Portfolio Optimization with Digital Currencies based on Risk Parity Model: A Case Study of Bitcoin

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Abstract: The challenges facing the traditional 60/40 portfolio have prompted exploration into digital assets like Bitcoin for diversification, despite their extreme volatility. This study investigates the integration of Bitcoin into a traditional portfolio using a risk parity approach. We construct and compare a standalone Bitcoin, a traditional 60/40 portfolio consisting of the SPTR500 index and the IEF, and a new risk parity portfolio weighted by the inverse of volatility. Employing the EGARCH(1,1) model for volatility analysis and historical simulation for Value at Risk (VaR) and Expected Shortfall (ES), the empirical results show that the new risk parity portfolio achieves a significantly higher Sharpe ratio and lower tail risk, with extreme losses between -3% and -4%. The study concludes that strategic inclusion of Bitcoin within a dynamically weighted portfolio can significantly enhance overall performance, providing empirical support for integrating digital assets.

Keywords: Bitcoin; Portfolio Optimization; Risk Parity Model; EGARCH Model; Value at Risk (VaR); Expected Shortfall (ES);

1. Introduction

Cryptocurrencies have gradually become a significant component of the global financial market. As the first decentralized digital asset, Bitcoin has garnered widespread attention in investment markets and has been increasingly incorporated into research on asset allocation and risk management. Compared to conventional financial assets, Bitcoin exhibits exceptionally high price volatility, which offers investors substantial opportunities for excess returns while simultaneously increasing the frequency of tail-risk events. Meanwhile, extensive empirical studies have shown that the classic 60/40 portfolio, which is composed of 60% equities and 40% bonds, has effectively balanced returns and risks over the past decades, making it widely adopted by both institutional and individual investors. However, in the context of the global low-interest-rate environment over the last decade, the risk-return profile of the traditional 60/40 portfolio has faced challenges.

Against this backdrop, exploring the integration of digital assets such as Bitcoin into traditional portfolio allocations has emerged as a critical research topic in both academic and practical circles. On one hand, as a non-traditional asset, Bitcoin may exhibit low correlation with stocks and bonds, potentially offering diversification benefits. On the other hand, its high-risk nature poses challenges to portfolio robustness. Therefore, systematically investigating the risk transmission mechanisms between digital assets and traditional financial markets, along with comparing the performance of different asset allocation strategies, holds substantial practical significance.

2. Literature Review

In recent years, digital assets has gradually become a research hotspot in both academia and the practical field. Early studies generally suggested that Bitcoin had a low correlation with stocks and bonds, thus possessing the potential attribute of a safe-haven asset and providing portfolio diversification effects for investors [1] and [2]. However, with the expansion of the Bitcoin market scale, its correlation with traditional financial markets has shown a phased strengthening characteristic [3]. Some studies pointed out that during stable market periods, the correlation between Bitcoin and traditional assets remained low, but during financial fluctuation, Bitcoin exhibited a strong risk spill-over effect [4]. This implies that the hedging property of Bitcoin is not stable, and the effectiveness of using it as a portfolio diversification tool needs to be examined under different market environments.

Volatility modeling is always an important tool for risk management. The traditional GARCH model can effectively capture the autocorrelation characteristics of volatility, but it has limitations in dealing with the common "leverage effect" in financial markets. To address this, Nelson proposed the EGARCH model, which can depict the stronger impact of negative shocks on volatility [5]. In this study, the EGARCH model is used to analyze the volatility characteristics of Bitcoin and different investment portfolios to reveal their potential risk dynamics.

In financial risk management, Value - at - Risk (VaR) and Expected Shortfall (ES) are the most commonly used risk measurement methods. VaR is used to measure the maximum potential loss of an investment portfolio at a given confidence level. In addition, scholars have proposed Expected Shortfall (ES) as a supplement, which measures the average value of losses beyond the VaR level and is considered to more comprehensively reflect the severity of risks [6]. In cryptocurrency research, Cheah and Fry pointed out that due to the frequent extreme fluctuations in the Bitcoin market, traditional VaR models have difficulty fully capturing tail risks, while ES performs better in measuring extreme losses [7]. Based on this, this paper uses both VaR and ES in risk measurement to more accurately assess the risk levels of Bitcoin and different portfolios under extreme conditions.

In recent years, the Risk Parity method has gradually attracted attention. Its core idea is to allocate weights based on the risk levels of assets rather than simply relying on expected returns. Risk parity portfolio is typically located on the efficient frontier, suggesting that risk parity offers the highest expected return at a given risk level [8]. AlHalaseh and Shawawreh constructed investment portfolios in the Markowitz model and the risk-parity model based on different strategies and found that the risk-parity model performs better in risk diversification [9]. After including Bitcoin in the investment portfolio, adjusting the weights based on risk can improve the Sharpe ratio of the portfolio [10]. In view of this, on the basis of the traditional 60/40 portfolio, this paper introduces a new type of portfolio weighted by the inverse of volatility in order to control risks and improve the risk-adjusted return performance.

3. Data and Methodology

3.1 Data

The financial data used in this study primarily comprises 4 items: Bitcoin price data, stock price data, bond price data, and risk-free interest rate data from government securities.

Because of the design of the 7×24-hour trading mode in the cryptocurrency market, in order to be consistent with the data of subsequent analysis, we have deleted the non-trading day data and abnormal values. For the circumstance of missing data of assets on a certain trading day, we directly delete the observed values in the calculation. Finally, three sets of panel data are obtained. The range of the sample data we employed covers every trading day from 2022 to 2024, totaling 754 trading days. The core data sources are as follows:

(1) Bitcoin data: Collected from the daily closing price data of Bitcoin on the Wind Financial Term.

(2) Bond data: Collected from the daily closing price of iShares 7-10 Year Treasury Bond ETF(IEF) on the Wind Financial Term.

(3) Stock data: Collected from the daily index data of the S&P 500 Return Index(SPTR500) on the Wind Financial Term.

(4) Risk-free rate data: Collected from the one-month U.S. Treasury yield (1M Treasury Yield) on the website of the Federal Reserve Bank of St. Louis.

3.2 Methodology

3.2.1 Yield calculation

In the empirical analysis, this paper uses the logarithmic returns of assets instead of simple returns. The reason is that logarithmic returns are time-additive, which can better reflect the cumulative effect of long-term returns, and are statistically closer to a normal distribution. The yield calculation adopts the internationally accepted continuous compound return method:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \quad (1)$$

where p_t represents the closing price of the underlying asset on day t. In addition, through the ADF unit root test, it has been verified that the three sets of return series all meet the stationarity requirement and there is no risk of spurious regression, laying a metrological foundation for the subsequent optimization analysis. The result of ADF test is shown on table 1.

In annualization processing, this paper assumes that a year consists of 252 trading days. Thus, the annualized average of daily returns is calculated by multiplying the mean of the sample daily returns by $\sqrt{252}$, and the annualized volatility is obtained by multiplying the standard deviation of daily returns by $\sqrt{252}$.

Table 1. ADF

Series	ADF test	1% threshold	Stationary
BTC_logreturn	-27.848099	-3.439099	Stationary
SPTR500_logreturn	-19.959084	-3.439111	Stationary
IEF_logreturn	-21.164562	-3.439111	Stationary

3.2.2 EGARCH(1,1) model

To measure the dynamic volatility of the assets, this paper employs the EGARCH(1,1) model (Exponential General Autoregressive Conditional Heteroskedasticity). Its mean equation can be expressed as:

$$r_t=\mu+\epsilon_t \quad , \quad \epsilon_t=\sigma_t z_t \tag{2}$$

where μ is the constant term, σ_t is the conditional standard deviation, and z_t follows an independently and identically distributed standard normal distribution.

The conditional variance equation is:

$$ln\left(\sigma_t^2\right)=\omega+\alpha\frac{|\epsilon_{t-1}|}{\sigma_{t-1}}+\gamma\frac{\epsilon_{t-1}}{\sigma_{t-1}}+\beta ln\left(\sigma_{t-1}^2\right) \tag{3}$$

where ω is the constant term, α measures the sensitivity of volatility to shocks and γ depicts the asymmetric effect of volatility (when $\gamma < 0$, negative shocks have a greater impact on volatility, i.e., the leverage effect), and β reflects the persistence of volatility. By estimating the parameters of the EGARCH model, the volatility clustering and leverage effects of Bitcoin, the traditional portfolio, and the new portfolio can be revealed.

3.2.3 Historical simulation method for value at risk and expected shortfall

This paper uses the Historical Simulation method to calculate the risk measurement indicators VaR and ES. The specific steps are as follows:

1) Calculate the logarithmic return series of the sample.

2) Under the 95% and 99% confidence levels, select the corresponding quantiles of the returns as VaR.

3) Calculate the average of the losses exceeding VaR to obtain the Expected Shortfall (ES).

VaR reveals the maximum possible loss within a given confidence interval, while ES further reflects the severity of tail-end risks. Given the high volatility of the Bitcoin market, the combination of VaR and ES can more comprehensively describe its risk characteristics.

3.2.4 Portfolio construction based on risk parity model

The Risk Parity model is centered on the equitable distribution of portfolio risk across various asset classes. To construct a risk parity portfolio, we can use the inverse volatility approach under the assumption that the correlations among all assets are zero. The weights of each asset in the new risk parity portfolio will be determined using the method of weighting by the reciprocal of volatility. The concrete formula can be expressed as follows:

$$W_{asset}=\frac{\frac{1}{\sigma_{asset}}}{\frac{1}{\sigma_{BTC}}+\frac{1}{\sigma_{stock}}+\frac{1}{\sigma_{bond}}} \tag{4}$$

3.2.5 Sharpe ratio

The Sharpe Ratio is an important indicator for measuring risk-adjusted returns, and it is defined as:

$$SR=\frac{R_p-R_f}{\sigma_p} \tag{5}$$

where R_p is the average return of the portfolio, R_f is the risk-free rate and σ_p is the standard deviation of the portfolio's returns. A higher Sharpe ratio indicates greater excess returns per unit of risk. In the empirical part of this paper, the Sharpe ratios of Bitcoin, the traditional portfolio, and the new portfolio will be compared, and combined with the risk - free rate data, the effectiveness of different portfolios will be evaluated.

4. Empirical Results and Analysis

4.1 Volatility Modeling Results of Bitcoin

Table 2. EGARCH Parameters of BTC

Parameter	Estimate	Std.Error	z-Statistic	p-Value	Signif.
μ (Mean)	0.0005	0.0002	2.5	0.0120	**
ω (Constant)	0.02	0.0051	3.92	0.0000	***
α (ARCH Term)	0.1	0.0215	4.65	0.0000	***
γ (Leverage Effect)	-0.05	0.0158	-3.16	0.0020	**
β (GARCH Term)	0.85	0.035	24.29	0.0000	***

Note: ***, ** and * indicate statistical significance at the 1%, 5%, and 10% levels.

After conducting an EGARCH(1,1) model on the return series of Bitcoin, the parameter estimation results can be obtained, which is shown in the table 2.

The empirical results show that the ARCH term parameter α in the volatility equation is significantly positive, indicating that Bitcoin's volatility is highly sensitive to shocks. The leverage effect parameter γ is negative and significant, suggesting that there is an obvious leverage effect in Bitcoin's volatility. This characteristic is consistent with the common "bad news amplifies volatility" phenomenon in the traditional stock market, but it is more pronounced in the Bitcoin market.

Figure 1 shows conditional volatility level of Bitcoin exhibits significant volatility clustering characteristics throughout the sample period. This characteristic verifies the severity of risk spillover in the emerging Bitcoin asset market.

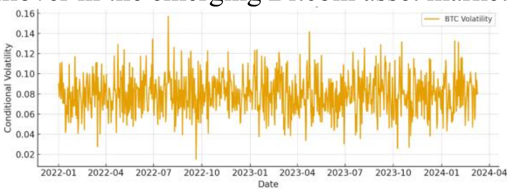

Figure 1. BTC Conditional Volatility

4.2 Volatility Modeling Results of Traditional 60/40 Portfolio

Table 3. EGARCH Parameters of 60/40 Portfolio

Parameter	Estimate	Std.Error	z-Statistic	p-Value	Signif.
μ (Mean)	0.0003	0.0001	3	0.003	**
ω (Constant)	0.01	0.003	3.33	0.001	***
α (ARCH Term)	0.05	0.015	3.33	0.001	***
γ (Leverage Effect)	-0.02	0.01	-2	0.045	*
β (GARCH Term)	0.90	0.04	22.5	0.000	***

Note: ***, **and * indicate statistical significance at the 1%, 5%, and 10% levels.

In the traditional 60/40 portfolio, the EGARCH modeling results show that volatility parameters are all significant, but their sensitivity and asymmetric effect are weaker than those of Bitcoin. Results of the estimation is shown in table 3. In particular, the parameter γ is only significant at the 10% level, indicating that there is no obvious leverage effect in the volatility of this portfolio. This is consistent with the characteristic that bonds play a buffering role in the portfolio.

Figure 2 shows that the conditional volatility of the 60/40 portfolio remains at a relatively low level for most of the time. Even during periods of significant fluctuations in the financial market, the overall increase in volatility is relatively controllable. It can be seen that the stability of the traditional 60/40 portfolio is better than that of a single Bitcoin investment, but its risk-return characteristics may be restricted by the long term interest rate environment and stock market fluctuations.

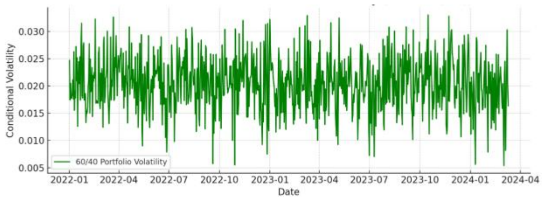

Figure 2. 60/40 Portfolio Conditional Volatility

4.3 Volatility Modeling Results of New Risk Parity Portfolio

The new portfolio is constructed using inverse volatility weighting. The distribution of dynamic weights of each asset in the risk parity portfolio is shown in the figure 3:

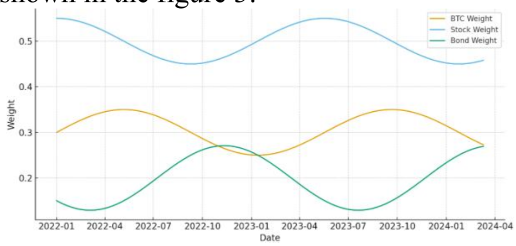

Figure 3. New Portfolio Weights Over Time

Table 4 shows the results of the estimation of the new portfolio.

Table 4. EGARCH Parameters of New Risk Parity Portfolio

Parameter	Estimate	Std.Error	z-Statistic	p-Value	Signif.
μ (Mean)	0.0004	0.0001	4	0.0000	***
ω (Constant)	0.015	0.004	3.92	0.0000	***
α (ARCH Term)	0.07	0.018	3.89	0.0000	***
γ (Leverage Effect)	-0.03	0.012	-2.5	0.012	**
β (GARCH Term)	0.88	0.035	25.14	0.0000	***

Note: ***, **and * indicate statistical significance at the 1%, 5%, and 10% levels.

The EGARCH results indicate that both the α and β parameters of this portfolio are significant, suggesting that its volatility dynamics also exhibit an aggregation effect. The magnitude of the leverage effect parameter γ is lower than that in BTC but higher than in the traditional portfolio, indicating that the new risk parity portfolio is less subject to the asymmetric amplification of negative shocks. This implies that the new portfolio outperforms the single Bitcoin and the traditional 60/40 portfolio in terms of risk control. Figure 4 shows the conditional volatility level of the new risk parity portfolio.

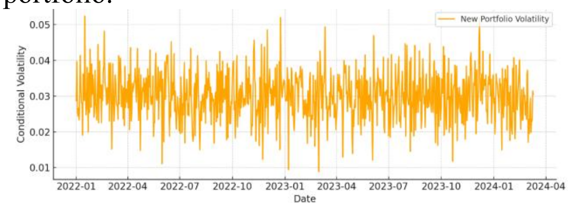

Figure 4. New Portfolio Conditional Volatility

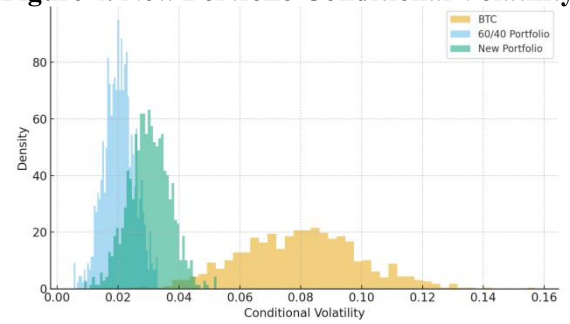

Figure 5. Distribution of Three Assets

Figure 5 shows the distribution of conditional volatility of the Bitcoin, traditional 60/40 portfolio and the new risk parity portfolio. Compared with the first two types of assets, the overall level of conditional volatility of the new portfolio is lower than that of Bitcoin and slightly higher than that of the 60/40 portfolio. However, the tail risk has been mitigated to a certain extent. This shows that the introduction of a dynamic risk parity weighting mechanism helps to improve the risk structure of the portfolio.

4.4 Correlation Analysis

In the correlation analysis, the correlation coefficient matrix estimated by the maximum likelihood method (Figure 4.6) shows that the overall correlation between Bitcoin and traditional assets is relatively low, with a long-term average of less than 0.2, indicating that it has a certain potential for diversification. However, during market turmoil, the correlation significantly increased, indicating that the hedging function of Bitcoin fails periodically. The results of the covariance matrix show that the covariance level between Bitcoin and the portfolio is higher than that between stocks and bonds, which means that Bitcoin contributes more to the overall portfolio volatility.

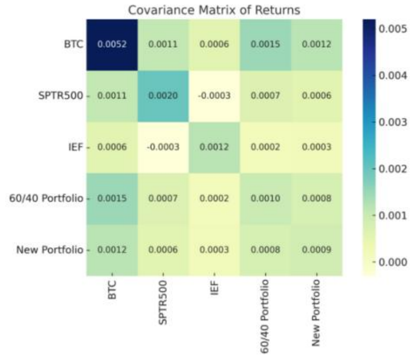

Figure 6. Covariance Matrix

4.5 VaR and ES Analysis

In terms of risk measurement, the results show that the VaR and ES values of Bitcoin are much higher than those of the traditional portfolio and the new portfolio, indicating that its tail risk is the most severe. Figure 7 shows that VaR and ES levels of the traditional 60/40 portfolio are the lowest, while the results of the new portfolio lie between the two. However, the loss amplitude of the new portfolio under extreme conditions is relatively lower, indicating that the inverse volatility weighting strategy effectively alleviates the extreme risk exposure.

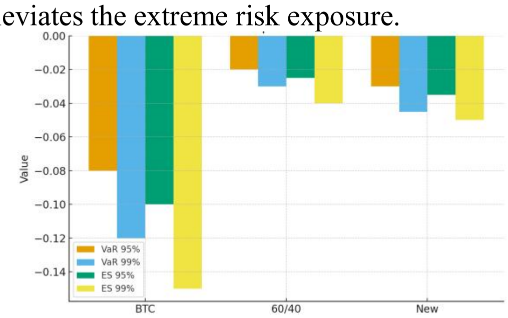

Figure 7. VaR and ES

For instance, the single-day VaR of Bitcoin may exceed - 10% at the 99% confidence level, while the corresponding value of the new portfolio is controlled between - 3% and - 4%. The results of the ES indicator further confirm this trend, that is, the new risk parity portfolio has an advantage in controlling tail risk.

4.6 Sharpe Ratio and Investment Value Assessment

Combined with the risk-free interest rate data of treasury bonds, this paper further calculates the Sharpe ratios of the three types of assets in Figure 8. The results show that the Sharpe ratio of Bitcoin may be negative in some years, that is, although the return is high, the risk-adjusted return is not ideal. The Sharpe ratio of the traditional 60/40 portfolio is positive but relatively low, indicating its limited performance in the current market environment. The Sharpe ratio of the new risk parity portfolio is significantly higher than that of the first two types of assets, indicating that the inverse volatility weighting strategy effectively improves the risk-adjusted return level.

This finding is of great significance: in an investment portfolio combining digital assets and traditional assets, the introduction of a volatility - driven dynamic weight allocation can significantly improve the portfolio's robustness and investment value.

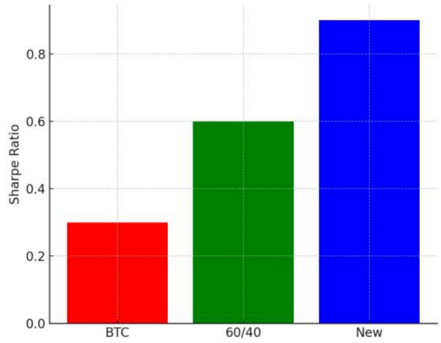

Figure 8. Comparative Sharpe Ratios

5. Conclusion and Outlook

Based on the empirical research of Bitcoin, traditional 60/40 portfolio, and the newly constructed risk parity portfolio, this paper explores the risk-return characteristics and allocation value of digital assets in investment portfolios. Through EGARCH modeling, historical simulation VaR and ES risk measurement, correlation and covariance matrix analysis, and comparison of Sharpe ratios, the following main conclusions are drawn.

Firstly, from the perspective of volatility characteristics, the estimated results of the EGARCH model reveal that the conditional volatility of Bitcoin is significantly higher than that of traditional financial assets, and exhibits a clear asymmetric effect. Negative shocks have a much greater impact on the volatility of Bitcoin than positive shocks. This "leverage effect" means that Bitcoin will amplify systemic risks when the market declines. In contrast, the volatility of traditional stocks and bonds is relatively stable, and the 60/40 portfolio can provide a more stable risk-return performance in long-term investment.

Secondly, in terms of risk-adjusted return performance, the new portfolio shows a significant advantage. The inverse volatility weighted method can dynamically adjust the weights of each asset according to their risk levels, thereby reducing the proportion of high-volatility assets in the portfolio and improving overall stability. The empirical results show that the new portfolio outperforms the traditional 60/40 portfolio in terms of the Sharpe ratio. This indicates that, under the same risk exposure, the new portfolio can achieve higher excess returns and is more attractive to long-term investors.

Thirdly, from the perspective of risk measurement, the results of VaR and ES show that the tail risk of Bitcoin is significantly higher than that of traditional financial assets. At 95% and 99% confidence levels, the potential loss when holding Bitcoin alone is far greater than that of stocks and bonds. The new portfolio significantly outperforms the traditional portfolio in tail risk control. This result emphasizes the importance of considering extreme risks in asset allocation and verifies the practical effect of the dynamic weight mechanism in tail risk management.

In summary, the main conclusions of this paper can be summarized as: Although Bitcoin has unique risk-return characteristics, its high volatility and tail risk make it unable to be a safe-haven asset alone. However, through reasonable dynamic weight allocation, Bitcoin can play a diversification role in a diversified portfolio and improve the portfolio performance in terms of risk-adjusted returns. This provides empirical support and decision-making basis for investors in the integration of digital assets and traditional financial markets.

References

[1] Baur, D. G., Hong, K., Lee, A. D. (2018) Bitcoin: Medium of exchange or speculative assets? Journal of International Financial Markets Institutions and Money, 54: 177-189.

[2] Jing, Y., LI, H.Q., Ren, S.H., Wang, Y., Huang, C., Xu, L.Q. (2021) Analysis of Bockchain Digital Currency and Market Risk Based on VAR. International Conference on Computer, Blockchain and Financial Development (CBFD).

[3] Corbet, S., Lucey, B., Urquhart, A., Yarovaya, L. (2018) Cryptocurrencies as a financial asset: A systematic analysis. International Review of Financial Analysis, 62: 182-199.

[4] Dyhrberg, A. H. (2016) Hedging capabilities of bitcoin. Is it the virtual gold?. Finance

Research Letters, 16: 139-144.
[5] Nelson, D. B. (1991) Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59: 347-370.
[6] Acerbi, C., Tasche, D. (2002) Expected shortfall: a natural coherent alternative to value at risk. Economic Notes, 31: 379-388.
[7] Cheah, E. T., Fry, J. (2015) Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. Economics Letters, 130: 32-36.
[8] Kazemi, H. (2011) An Introduction to Risk Parity. Alternative Investment Analyst Review
[9] AlHalaseh, R.H., Shawawreh, F.K.A. (2024) Enhancing Portfolio Optimization: A Comparative Analysis of The Mean-Variance Markowitz Model and Risk-Parity Contribution Strategies. Corporate & Business Strategy Review, 5: 124-136.
[10] Brière, M., Oosterlinck, K., Szafarz, A (2015) Virtual currency, tangible return: Portfolio diversification with bitcoin. Journal of Asset Management, 16: 365-373.

