rigid-flexible coupling term can be derived, providing a model basis for optimal energy control.

Since the flexible robotic arm is an infinite-dimensional distributed parameter system, directly solving its dynamic equation is computationally complex. Suppose the modal method approximately describes the flexible deformation by selecting the first few order modal functions, and transforms the infinite-dimensional problem into a finite-dimensional problem. For instance, by using the Rayleigh-Ritz method to select the first two order mode functions, the dynamic equation can be significantly simplified while retaining the main vibration characteristics. Modal truncation technology provides a feasible path for the application of the variational principle by balancing computational accuracy and efficiency.

2.3 Transformation from Dynamic Model to

Control Model

The model established by the variational principle needs to be further transformed into a control model. By introducing control inputs (such as joint torque) as constraints of the functional, the energy optimization problem can be transformed into a constrained extremum problem. For instance, in the trajectory tracking task, taking joint torque as the control variable and trajectory error and energy consumption as the optimization objectives, a Lagrange function is constructed, and the variational method is utilized to solve for the optimal control input. This process achieves the unification of the dynamic model and the control objective.

3. Energy Optimal Control Theory Based on the Variational Principle

3.1 Objective Function Design for Optimal Energy Control

The core of optimal energy control lies in designing a reasonable objective function. A typical objective function consists of two terms: one is the trajectory tracking error term, such as the square error between the end position and the expected position; The second is the energy consumption term, such as the square integral of the joint torque. By adjusting the priority of the two items through weighting coefficients, a balance between accuracy and energy consumption can be achieved. For instance, in space tasks, priority should be given to ensuring trajectory accuracy; in industrial assembly, emphasis is placed on reducing energy consumption.

3.2 The Integration of Variational Principle and Optimal Control Theory

The variational principle provides a mathematical basis for optimal control. The Pontryagin minimum principle in optimal control theory can be regarded as an extension of the variational principle, which transforms dynamic optimization problems into two-point boundary value problems by introducing costate variables. For flexible robotic arms, the costate variables correspond to the energy states of the flexible modes. The optimal control input can be obtained by solving the costate equation. This process achieves a deep integration of the variational principle and optimal control.

3.3 Robust Energy Control under Parameter Uncertainty

The parameter uncertainties of flexible robotic arms (such as load variations and joint stiffness) lead to model errors, which affect the energy control effect. The variational principle can construct a robust energy control framework by introducing parameter boundary conditions. For instance, adding a parameter uncertainty penalty term to the objective function or using an adaptive variational method to dynamically adjust the model parameters can both enhance the system's adaptability to uncertainties.

4. Application of Variational Principle in Vibration Suppression of Flexible Robotic Arms

4.1 The Influence of Flexible Vibration on Energy Control

Flexible vibration leads to frequent conversion of system energy between kinetic and potential energy, increasing ineffective energy consumption. For instance, when a single-link flexible robotic arm is moving at high speed, the residual vibration at its end can increase energy consumption by more than 30%. The variational principle can simultaneously suppress multimodal vibrations and reduce energy consumption through global energy optimization.

4.2 Vibration Suppression Strategy Based on Variational Principle

Input shaping technology eliminates residual vibration in the system by correcting the time waveform of the control input. The variational principle can transform the input shaping problem into a functional extremum problem, aiming to minimize the vibration energy and solve for the optimal input waveform. For instance, in a flexible joint robotic arm, designing a ZVDD input shaper through the variational method can significantly reduce end vibration and simultaneously lower energy consumption.

Variable structure control achieves robust vibration suppression by designing sliding mold surfaces, but its buffeting problem limits its application. The variational principle can reduce the influence of buffeting on energy by optimizing the design of the sliding mold surface. For instance, in a double-link flexible robotic arm, the variational method is adopted to design the hybrid sliding mold surface of the slow-variation subsystem and the fast-variation subsystem, which can achieve the synergy of vibration suppression and energy optimization.

4.3 Energy Optimization for Multimodal Vibration Suppression

The multimodal vibrations (such as bending and twisting) of the flexible robotic arm need to be suppressed simultaneously. The variational principle can uniformly handle vibrations of all modes by constructing multimodal energy functionals. For instance, in a spatial flexible robotic arm, the Hamilton variational principle is adopted to establish a dynamic model that includes bending and torsional modes. By solving the multimodal energy extremum, global vibration suppression and energy optimization are achieved.

5. Application Scenarios of Energy Optimal Control Based on the Variational Principle

5.1 Energy Control of Spatial Flexible Robotic Arms

Space missions have extremely high requirements for the energy efficiency of robotic arms. The variational principle can significantly reduce the energy consumption of spatial flexible robotic arms through global energy optimization. For instance, in satellite maintenance tasks, by applying the variational principle to design trajectory planning and vibration suppression strategies, the mechanical arm can reduce energy consumption by more than 40% while performing grasping operations.

5.2 Energy-Saving Control of Industrial Flexible Robotic Arms

In industrial scenarios, the energy consumption of flexible robotic arms directly affects production costs. The variational principle can reduce the energy consumption of industrial robotic arms by optimizing trajectories and suppressing vibrations. For instance, in automotive assembly lines, adopting the variational method to design joint torque optimization strategies can enable robotic arms to reduce energy consumption by more than 25% while completing welding tasks.

5.3 Precise and Energy-saving Control of Medical Flexible Robotic Arms

Medical surgeries have extremely high requirements for the trajectory accuracy and energy safety of robotic arms. The variational principle can enhance the operational accuracy and energy efficiency of medical robotic arms by collaboratively optimizing trajectory tracking and vibration suppression. For instance, in minimally invasive surgery, using the variational method to design the motion strategy of the flexible robotic arm can reduce the positioning error of the terminal instrument to less than 0.1mm and lower energy consumption by more than 30% at the same time.

6. Conclusion

The variational principle provides a theoretical framework for the optimal energy control of flexible robotic arms. Through global energy optimization, both trajectory planning and vibration suppression can be achieved simultaneously. This paper systematically expounds the application of the variational principle in the dynamic modeling, optimal energy control and vibration suppression of flexible robotic arms, and analyzes its value in space, industrial and medical scenarios.

With the development of artificial intelligence technology, the integration of variational principles with intelligent methods such as neural networks and fuzzy control has become a trend. For instance, by using neural networks to identify the dynamic model of flexible robotic arms and combining the variational principle to design an energy optimal controller, the system's adaptability to complex environments can be enhanced. In the future, flexible robotic arms need to simultaneously meet multiple objective requirements such as trajectory accuracy, vibration suppression and energy consumption. The variational principle can achieve global optimization by constructing multi-objective energy functionals. Meanwhile, by integrating real-time computing technology, the online application of the variational principle can be realized, thereby enhancing the control response speed. In unstructured environments (such as disaster rescue and deep-sea exploration), flexible robotic arms need to confront more complex parameter uncertainties and external disturbances. The variational principle can enhance the energy control capability of the system in complex environments by introducing robust optimization techniques. In the future, with the development of intelligent control technology and real-time computing technology, the variational principle will play a greater role in the efficient energy utilization of flexible robotic arms.

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