

# Pricing Municipal Investment Bonds with Implicit Government Guarantees: A Fractional Jump–Diffusion Approach

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**Abstract:** Over the past decade, local administration financing vehicles (LGFVs) in China have issued a large volume of municipal investment bonds to support infrastructure development and urban expansion. Despite their economic relevance, the credit risk of these bonds is largely shaped by implicit government guarantees, which substantially complicates risk assessment and valuation. Motivated by the presence of strategy persistence in administration asset management and the non-market-driven shocks affecting state-owned enterprise asset values, this paper models state-owned enterprise asset values dynamics using a fractional Brownian motion to capture long-range dependence and a jump–diffusion process to account for abrupt structural interventions. Within a structural credit risk framework, we develop a valuation model for municipal investment bonds that explicitly incorporates an implicit guarantee intensity parameter, and employ the fast Fourier transform (FFT) to obtain efficient numerical solutions. The results indicate that bond prices are positively related to the strength of implicit administration guarantees, yet the relationship is highly nonlinear. When the implicit guarantee intensity is relatively low, bond prices exhibit limited sensitivity to changes in guarantees; however, once the implicit guarantee intensity exceeds a critical threshold, price sensitivity increases markedly. These findings provide a new modeling perspective for pricing corporate bonds with implicit administration backing and contribute to the broader literature on credit risk under state intervention.

**Keywords:** Municipal Investment Bonds; Implicit Administration Guarantees; Fractional Brownian Motion; Jump–Diffusion Model; Fast Fourier

## Transform

### 1. Introduction

With the acceleration of urbanization in China and the growing financing needs of local governments, municipal investment bonds have increasingly become an important financing instrument. Owing to their close association with local government-backed public service projects, the credit risk of these bonds is substantially shaped by implicit government guarantees. As an informal yet widely perceived form of support, such guarantees enhance investor confidence and, to some extent, lower issuers' financing costs. However, implicit government guarantees may also distort market discipline, foster investor overreliance, and contribute to excessive debt accumulation. Beyond their effects on borrowing costs, these guarantees play an important effect on default risk. Despite their importance, the existing literature has not yet fully examined how implicit government guarantees affect the price of municipal investment bonds (MIBs). Incorporating implicit government guarantees into theoretical frameworks and quantitatively assessing their impact on financing costs and default risk therefore remains an important and unresolved research challenge.

The practice of governments rescuing large financial institutions during financial crises to contain systemic risk is well documented, providing clear evidence of implicit government guarantees [1–4]. A growing body of research demonstrates that such guarantees significantly influence credit spreads and facilitate the transmission of financial risk across institutions [5–7].

A salient feature of China's credit bond market is the widespread presence of bonds perceived to be backed by implicit government guarantees. In the absence of explicit guarantees, investors commonly presume that MIBs benefit from

government support. As a result, MIBs are generally regarded as bearing lower credit risk than those issued by privately owned enterprises (POEs) [8–10]. Consistent with this perception, implicit government guarantees reduce financing costs for MIBs, as reflected in substantially lower yield spreads.

Accurate pricing of MIBs requires a reliable valuation of the underlying firm assets. Early asset-pricing theory was largely developed within the continuous-time framework of Black and Scholes (1973) [11], which assumes that asset prices follow a geometric Brownian motion (GBM), implying symmetrically normally distributed returns. Nevertheless, a substantial body of empirical evidence documents that financial asset returns deviate markedly from normality, exhibiting pronounced skewness and excess kurtosis. Such empirical regularities suggest that asset price dynamics are characterized by abrupt and economically significant jumps, which cannot be adequately captured by a pure diffusion process. To address these limitations, Merton (1976) introduced the jump–diffusion model, which augments a standard Brownian diffusion with a Poisson-driven jump component [12]. In this framework, asset prices evolve continuously but are subject to discrete jumps arriving at a constant intensity, thereby capturing the fat tails and discontinuities observed in return distributions. The Merton model has since served as a cornerstone for a large body of subsequent research. Nevertheless, the model assumes that jump sizes follow a symmetric lognormal distribution, implying identical statistical properties for upward and downward jumps. This assumption is at odds with extensive empirical evidence showing that negative jumps are more frequent and more pronounced than positive ones, especially during market downturns, stock price crashes, and financial crises.

To better align models with empirical observations, a variety of asymmetric jump–diffusion models have been developed. These extensions introduce skewness parameters into the jump distribution—through approaches such as lognormal mixtures, exponential distributions, or generalized normal distributions—or allow jump intensity to vary dynamically with market conditions, thereby more accurately capturing the skewness of return distributions and the volatility smile [13–14].

These enhancements have yielded notable improvements in derivative pricing, risk management, and volatility modeling. Despite these advances, traditional jump–diffusion models retain key limitations. In particular, the Brownian diffusion component is still assumed to have independent increments and short memory, rendering it insufficient for capturing volatility clustering and long-range dependence, which are pervasive in financial time series. To address this limitation, Mandelbrot and Van Ness (1968) proposed fractional Brownian motion (FBM) as a theoretical framework [15]. The FBM’s self-similarity and long-memory properties enable a more accurate representation of return persistence and correlation structures. However, the continuity inherent in FBM prevents it from capturing the abrupt price jumps observed in financial markets.

Based on this, researchers in recent years have attempted to combine the FBM or fractal time transformation mechanism with jump processes, proposing the Fractional Jump–Diffusion model [16–18]. By simultaneously incorporating a long-memory diffusion component and a discontinuous jump component into price dynamics, this model theoretically unifies the advantages of traditional Brownian diffusion, jump processes, and fractal dynamics. The Fractional Jump–Diffusion model not only demonstrates higher fitting accuracy in option pricing and risk management, but also provides a flexible analytical framework for studying the multi-scale dynamics of asset prices.

In the municipal bond market, understanding pricing mechanisms is particularly important. To address these limitations, this paper models state-owned enterprise asset values dynamics using a FBM to capture long-range dependence and a jump–diffusion process to account for abrupt structural interventions. Within a structural credit risk framework, we develop a valuation model for municipal investment bonds that explicitly incorporates an implicit guarantee intensity parameter, and employ the fast Fourier transform (FFT) to obtain efficient numerical solutions.

The remainder of the paper is organized as follows. Section 2 constructs a basic model for bond prices with implicit government guarantees from an options perspective. Section 3 uses a FBM with an asymmetric double-exponential jump process to measure asset value and thus obtain the bond price based on the basic model;

Section 4 applies the Fourier transform to the bond price model to construct a numerical algorithm and analyzes the resulting calculations. The final section presents the conclusions of this paper.

## 2. Basic Model

According to Merton's structural model, the value of a zero-coupon bond can be expressed as:

$$P_{bond} = e^{-rT}K - E[\min(K - V_T)^+] \quad (1)$$

In equation (1),  $K$  denotes the bond's face value payable at maturity,  $V_T$  represents the firm's asset value,  $r$  is the risk-free rate, and  $T$  is the maturity of bond.

Equation (1) can alternatively be interpreted from an option-theoretic perspective. Specifically, the value of a corporate bond can be decomposed into the value of a default-free bond with the same maturity minus the value of a European put option held by the firm's shareholders. The underlying asset of this put option is the total value of the firm's assets, while the strike price equals the face value of the outstanding debt due at maturity. Within the structural framework of default modeling, if the asset value at maturity falls below the debt obligation, shareholders optimally choose to default by refraining from further capital injections, thereby triggering a default event. Consequently, corporate default can be interpreted as the exercise of this implicit put option. Accordingly, the price of a corporate bond is equal to the value of a risk-free bond minus the value of the embedded shareholder put option.

However, the presence of implicit government guarantees fundamentally alters the value decomposition of MIBs. Such guarantees imply that, in the event of default by a local government financing vehicle, bondholders may effectively "put" the bond back to the government at  $K$ , thereby mitigating, or even eliminating, potential losses. From an economic perspective, this implicit guarantee is analogous to the provision of a put option to bondholders, although it does not constitute a standardized or legally enforceable financial contract. This is because government intervention in the event of default is discretionary rather than mandatory, and depends on factors such as the local government's fiscal capacity, incentive structure, and prevailing policy stance, giving rise to substantial uncertainty. Consequently, when

implicit guarantees are present, the value of MIBs reflects not only the fundamental bond value described in equation (1), but also an additional component,  $P_{put}$ , representing the value of the implicit put option enjoyed by bondholders.

$$P_{bond} = e^{-rT}K - E[\min(K - V_T)^+] + P_{put} \quad (2)$$

Building on this framework, we introduce a parameter  $\theta$  ( $0 \leq \theta \leq 1$ ) to capture the strength of the implicit government guarantee, which reflects the probability or willingness of the government to intervene and provide a bailout when a local government financing vehicle defaults. The magnitude of the implicit guarantee,  $\theta$  directly determines the effective strike price of the associated put option. When government support is uncertain, the protection available to bondholders is equivalent to a put option with a stochastic effective strike price of  $\theta K$ , rather than a deterministic strike price equal to the bond's par value  $K$ . Accordingly, the value of the put option embedded in the implicit government guarantee can be expressed as:

$$P_{put} = E[\min(\theta K - V_T)^+] \quad (3)$$

Here,  $P_{put}$  denotes the value of a put option with an exercise price of  $\theta K$ , and its magnitude varies monotonically with the strength of the implicit government guarantee  $\theta$ . When  $\theta = 0$ , no implicit guarantee is present, and the bond price is entirely determined by market-implied default risk. As  $\theta \rightarrow 1$ , the implicit guarantee becomes effectively fully binding, and the bond value converges to that of a risk-free bond with the same maturity. For intermediate values,  $0 < \theta < 1$ , government support for local government financing vehicles (LGFVs) is uncertain and depends on factors such as the local government's fiscal capacity and policy incentives. Accordingly, the price of LGFV bonds subject to implicit government guarantees can be expressed as:

$$P_{bond} = e^{-rT}K - E[\min(K - V_T)^+] + E[\min(\theta K - V_T)^+] \quad (4)$$

The next section derives the value of the option component in equation (4) under the assumption that the asset value of the local government financing vehicle follows a FBM with jumps, thereby completing the pricing framework for MIBs.

## 3. Pricing MIBs

The adoption of FBM to model the evolution of state-owned enterprise (SOE) asset values is

motivated by two main considerations. First, SOE asset values are continuously affected by interventions from state-owned asset management authorities, which induce pronounced temporal dependence in asset dynamics and violate the independent-increment and memoryless assumptions underlying standard Brownian motion. FBM is capable of capturing long-range dependence and persistence, and is therefore more appropriate for describing the dynamic behavior of SOE asset values.

Second, SOEs frequently engage in non-market-oriented asset reallocations, such as gratuitous asset transfers. These administrative interventions generate abrupt and discrete changes in asset values: the asset base of the receiving entity increases sharply over a short horizon, while that of the transferring entity declines correspondingly. To capture such directional and asymmetric discontinuities, we augment the asset value process with an asymmetric double-exponential jump component.

$Q_H$  is the risk-neutral probability, the asset value of state-owned enterprises (SOEs), denoted by  $V_t$ , is assumed to evolve according to the following stochastic differential equation (SDE):

$$\frac{dV_t}{V_t} = (r - \lambda m)dt + \sigma dB_t^H + (Y_j - 1)dN_t^Q \quad (5)$$

Where  $\lambda$  is the random jump intensity,  $\sigma$  is the volatility of the return on  $V_t$ .  $\{B_t^H: t \geq 0\}$  follows a Fractional Brownian Motion with a Hurst parameter  $H \in (0,1)$ , and  $m$  is the mean adjustment term for the jump term.  $Y_j$  is used to describe the random jump amplitude. It is a series of independent and identically distributed non-negative random variables.  $Y_j$  follows an asymmetric double exponential distribution with a probability density [13]:

$$f(y) = p\eta_1 e^{-\eta_1 y} I_{\{y \geq 0\}} + q\eta_2 e^{-\eta_2 y} I_{\{y < 0\}}, \quad (p \geq 0, \eta_1 \geq 1, \eta_2 \geq 0) \quad (6)$$

Where  $p, q (p + q = 1)$  represent the probabilities of jumping upwards and downwards, respectively, and  $\eta_1$  and  $\eta_2$  are the positive and negative mean values of the jump, respectively.  $m$  is the mean of the jump, satisfying  $E(Y_j - 1) = m$ , which can be expressed by the following formula:  $m = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1$

Following Rostek (2012)[19], the asset value of local government financing vehicles (LGFVs) can be expressed as:

$$V_T = V_0 \exp((r - \lambda m)T - \frac{1}{2}\sigma^2 T^{2H} + \sigma B_T^H) \prod_{j=1}^{N(T)} e^{Y_j}$$

Let  $X_T = \ln V_T$ , then:

$$X_T = X_0 + (r - \lambda m)T - \frac{1}{2}\sigma^2 T^{2H} + \sigma B_H(T) + \sum_{j=1}^{N(T)} Y_j \quad (7)$$

When  $H \neq \frac{1}{2}$ ,  $B_t^H$  is neither a semi-martingale nor a martingale. As a result, classical stochastic calculus techniques based on Itô integrals are no longer applicable for pricing derivative securities written on such processes. To overcome this limitation, we instead rely on the characteristic function of the asset price distribution and employ Fourier transform-based numerical methods to compute option values. The Fourier transform approach is a widely used technique for handling complex characteristic functions and is particularly well suited to asset pricing models that incorporate both jump-diffusion features and fractal dynamics. The central idea is to recover the asset price (or return) distribution via an inverse Fourier transform of its characteristic function, and subsequently evaluate the discounted expected payoff of the option.

Let  $\psi(u) = E(e^{iuX_T})$  denotes the characteristic function of  $X_T$ . As  $\{Y_j - 1\}$ ,  $\{B_t^H: t \geq 0\}$ ,  $\{N_t: t \geq 0\}$  are independent of each other, the  $\psi(u)$  is:

$$\psi(u) = e^{iu(X_0 + (r - \lambda m)T - \frac{1}{2}\sigma^2 T^{2H})} \cdot E^Q \left[ e^{iu\sigma B_H(T)} \right] \cdot E^Q \left[ e^{iu \sum_{j=1}^{N(T)} Y_j} \right] \quad (8)$$

Where

$$E^Q \left[ e^{iu\sigma B_H(T)} \right] = e^{-\frac{(iu)^2 \sigma^2 T^{2H}}{2}}, \quad E^Q \left[ e^{iu \sum_{j=1}^{N(T)} Y_j} \right] = e^{\frac{p\eta_1}{\eta_1 - iu} + \frac{q\eta_2}{\eta_2 + iu} T} \quad (9)$$

Let  $C_T(K)$  be a European call option with an underlying asset  $V_T = e^{X_T}$  and a strike price of  $K$ . Assume  $k = \ln(K)$ . To ensure integral convergence, define a parameter  $\alpha > 0$ , satisfying:

$$c_\alpha(k) = e^{\alpha k} C_T(K) \quad (10)$$

Then the characteristic function after the Fourier transforms of the European call option  $\varphi(\xi)$  is:

$$\varphi(\xi) = \int_{-\infty}^{\infty} e^{i\xi k} c_\alpha(k) d\xi = \frac{e^{-rT} \psi(\xi - (1+\alpha)i)}{\alpha^2 + \alpha - \xi^2 + (1+2\alpha)i\xi} \quad (11)$$

Therefore,  $c_\alpha(k)$  can be obtained through the inverse Fourier transform.

$$c_\alpha(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik\xi} \varphi(\xi) d\xi \quad (12)$$

Then get:

$$C_T(K) = e^{-\alpha k} c_\alpha(k) = e^{-\alpha k} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik\xi} \varphi(\xi) d\xi \quad (13)$$

Substitute equation (8) into equation (11) to obtain the characteristic function expression corresponding to  $c_T(k)$  under the risk-neutral probability measure.

Because values of put options and value of call options have the following relationship:

$$P_T(K) = C_T(K) + e^{-rT}K - V_0 \quad (14)$$

Therefore, the value of the put option is:

$$E[\min(K - V_T)^+] = P_T(K) = e^{-\alpha k} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik\xi} \varphi(\xi) d\xi + e^{-rT}K - V_0 \quad (15)$$

If the execution price becomes  $\theta K$ , let  $k_\theta = \ln(\theta K)$ , then we have:

$$E[\min(\theta K - V_T)^+] = P_T(\theta K) = e^{-\alpha k} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik_\theta \xi} \varphi(\xi) d\xi + e^{-rT}\theta K - V_0 \quad (16)$$

Substituting equations (14) and (15) into equation (3), we get:

$$P_{bond} = e^{-rT}\theta K - e^{-\alpha k} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik\xi} \varphi(\xi) d\xi + e^{-\alpha k_\theta} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik_\theta \xi} \varphi(\xi) d\xi \quad (17)$$

#### 4. Summation Results by FFT

We employ the fast Fourier transform (FFT) method proposed by Carr and Madan (1999) to compute the European call option in equation (17)[20]. Subsequently, the value of bonds incorporating implicit government guarantees can be determined.

Let  $\Delta k$  denotes the regular spacing of size, and  $\Delta v = \frac{2\pi}{\Delta k N}$  is the frequency step size, and let:

$$\xi_j = j\Delta v, \quad j = 0, 1, \dots, N - 1 \quad (18)$$

The log strike  $k_m$  ranging from  $-b$  to  $b$ , and  $k_m$  is:

$$k_m = -b + m\Delta k \quad m = 0, 1, \dots, N - 1 \quad (19)$$

Where  $b = \frac{N\Delta k}{2}$ . Following Huang and Zhu (2014)[21], the value of a European call option is:

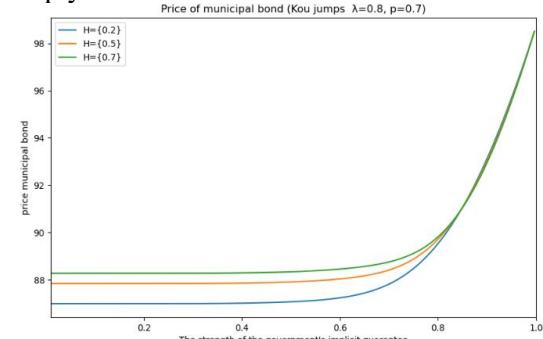
$$C_T(k_m) \approx \frac{e^{-\alpha k}}{\pi} \left[ \sum_{j=1}^N e^{-i\xi_j k_m} e^{ib\xi_j} \psi(\xi_j) \omega_j \right] \quad (20)$$

Where  $\omega_j$  are integration weights. We incorporate Simpson's rule weightings into our algorithm.

To obtain a square-integrable function, the parameter  $\alpha$  is typically chosen to be between 0.5 and 2. Based on experience, we set the value of  $\alpha=1$  and  $N = 1024$ . Finally, Other parameters are set as follows:  $\lambda = 0.8, p = 0.7, q = 0.3, \eta_1 = 3, \eta_2 = 5, r = 0.03, \sigma = 0.2$ , and  $T = 0.5$ .

As illustrated in Figure 1, the simulation results indicate a pronounced positive relationship between the strength of the implicit government

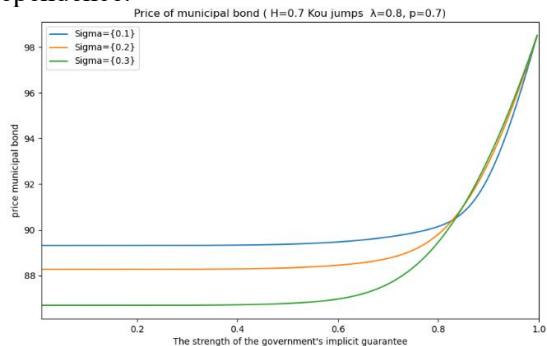
guarantee and the price of MIBs. As the implicit guarantee strengthens, bond prices increase steadily, gradually approaching the par value of 100. At lower levels of guarantee strength, the increase in bond prices is relatively modest; however, once the implicit guarantee exceeds approximately 0.8, the sensitivity of bond prices to changes in guarantee strength rises sharply, producing a noticeably steeper upward slope. This behavior reflects the underlying nature of implicit guarantees: they do not constitute formal, legally binding commitments, but rather represent contingent support contingent on the government's willingness to intervene. When the implicit guarantee is weak, market expectations regarding potential government bailouts are limited, and MIBs prices resemble those of conventional corporate bonds subject to default risk, resulting in muted responsiveness to changes in guarantee strength. In contrast, when the guarantee surpasses a critical threshold and signals of potential government intervention become clearer, market expectations of government backing increase rapidly, causing bond prices to become highly sensitive to further increases in guarantee strength and to rise sharply.



**Figure 1. Strength of the Implicit Government Guarantee and the Bond Price with Different H**

Figure 1 further demonstrates that, regardless of whether the fractional Hurst parameter  $H$  is less than 0.5, equal to 0.5, or greater than 0.5, bond values converge to the same level as the implicit guarantee strength approaches 1. However, at lower levels of implicit guarantee strength, the dynamics of asset prices associated with different Hurst parameters produce notable differences in bond values. Holding other parameters constant, a higher Hurst parameter corresponds to a higher bond value. This finding highlights the significant role of long-term dependence in asset prices for the valuation of MIBs. A larger Hurst parameter indicates

stronger trend persistence or long-memory effects, resulting in a higher expected asset value trajectory and, consequently, an elevated bond value. From a modeling perspective, a higher Hurst parameter may more accurately capture the behavior of state-owned enterprise (SOE) asset prices, which are not solely driven by random market fluctuations but are also influenced by factors such as policy interventions and government support, thereby exhibiting stronger persistence or structural dependence.



**Figure 2. Strength of the Implicit Government Guarantee and the Bond Price with Different  $\sigma$**

Furthermore, we examine the relationship between asset volatility and bond prices, as illustrated in Figure 2. When the implicit guarantee strength is low, bonds with higher asset volatility exhibit substantially lower prices, whereas bonds with lower volatility maintain relatively higher prices. This observation is consistent with the traditional credit risk pricing framework: in the absence of strong guarantees, higher asset volatility corresponds to greater default risk, thereby reducing bond value.

As the implicit guarantee strength increases, however, the prices of high-volatility bonds rise more rapidly than those of low-volatility bonds. Once the implicit guarantee exceeds approximately 0.8, this pattern reverses, and high-volatility bonds begin to surpass low-volatility bonds in value. This phenomenon can be attributed to the fact that higher asset volatility enhances the value of the “rescue option” embedded in the implicit guarantee. In other words, increased volatility raises the likelihood of triggering government intervention, thereby amplifying the effect of the implicit guarantee and providing an additional boost to bond value.

Finally, as the implicit guarantee strength approaches 1 (i.e., a full guarantee), bond prices converge to the level of risk-free bonds

regardless of volatility, consistent with the theoretical expectation that a complete guarantee effectively eliminates default risk.

## 5. Conclusion

The numerical simulation results further support the practical relevance of modeling the asset dynamics of SOEs in China using fractional Brownian motion combined with asymmetric jump processes. First, the asset prices of local SOEs often exhibit pronounced long-term dependence and trend persistence. Their operational performance, financing capacity, and asset expansion are continuously influenced by factors such as local government policy guidance, fiscal support, and administrative coordination. Fractional Brownian motion, through the Hurst parameter, captures the long-memory and autocorrelation structure of asset prices, thereby more accurately reflecting the persistent dynamics arising from “policy inertia,” “credit continuation,” and other non-market-driven fluctuations in SOE asset changes.

Second, the local fiscal system and urban investment bond market in China are characterized by pronounced jump risk. For instance, tightening of regulatory supervision over local government financing platforms, fluctuations in land revenues, investigations of hidden debt, credit events, and policy-induced bailouts can all trigger abrupt jumps in asset values. Notably, these jumps typically exhibit directional asymmetry: downward jumps, often resulting from policy tightening, default events, or fiscal pressures, tend to be large in magnitude and clustered in occurrence; in contrast, upward jumps, mainly stemming from government bailouts, relaxed refinancing policies, or special support, are intermittent but can have substantial impacts. Incorporating asymmetric jump processes therefore not only captures the extreme risk features of local government financing vehicle asset prices, but also reflects the structural volatility asymmetry induced by policy interventions and credit contagion.

In summary, the integration of fractional Brownian motion with asymmetric jump processes not only provides a realistic representation of the asset dynamics of SOEs, but also establishes a theoretically richer stochastic framework. This framework is well aligned with the Chinese institutional context and is particularly suitable for analyzing implicit

guarantee mechanisms, the evolution of default risk, and the pricing behavior of MIBs.

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