Research on Universal Life Insurance Pricing under Stochastic Interest Rates and the Ornstein-Uhlenbeck Process

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Abstract: Universal life insurance typically consists of two components: a protection account and an investment account. The protection account is established to cover insurance liabilities and is used to pay risk charges and claim benefits. The investment account, funded by premiums paid by policyholders, is managed by the insurer for investment purposes in order to achieve asset appreciation. Within this framework, this paper assumes that the asset value of the investment account follows an exponential Ornstein–Uhlenbeck (O-U) process, while the risk-free interest rate follows the Vasicek stochastic interest rate model. A pricing model for universal life insurance is then developed under a stochastic interest rate environment. Since the value structure of the investment account exhibits characteristics of an embedded option, it can be regarded as an embedded option pricing problem. By combining actuarial pricing methods with option pricing theory, this study analyzes the pricing mechanism of universal life insurance products without relying on the assumption of a complete market, aiming to provide theoretical support for product design and risk management.

Keyword: Universal Insurance; O-U Process; Random Interest Rate; Insurance Actuarial Method

1. Introduction

As economies grow and markets evolve, insurance products have expanded beyond their traditional role as tools for risk protection. Increasingly, they serve as instruments for investment and asset management. Among these, universal life insurance stands out as a hybrid product that combines the protective features of insurance with the wealth accumulation potential of investment. However, the pricing of universal life insurance is influenced by complex factors, such as stochastic interest rates, which introduce significant uncertainty. To address these challenges, actuarial methods must be employed to develop accurate pricing models that enhance both the investment value and risk management capabilities of these products. This paper aims to explore the pricing of universal life insurance under stochastic interest rate models, providing the insurance industry with scientifically grounded and practical pricing strategies.

Stochastic interest rates, which reflect the random fluctuations in asset prices, have a profound impact on the investment portfolios and asset allocation strategies of insurance companies. These fluctuations directly affect the pricing and risk management of insurance products. Universal life insurance, in particular, faces additional challenges, such as uncertainties in premium income and the complexities of profit distribution. For insurance companies, establishing a rational and stable pricing strategy for universal life insurance is essential for improving economic efficiency and strengthening risk management. At the same time, understanding the principles behind universal life insurance pricing and risk control can help investors make more informed and rational decisions. Therefore, research on universal life insurance pricing under stochastic interest rate models is of both theoretical and practical importance [1].

Actuarial methods, which are based on mathematical statistics and risk analysis, provide a systematic approach to evaluating the risk levels and premium income of insurance products. In this study, we apply actuarial techniques to develop pricing models for universal life insurance under stochastic interest rate framework [2]. Specifically, we construct mathematical models to identify optimal pricing strategies for universal life insurance. The research considers the impact of stochastic interest rates on the asset pricing of universal life insurance and examines the close relationship between universal life insurance investment accounts and option pricing. Using actuarial

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methods, we further analyze the pricing of European call and put options under the Ornstein-Uhlenbeck (O-U) process [3].

2. Preliminary Knowledge

2.1 Brownian Motion, Martingales, and the Black-Scholes Model

Definition 1 A Brownian motion $\{B_1(t), t \geq 0\}$ satisfies the following properties:

(1) For $\forall t > s$, $B_1(t) - B_1(s)$ is independent of $\{B_1(u), 0 \leq u \leq s\}$.

(2) For $\forall t > s$, $B_1(t) - B_1(s)$ follows a normal distribution with mean 0 and variance $t - s$, i.e., $B_1(t) - B_1(s) \sim N(0, t - s)$.

(3) $B_1(t)$ is a continuous function of time t for $t \geq 0$.

(4) $B_1(0) = 0$.

Lemma 1 Suppose $B_1(t) \sim N(0, {\sigma_1}^2)$, $B_2(t) \sim N(0, {\sigma_2}^2)$, and the correlation coefficient between $B_1(t)$ and $B_2(t)$ is ρ. Then, for $\forall\, a, b, c, d, m \in R$, the following holds:

$$E\left[e^{cB_1+dB_2} I_{\{aB_1+bB_2 \geq m\}}\right] = exp\left\{\frac{c^2{\sigma_1}^2 + d^2{\sigma_2}^2 + 2\rho cd\sigma_1\sigma_2}{2}\right\} * \Phi\left(\frac{ac^2{\sigma_1}^2 + bd^2{\sigma_2}^2 + \rho(ad+bc)\sigma_1\sigma_2 - m}{\sqrt{a^2{\sigma_1}^2 + b^2{\sigma_2}^2 + 2\rho ab\sigma_1\sigma_2}}\right) \quad (1)$$

Definition 2 Let $\{X_1(t)\}$ be a stochastic process adapted to the filtration $\{F_t\}$, and suppose $E[X_1(t)] < \infty$ for $\forall\, t$. If for $\forall\, 0 < u < s$, $E[X_1(s)|\, F_u] = X_1(u)$, then $\{X_1(s)\}$ is a martingale.

To facilitate the later valuation of option-type components embedded in universal life insurance, we briefly recall the standard Black–Scholes framework [4]. Rather than repeating all classical assumptions in detail, we only retain the ingredients that are needed in the subsequent analysis. We consider a frictionless market with continuous trading, no arbitrage, and no dividend payments during the life of the option. Under this setting, the price of the risk-free asset $B_1(t)$ and the risky asset $S_1(t)$ evolves as:

$$\begin{aligned} dB_1(t) &= rB_1(t)dt \\ dS_1(t) &= S_1(t)[\mu dt + \sigma dB_1(t)] \end{aligned} \quad (2)$$

where r denotes the constant risk-free rate, μ is the drift of the risky asset, and σ is its volatility. Here $B_1(t)$ in the diffusion term represents a standard Brownian motion.Within this framework, the time-0 prices of a European call option and a European put option with strike price K and maturity T are given by:

$$\begin{aligned} C &= S_1(0)\Phi(d_1) - Ke^{-rT}\Phi(d_2) \\ P &= Ke^{-rT}\Phi(-d_2) - S_1(0)\Phi(-d_1) \end{aligned} \quad (3)$$

Where,

$$\begin{aligned} d_1 &= \frac{ln\left(\frac{S_1(0)}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \\ d_2 &= \frac{ln\left(\frac{S_1(0)}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \end{aligned} \quad (4)$$

In the above expressions, $S_1(0)$ is the initial price of the underlying asset, K is the strike price, T is the maturity, and $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. In this paper, these formulas are not introduced for their own sake; instead, they serve as a benchmark for understanding the option-based decomposition that will later appear in the pricing of universal life insurance. The subsequent model departs from the constant-rate Black–Scholes environment by allowing both the investment account and the interest rate to evolve stochastically.

2.2 Actuarial Pricing of Options

We next introduce the actuarial pricing framework that will be used in the valuation of the embedded option features contained in the universal life contract. Consider a financial market on the time interval $[0, T]$ with two tradable assets. One is a risk-free asset associated with the short rate process, and the other is a risky asset with price $S_1(t)$ at time t. The process $\{S_1(t), t \geq 0\}$ is assumed to be defined on a complete probability space, and T denotes the maturity date.

Definition 3 Let $r(s)$ denote the risk-free interest rate. The cumulative expected return of the risky asset over $[0, T]$, written as $\int_0^T \beta(s)\, ds$, is defined through:

$$e^{\int_0^T \beta(s)ds} = \frac{E[S_1(T)]}{S_1(0)} \quad (5)$$

This definition characterizes the average growth rate of the risky asset over the contract horizon and will be used to express the actuarial values of contingent claims without imposing a complete-market replication argument.

Definition 4 The actuarial price at time $t = 0$ of a European call option with strike price K and

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maturity T is defined by:

$$C(K,T,S_1(0)) = E\left[max\left\{e^{-\int_0^T \beta(s)ds}S_1(T) - Ke^{-\int_0^T r(s)ds}, 0\right\}\right] \quad (6)$$

Definition 5 Similarly, the actuarial price at time $t = 0$ of a European put option with strike price K and maturity T is defined by:

$$P(K,T,S_1(0)) = E\left[max\left\{Ke^{-\int_0^T r(s)ds} - e^{-\int_0^T \beta(s)ds}S_1(T), 0\right\}\right] \quad (7)$$

Compared with the classical risk-neutral pricing approach, the actuarial method values the option payoff by combining the stochastic discount factor with the expected accumulation behavior of the underlying asset. This representation is particularly convenient in the present study, because the payoff structure of the universal life investment account will later be decomposed into call- and put-type components under stochastic interest rates.

2.3 Exponential O-U Process

The basic assumptions of the exponential O-U process model [5] are as follows: Consider a risky security (e.g., a stock) in a complete, arbitrage-free financial market with continuous trading. The price process of this security at time t satisfies the following stochastic differential equation:

$$dS_1(t) = S_1(t)[(\mu - \alpha \ln S_1(t))dt + \sigma_s dB_s(t)] \quad (8)$$

where $\{B_s(t), 0 \le t \le T\}$ is a standard Brownian motion, σ_s is the volatility parameter, and μ and α are constants. The parameter α governs the speed of mean reversion in the log-price process. When α is close to zero, the mean-reverting effect becomes weak, and the model approaches the Black–Scholes-type setting [6].

Lemma 2 If the price $S_1(t)$ of the underlying asset follows the exponential O-U process, then:

$$S_1(T) = S_1(t)^{e^{\alpha(t-T)}}\, exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)\int_t^T e^{\alpha(s-T)}ds + \sigma\int_t^T e^{\alpha(s-T)}dB_s(s)\right\} \quad (9)$$

Proof: Using $Itô$ formula, we have: $d\ln S_1(t) = \left(\mu - \sigma\ln S_1(t) - \frac{1}{2}\sigma^2\right)dt + \sigma dB_s(t)$. Thus, $d(e^{\alpha t}\ln S_1(t)) = e^{\alpha t}\left[\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dB_s(t)\right]$. Integrating both sides from t to T and rearranging terms, **Lemma 2** is proven.

Lemma 3 If the price $S(t)$ of the underlying asset follows the exponential O-U process, then the expected return $\int_t^T \beta(s)ds$ of $S(t)$ over the interval $[t,T]$ is given by:

$$\int_t^T \beta(s)ds = \left(e^{\alpha(t-T)} - 1\right)\ln S(t) + \left(\mu - \frac{1}{2}\sigma^2\right)\int_t^T e^{\alpha(s-T)}ds + \frac{1}{2}\sigma^2\int_t^T e^{2\alpha(s-T)}ds \quad (10)$$

Proof: $E[S(T)]$

$$= S(t)^{e^{\alpha(t-T)}}\, exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)\int_t^T e^{\alpha(s-T)}ds\right\}E\left[exp\left\{\sigma\int_t^T e^{\alpha(s-T)}dB_s(s)\right\}\right]$$

$$= S(t)^{e^{\alpha(t-T)}}\, exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)\int_t^T e^{\alpha(s-T)}ds + \frac{1}{2}\sigma^2\int_t^T e^{2\alpha(s-T)}ds\right\}$$

That is: $\frac{E[S(T)]}{S(t)} = \int_t^T \beta(s)ds$

$$= \left(e^{\alpha(t-T)} - 1\right)ln\, S(t) + \left(\mu - \frac{1}{2}\sigma^2\right)\int_t^T e^{\alpha(s-T)}ds + \frac{1}{2}\sigma^2\int_t^T e^{2\alpha(s-T)}ds$$

Lemma 3 is proven.

2.4 Stochastic Interest Rate Models

2.4.1 Vasicek Model

(1) Introduction to the Vasicek Model

To incorporate uncertainty in discounting, the short-term risk-free interest rate is assumed to follow a Vasicek Model [7]. This specification is adopted because it captures mean reversion in a tractable way, which is particularly useful in the valuation of long-term insurance liabilities. In the present setting, the stochastic behavior of the short rate affects both the discount factor applied to future benefits and the actuarial values of the option-type components embedded in the universal life contract.

The short rate $r(t)$ is assumed to satisfy:

$$dr(t) = [a - br(t)]dt + \sigma_r dB_r t \quad (11)$$

where a and b are constants, σ_r is the volatility of the interest rate, and $\{B_r(t), 0 \leq t \leq T\}$ is a standard Brownian motion. The parameter b measures the speed of mean reversion, while a/b represents the long-run level toward which the short rate tends to move.

In the Vasicek model, the parameter $\frac{a}{b}$ represents the long-term average value of the short-term interest rate. From the model, it can be observed that when $r(t) > \frac{a}{b}$, the equation exhibits a negative drift term, meaning that, on average, the short-term risk-free interest rate $r(t)$ will decrease. Conversely, when $r(t) < \frac{a}{b}$, the equation has a positive drift term, indicating that the short-term risk-free interest rate $r(t)$ will increase. Thus, the changes in the interest rate $r(t)$ tend to follow a trajectory that converges toward the mean level $\frac{a}{b}$. From this perspective, the Vasicek model has strong economic foundations: when the interest rate $r(t)$ is high, borrowers reduce their demand for funds, leading to a decrease in $r(t)$; when the interest rate $r(t)$ is low, borrowers have a higher demand for funds, causing $r(t)$ to rise.

In practice, the instantaneous interest rate $r(t)$ is generally positive. However, in the Vasicek model, $r(t)$can potentially take negative values, which is inconsistent with market norms. Nevertheless, compared to the Merton model, the Vasicek model accounts for the impact of volatility, making it a more precise representation of how the mean of interest rates fluctuates over time.

(2) Actuarial Pricing of Zero-Coupon Bonds under the Vasicek Model

Lemma 4 If the short rate $r(t)$ follows the Vasicek process in (11), then for any $t < s$:

$$r(s) = e^{-b(s-t)}r(t) + \int_t^s a\, e^{-b(s-t)}du + \int_t^s \sigma_r\, e^{-b(s-t)}dB_r(u) \quad (12)$$

Proof: Using $It\hat{o}$'s formula, we have: $d(e^{bt}r(t)) = ae^{bt}dt + e^{bt}dB_r(t)$. Thus for any $t < s$, integrating both sides from t to s, we obtain: $r(s) = e^{-b(s-t)}\, r(t) + \int_t^s a\, e^{-b(s-t)}\, du + \int_t^s \sigma_r\, e^{-b(s-t)}\, dB_r(u)$

Thus, Lemma 4 is proven.

Lemma 5 Let $p(t,T)$ denote the actuarial price at time t of a zero-coupon bond maturing at time T. Then:

$$\int_t^T r(s)ds = -\ln p(t,T) + \frac{1}{2}\int_t^T (\sigma_r\theta(s,T))^2 ds + \int_t^T \sigma_r\, \theta(s,T)dB_r(s) \quad (13)$$

Where, $\theta(s,T) = \int_s^T e^{b(s-u)}du$.

Proof: From Lemma 4, we know: $\int_t^T r(s)ds = r(t)\,\theta(s,T) + \frac{a}{b}(T - t - \theta(s,T)) + \int_t^T \sigma_r\, \theta(s,T)dB_r(s)$ Since: $p(t,T) = E\left[e^{-\int_t^T r(s)ds} \mid F_t\right] = exp\left\{-r(t)\theta(s,T) + \frac{a}{b}(T - t + \theta(s,T)) + \frac{1}{2}\int_t^T (\sigma_r\theta(s,T))^2 ds\right\}$.Thus, Lemma 5 is proven.

3. Pricing Model for Universal Life Insurance under the O-U Process Stochastic Interest Rate

3.1 Basic Introduction to the Model

It is well known that the policy value of universal life insurance is significantly affected by interest rate fluctuations, particularly since the asset value of the investment account is closely linked to market performance. To mitigate the risks associated with fixed interest rates, it is necessary to analyze the policy value of universal life insurance under stochastic interest rates. The specific content of this model is as follows: assume that an insured individual, aged x, purchases a universal life insurance policy at the initial time with a term of T. The policy value is denoted as U. If the insured passes away at some time t during the insurance period, the insurance company will pay the death benefit at the end of that year. The death benefit is the greater of the risk sum insured and the asset value of the investment account. If the insured survives until the end of the insurance term, the insurance company will pay the asset value of the investment account, which is guaranteed to have a minimum fixed return.

3.2 Assumptions and Notations of the Model

3.2.1 Basic Assumptions of the Model

The assumptions regarding the value of the universal life insurance investment account and

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the interest rate are as follows:

(1) The book value of the universal life insurance investment account, $S_1(t)$, follows an exponential O-U process, expressed as:

$$dS_1(t) = S_1(t)[(\mu - \alpha \ln S_1(t))dt + \sigma_s dB_s(t)] \quad (14)$$

Where, σ_s is the volatility of the investment account assets, $B_s(t)$ is a standard Brownian motion, and μ and α are constants.

(2) The risk-free interest rate, $r(t)$, follows the Vasicek model, expressed as:

$$dr(t) = [a - br(t)]dt + \sigma_r dB_r(t) \quad (15)$$

Where, σ_r is the volatility of the interest rate, $B_r(t)$is a standard Brownian motion, and a and b are constants.

3.2.2 Notations of the Model

The notations used in this chapter for the model are as follows:

$S_1(t)$ ——denotes the book value of the investment account at time t. It is the main state variable governing the investment component of the contract;

g——denotes the minimum guaranteed annual interest rate credited to the investment account;

$G(t)$ ——denotes the guaranteed accumulation value at time t. Under the minimum guarantee mechanism, it represents the benchmark value that the account must at least attain over time;

$A(t)$ ——The asset value of the investment account of the universal life insurance at time t, where $A(t) = max\{S(t), G(t)\}$, i.e., the greater of $S(t)$ and $G(t)$;

F——denotes the risk sum insured specified in the policy. It is the guaranteed protection amount used in determining the death benefit;

U——denotes the single premium paid by the policyholder at time 0, which is also the quantity to be determined in the pricing model;

$b(t)$——denotes the death benefit paid by the insurance company to the beneficiary when the insured dies at time t, where $b(t) = max\{A(t), F\}$;

$r(t)$ ——denotes the risk-free interest rate, which follows the Vasicek model;

q_{x+t}——denotes the probability that an insured aged x dies within t years after policy issue;

${}_tp_x$——denotes the probability that an insured aged x survives for at least t years after policy issue;

T——denotes the insurance term of the policy;

σ_s——denotes the volatility of $S_1(t)$, which is constant;

σ_r——denotes the volatility of $r(t)$, which is constant;

$p(t,T)$ ——denotes the actuarial price of a zero-coupon bond with maturity T at time t;

P_t——denotes the actuarial price of a European put option with strike price K and maturity T at time t;

C_t——denotes the actuarial price of a European call option with strike price K and maturity T at time t.

3.3 Model Construction and Derivation

$$\begin{cases} C_t = C(K,T,S_1(t),r(t)) = S_1(t)\Phi(d_1) - Kp(t,T)\Phi(d_2) \\ P_t = P(K,T,S_1(t),r(t)) = Kp(t,T)\Phi(-d_2) - S_1(t)\Phi(-d_1) \end{cases} \quad (16)$$

When the insured dies, the death benefit is $b(t)$. The expression and simplification of $b(t)$ are as follows:

$$\begin{aligned} & b(t) \\ & = max\{F, A(t)\} = A(t) + max\{F - A(t), 0\} \\ & = A(t) + max\{F - max\{S_1(t), G(t)\}, 0\} \end{aligned} \quad (17)$$

From the above simplification, it can be seen that when $F > G(t)$, $b(t)$ can be regarded as the asset value of the investment account $A(t)$ plus the difference between the payoff values of two European put options with strike prices F and $G(t)$at maturity [8-10].

On the other hand, for $b(t)$, we have:

$$\begin{aligned} b(t) &= A(t) + max\{F - A(t), 0\} \\ &= \begin{cases} A(t), & F \le G(t) \\ A(t) + max\{F - S_1(t), 0\} - max\{G(t) - S_1(t), 0\}, & F > G(t) \end{cases} \\ &= \begin{cases} A(t), & F \le G(t) \\ S_1(t) + max\{F - S_1(t), 0\}, & F > G(t) \end{cases} \end{aligned} \quad (18)$$

From the above, it is evident that when $F \le G(t)$, $b(t)$ is equivalent to a "fund" with a guaranteed return, and the risk sum insured F does not play a role. When $F > G(t)$, $b(t)$ is equivalent to the actual book value of the investment account $S_1(t)$ plus a European put option with a strike price of F.

In summary, the risk sum insured F only takes effect when $F > G(t)$. Therefore, the following discussion assumes $F > G(t)$. Based on the

described death benefit and maturity benefit of the model, the policy value at the initial time can be expressed as:

$$U = \sum_{t=1}^{T} {}_{t-1|}p_x q_{x+t-1} E\left[e^{-\int_0^t r(s)ds} b(t)\right] + {}_{T|}p_x E\left[e^{-\int_0^T r(s)ds} A(T)\right] \tag{19}$$

Substituting the above results, we obtain:

$$\begin{aligned} U &= \sum_{t=1}^{T} {}_{t-1|}p_x q_{x+t-1} E\left[e^{-\int_0^t r(s)ds} b(t)\right] + {}_{T|}p_x E\left[e^{-\int_0^T r(s)ds} A(T)\right] \\ &= \sum_{t=1}^{T} {}_{t-1|}p_x q_{x+t-1} [S(0) + P(F, T, S(t), r(t))] + {}_{T|}p_x [S(0) + P(G(T), T, S(T), r(T))] \\ &= S(0) + \sum_{t=1}^{T} {}_{t-1|}p_x q_{x+t-1} P(F, T, S(t), r(t)) + {}_{T|}p_x P(G(T), T, S(T), r(T)) \end{aligned} \tag{20}$$

4. Numerical Simulation and Sensitivity Analysis

4.1 Parameter Assumptions

In this section, we take a 30-year-old male as an example. The mortality rate is based on the newly released fourth life table for non-pension business (CL1) by the National Financial Regulatory Administration (NFRA). The following parameter assumptions are made: $\sigma_s = 0.1$, $g = 1.5\%$, $T = 20$, $S(0) = 1000$, $G(0) = 1000$, $F = 10000$, $\frac{a}{b} = 0.02$, $b = 0.2$, $\sigma_r = 0.2$

4.2 Simulation Results

Using the single premium formula (20) derived in the previous section and the baseline parameter settings, 50,000 simulated paths were generated to jointly simulate the interest rate and the investment account value. The resulting single premium for the universal life insurance is: $U = 1150.15$

Further decomposition of the single premium into the present value of death benefits and the present value of maturity benefits reveals that: The present value of death benefits is approximately 212.80, accounting for about 18.5% of the total premium; The present value of maturity benefits is approximately 937.35, accounting for about 81.5% of the total premium. This result indicates that under the assumption of a long insurance term, the majority of insured individuals survive until maturity. Consequently, the value of the universal life insurance contract is primarily determined by the maturity benefit, while the death benefit serves as a supplementary component with option-like characteristics. This numerical result is highly consistent with the theoretical structure of the model and the actual features of insurance contracts.

4.3 Sensitivity Analysis of Key Parameters

To further analyze the impact of parameter changes on the pricing results of universal life insurance, we selected four parameters with significant economic implications for pricing: the minimum guaranteed interest rate g, the risk amount insured F, the interest rate volatility σ_r, the mean reversion speed of the interest rate b. A one-factor sensitivity analysis was conducted for each parameter while keeping all other parameters constant. Each parameter was assigned 20 evenly spaced values within a reasonable range, and 50,000 simulated paths were used to calculate the corresponding single premium for each value.

When all other parameters remain unchanged, Figure 1 shows that the single premium increases significantly as the minimum guaranteed interest rate g rises, and the increase in the premium mainly comes from the increase in the present value of the maturity benefit, indicating that the minimum guaranteed interest rate has a decisive influence on the pricing of universal insurance; Figure 2 shows that as the risk amount F increases, the lump-sum premium of universal insurance increases approximately linearly. The numerical results indicate that the risk coverage amount only affects the premium through the death benefit portion, verifying the guarantee attribute of the risk coverage amount in the model; Figure 3 shows that the single premium increases significantly as the interest rate volatility σ_r rises, and it presents a non-linear characteristic

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at higher volatility levels, reflecting the significant impact of random interest rate uncertainty on long-term insurance pricing; Figure 4 shows that an increase in the rate mean recovery speed b leads to a monotonic increase in the lump-sum premium of universal insurance, indicating that the interest rate regression characteristic plays an important role in insurance pricing by influencing the long-term discount structure.

From Figure 5 and Figure 6, it can be observed that the minimum guaranteed interest rate and the interest rate volatility are the most crucial factors influencing the pricing of universal insurance under a random interest rate environment; the risk coverage amount mainly affects the death benefit portion, reflecting the protection function of the insurance contract; the speed of the interest rate mean recovery has a secondary but stable impact on the premiums through its influence on the long-term discount structure. Overall, the numerical results not only validate the rationality of the model structure but also provide useful quantitative references for the parameter setting and risk management of universal insurance products.

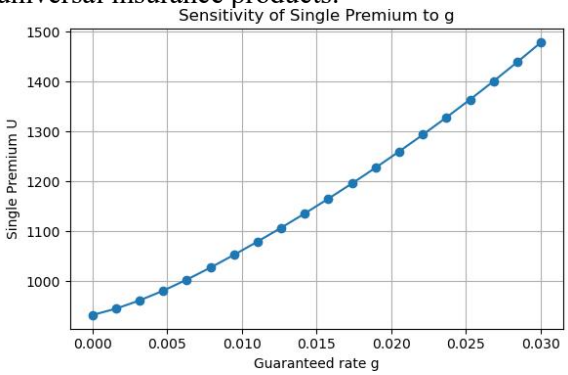

Figure 1. The Impact of the Minimum Guaranteed Interest Rate g on Single Premium

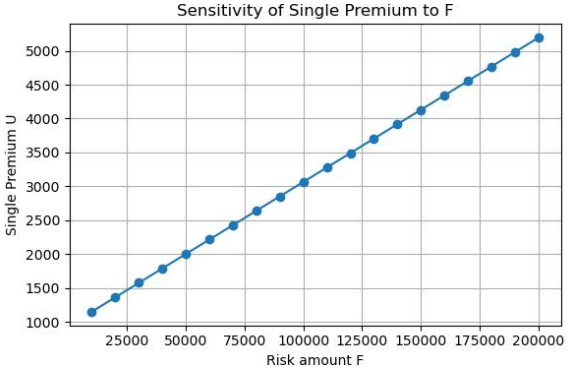

Figure 2. The Impact of the Risk Amount Insured F on Single Premium

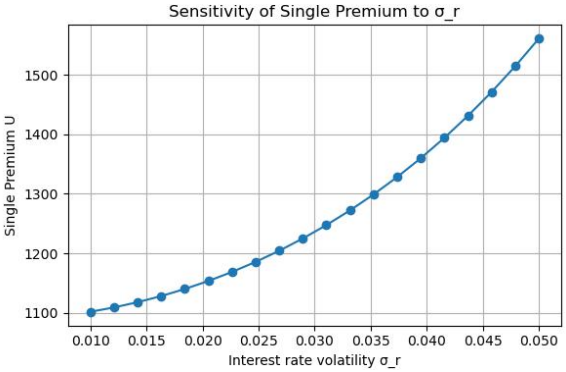

Figure 3. The Impact of the Interest Rate Volatility σ_r on Single Premium

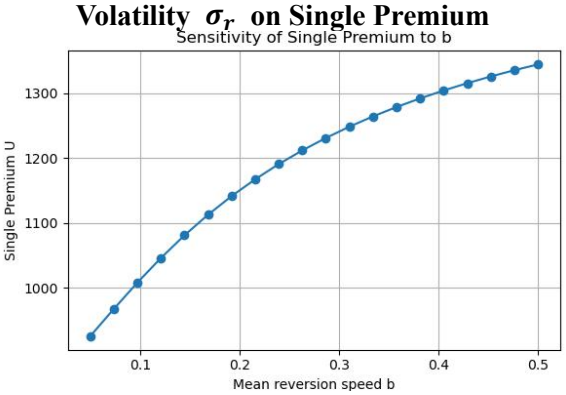

Figure 4. The Impact of the Mean Reversion Speed of the Interest Rate b on Single Premium

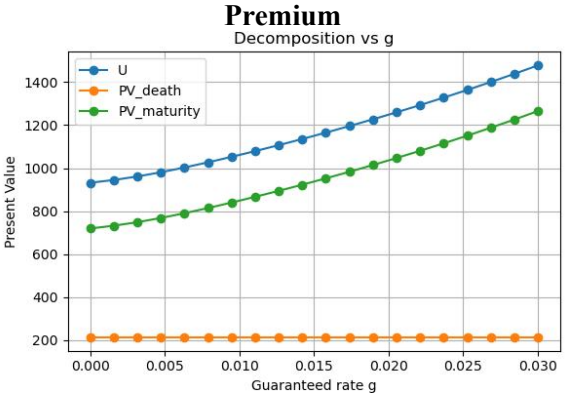

Figure 5. The Decomposition Results of the Impact of the Minimum Guaranteed Interest Rate

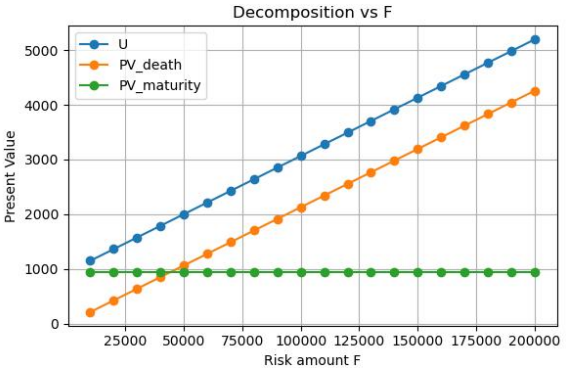

Figure 6. The Decomposition Results of the Impact of the Risk Sum Insured F

5. Conclusion

The financial markets in real life are quite different from the market assumptions set in traditional option pricing methods. They often involve arbitrage, imbalance, and incompleteness. The emergence of actuarial insurance methods has brought about a method that can price options without any market assumptions. Up to now, many experts and scholars have conducted extensive research on the option pricing problem under the actuarial insurance method and the close relationship between universal insurance investment accounts and options. They have achieved fruitful research results. Based on these studies, this paper briefly introduces the origin and development of universal insurance, and focuses on the close relationship between the value of universal insurance investment accounts and option pricing. By using the actuarial insurance method, the value of universal insurance investment accounts is made to follow the exponential O-U process, and under the assumption that the risk-free interest rate follows the Vasicek model, the pricing formula of the universal insurance pricing model is obtained.

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