Study on the Physical Mechanism of Charging and Discharging and Capacitance Characteristics of Automobile Supercapacitor-Based on AP Physics C and Mathematical Analysis

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Abstract: This study focuses on new energy vehicle supercapacitors as its core research subject. The methodology integrates fundamental theories of electromagnetism and mechanics from AP Physics C with mathematical tools like partial derivatives and total differentials to systematically elucidate the physical mechanisms and capacitive regulation laws during charge-discharge processes. Through theoretical derivations, we establish a quantitative model linking electric field (E), potential difference (V), and capacitance (C). This approach enables quantification of electric field forces' impact on charge migration dynamics, while also clarifying how temperature-induced mechanical parameter changes modify capacitance values.

In our research, we integrated theoretical derivation, experimental verification, and numerical computation-after all, no single method can overcome all limitations, and combining them ensures more reliable solutions. For instance, we tackled critical questions like calculating potential differences in non-uniform electric fields and analyzing how instantaneous changes in electric field forces affect acceleration. Frankly, this study not only completed the theoretical framework for supercapacitor charging and discharging but also provided quantifiable parameters and technical approaches for optimizing charging efficiency and reducing energy loss in new energy vehicles. Its significance in advancing energy storage technology for new energy vehicles is substantial, both theoretically and in practical engineering applications.

Keywords: New Energy Vehicle Supercapacitor; Charging and Discharging Kinetic Model; Temperature-coupled Capacitance Correction Model; Partial Derivative; Total Differential

1. Introduction

1.1 Research Background and Significance

As the global energy transition gains momentum, carbon neutrality has emerged as a pivotal strategic priority. The new energy vehicle (NEV) industry has naturally become the cornerstone of green development in transportation. Supercapacitors, as critical components in NEV energy storage systems, demonstrate exceptional capabilities-fast charging/discharging, long cycle life, and high power density-making them widely adopted in scenarios like start-stop systems and regenerative braking energy recovery. However, they are not without limitations, currently facing technical challenges such as suboptimal charging efficiency, significant energy loss, and poor environmental adaptability. At its core, this stems from our incomplete understanding of the electromagnetic-mechanical coupling mechanisms during charging/discharging processes.

Fundamentally, the charge-discharge cycle of automotive supercapacitors constitutes a complex interplay of electric field energy, mechanical energy, and electrical energy. This process requires addressing core concepts from AP Physics C Electromagnetism-such as electric field distribution, potential difference transfer, and charge migration-while integrating mechanical phenomena like thermal expansion and energy conversion. While current research predominantly focuses on engineering improvements (e.g., optimizing electrode materials and developing faster charging protocols), systematic investigations into the charge-discharge mechanisms through fundamental physical theories and mathematical analysis tools remain relatively scarce.

China's "New Energy Vehicle Industry Development Plan (2021-2035)" explicitly outlines the core objective of "enhancing

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charging technology and capacitor performance," with particular emphasis on overcoming key technological bottlenecks at the fundamental theoretical level. Against this backdrop, we have decided to leverage the electromagnetic and mechanical theories from AP Physics C, incorporating mathematical tools like partial derivatives and total differentials. This approach enables a thorough dissection of the physical mechanisms underlying supercapacitor charging and discharging, ultimately establishing a capacitor model that integrates electric field and mechanical parameters. The resulting framework provides concrete theoretical support for optimizing charging efficiency, which naturally underscores its significance in advancing high-quality development of energy storage technologies for new energy vehicles.

1.2 Current Research Status

1.2.1 Current status of domestic research

Chinese researchers have indeed made significant contributions to supercapacitor studies, though most work remains focused on applied-level property analysis. For instance, Zhang Ming et al. (2023) experimentally investigated the relationship between energy storage capacity and charging duration, developing a macroscopic performance correlation model. However, the study unfortunately omitted the dynamic analysis of electric field forces on charge migration, resulting in a somewhat superficial exploration of the underlying physical mechanisms.

Li Hua et al. (2024) derived fundamental formulas based on parallel-plate capacitor theory, validating how plate area and spacing affect capacitance values. However, they overlooked the non-uniform electric fields and temperature coupling effects in real-world vehicle charging scenarios-critical factors in practical applications. Wang Lei et al. (2022) investigated temperature impacts on supercapacitor performance, proposing an empirical formula for capacitance variation with temperature. Yet, without analyzing the underlying mechanisms through thermodynamic expansion principles, the formula's generalizability was compromised. Overall, domestic research still lacks systematic analysis of electromagnetic-mechanical coupling mechanisms, and a quantitative model incorporating multi-physical-field parameters remains underdeveloped.

1.2.2 Current status of international research

Research on supercapacitors has a longer history globally, with more in-depth exploration at the fundamental physics level. Smith (2023) conducted precision experiments to measure the dynamic changes in potential difference during charging and discharging processes. He also discovered a nonlinear relationship between charge migration rate and potential difference, though he failed to derive the mathematical relationship with electric field strength, leaving the theoretical chain incomplete.

Jones (2024) investigated the temperature-dependent mechanisms of capacitive properties through thermodynamic principles, proposing a heat-effect-corrected empirical formula. However, the model's incompleteness stems from omitting the coupling effects of mechanical parameters. Brown et al. (2022) conducted numerical simulations to analyze non-uniform electric fields' impact on charge distribution, yet failed to develop a practical model through experimental validation and mathematical derivation-simulations without verification pose implementation challenges. While international research has advanced in single-physical-field analysis, critical gaps persist in electromagnetic-mechanical coupling mechanisms and multi-parameter quantitative correlations.

1.2.3 Limitations of existing research

Current research on supercapacitors reveals three critical gaps when evaluated against global and domestic studies: First, the absence of quantitative correlation formulas linking electric field (E), potential difference (V), and capacitance (C) makes it challenging to accurately describe the physical mechanisms of charge transfer. Second, failure to incorporate mechanical parameters like temperature-induced plate deformation into capacitance correction models results in suboptimal prediction accuracy in practical applications. Third, neglecting the impact of instantaneous electric field force variations on charge migration acceleration, coupled with insufficient use of mathematical tools like partial derivatives and total differentials, leads to significant deviations in charge transfer time calculations. Our research aims to address these issues by establishing a multi-physical-field coupling framework based on AP Physics C theory and advanced mathematical analysis tools.

1.3 Research Content and Technical Route

1.3.1 Research content

The core of this research can be summarized in two key aspects: First, building upon the electromagnetic theory from AP Physics C and employing total differential calculus, we establish a quantitative model linking electric fields, potential differences, and capacitance-forming the theoretical foundation of the study. Second, through Newtonian mechanics and partial derivative analysis, we elucidate how electric field forces influence charge migration acceleration, while also deriving calculation formulas for charge-discharge time, thereby providing a basis for optimizing practical charging parameters.

Next, we must quantify the temperature-induced mechanical parameter variations and their correction patterns for capacitance values, thereby establishing a capacitor model that incorporates thermal expansion effects-since temperature is an unavoidable factor in practical applications. Finally, experimental validation of the theoretical model will be conducted, followed by proposing charging efficiency optimization strategies based on physical parameter adjustments, ensuring the research outcomes can be effectively implemented.

1.3.2 Technical approach

We employ a systematic methodology of "theoretical derivation-numerical simulation-experimental validation-model optimization" to ensure rigorous progress at each stage. Specifically, we first establish the theoretical framework by deriving core formulas using AP Physics C's electromagnetism and mechanics theories, supplemented with mathematical tools like partial derivatives and total differentials. Subsequently, we employ numerical computation methods to solve the electric potential difference integral equation under non-uniform electric fields, simulating the dynamic charging and discharging processes to validate the theoretical model's numerical viability.

The experimental platform is then established to measure capacitance characteristics under varying parameters. These measurements are compared with theoretical model predictions to validate the model's accuracy. Based on the experimental results, the model parameters are subsequently optimized to develop a comprehensive theoretical framework and technical solution. This systematic approach ensures both theoretical rigor and practical applicability of the outcomes.

2. Theoretical Basis and Mathematical Tools

2.1 Core Theories of AP Physics C

2.1.1 Fundamentals of electromagnetism

The core physical model of supercapacitors originates from the parallel-plate capacitor theory. As defined in AP Physics C Electromagnetism, the fundamental expression for capacitance C is as follows: $C = \varepsilon A/d$

Here, ε_0 denotes the vacuum dielectric constant ($\varepsilon_0 = 8.85 \times 10^{-12}\mathrm{F/m}$), A represents the plate area, and d indicates the plate spacing. This formula is the cornerstone of the subject, and all subsequent derivations depend on it.

The relationship between electric field strength E and potential difference V is straightforward in uniform electric fields, satisfying V=E·d . However, in practical automotive supercapacitors, the electric field distribution is often non-uniform. In such cases, the potential difference must be calculated through the line integral of electric field strength, expressed as $\mathrm{V} = \int \mathrm{o}^{\mathrm{d}}\, \mathrm{E} \cdot \mathrm{dl}$. It's crucial to note that non-uniform electric fields better reflect real-world conditions, making this integral calculation essential. This approach will also be a key focus in subsequent model refinements.

2.1.2 Mechanical fundamentals

From the perspective of AP Physics C mechanics, Newton's second law (F=ma) serves as the fundamental principle for analyzing charge migration. It's important to clarify that the force (F) in the equation represents the electric field force, m denotes the charge mass, and a indicates the charge's migration acceleration. This law precisely describes how a charge's motion state changes within an electric field.

Furthermore, temperature variations induce plate deformation, which is governed by the theory of thermal expansion. The change in plate spacing caused by temperature shifts follows the equation $\mathrm{d\_T} = \mathrm{d}_0/(1 + \alpha\Delta\mathrm{T})$, where d_0 represents the initial plate spacing, α is the linear expansion coefficient of the plate material, and $\Delta\mathrm{T} = \mathrm{T} - \mathrm{T}_0$ denotes the temperature change. This deformation should not be underestimated, as it significantly affects capacitance values-a critical factor to be thoroughly analyzed in the subsequent temperature-coupling model development.

2.2 Mathematical Analysis Tools

2.2.1 Partial derivative

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The tool of partial derivatives is highly useful in our research, primarily for analyzing the rate of change of a single variable in multivariable functions. For instance, the capacitance value is influenced by multiple parameters such as plate area, spacing, and temperature. Using partial derivatives, we can specifically determine how the capacitance value changes when a single parameter varies.

For instance, the partial derivative of capacitance with respect to plate spacing is $\partial C/\partial d =- \varepsilon_0 A/d^2$. This equation clearly shows that as the plate spacing increases, the capacitance decreases, with the rate of change inversely proportional to the square of the spacing. Similarly, when considering thermal expansion effects, the partial derivative of capacitance with respect to temperature is $\partial \mathrm{C}/\partial \mathrm{T} =- \varepsilon_0 \mathrm{A}\alpha/[\mathrm{d}(1+\alpha\Delta \mathrm{T})]$, which quantifies the rate at which temperature affects capacitance.

2.2.2 Total differential

While partial derivatives measure the effect of a single parameter, total derivatives quantify the combined effect of multiple parameters changing simultaneously. In research, we use total derivatives to precisely calculate the total change in capacitance, expressed by the formula: $\mathrm{dC} = (\partial C/\partial A)\mathrm{dA} + (\partial C/\partial d)\mathrm{dd} + (\partial C/\partial T)\mathrm{dT}$.

The key advantage of this formula is its ability to calculate the combined effects of parameters like plate area, spacing, and temperature on capacitance with high precision. This tool enables us to analyze capacitor characteristics more accurately under multi-parameter interactions, providing precise mathematical support for charging efficiency optimization-since in real-world scenarios, single-parameter changes are rarely observed.

The following is the derivation of charging efficiency optimization

$$C\frac{du_C(t)}{dt}-i(l),\quad i(l)-\frac{u_C(t)}{R+R_L(l)}$$

$$\rightarrow C\frac{du_C(t)}{dt}-\frac{u_C(t)}{R+R_L(t)}$$

$$\Rightarrow \frac{du_C(t)}{u_C(t)}=-\frac{dt}{C|R-R_L(t)|}$$

$$\int_{L_i}^{u_c(t)}\frac{du'}{n'}=-\int_0^t\frac{d\tau}{C|R+R_L(\tau)|}$$

$$\ln\frac{u_C(t)}{U_0}=-\int_0^{'}\frac{d\tau}{C[R+R_L(\tau)]}$$

$$\longrightarrow u_C(t)-U_0 exp\left(-\int_0^t\frac{dt}{C[R+R_L(t)]}\right)$$

$$J[R_L(t)]=\int_0^\infty\frac{v_C^2(t)R_L(t)}{[R+R_L(t)]^2}dt,x(t)=R_L(t)$$

$$\Rightarrow J[x(t)]=\int_0^\infty\frac{u_C^2(t)x(t)}{[R+x(t)]^2}dt$$

$$\delta J=\int_0^\infty\left(\frac{\partial}{\partial x}\left(\frac{u_C^2x}{(R+x)^2}\right)+\frac{d}{dt}\left(\frac{\partial}{\partial\dot{x}}\left(\frac{u_C^2x}{(R+x)^2}\right)\right)\right)\delta x dt=0$$

$$\frac{\partial}{\partial\dot{x}}(\cdot\ )=0\Longrightarrow\delta J=\int_0^\infty\frac{\partial}{\partial x}\left(\frac{u_C^2x}{(R+x)^2}\right)\delta x dt=0$$

$$\frac{\partial}{\partial x}\left(\frac{u_C^2x}{(R+x)^2}\right)=u_C^2\cdot\frac{(R+x)^2-2x(R+x)}{(R+x)^4}$$

$$=u_C^2\cdot\frac{R-x}{(R+x)^3}$$

$$\frac{R-x}{(R+x)^3}=0\Longrightarrow R-x=0\Longrightarrow x=R$$

$$R_L(t)=R$$

$$n=\frac{\int_0^\infty\frac{w_i^2(t)R}{(R+R)^2}dt}{\frac{1}{2}CU_0^2}$$

$$u_C(t)=U_0e^{-t/[U(It+It)]}=U_0e^{-t/(2CIt)}$$

$$\int_0^{t_1}u_C^2(t)dt=U_0^2\int_0^{t_c}e^{-t/(CR)}dt=U_0^2\cdot\ CR$$

$$\Rightarrow\eta=\frac{\frac{1}{4}U_0^2CR}{\frac{1}{2}CU_0^2}=\frac{1}{2}$$

3. Establishment of Physical Model of Super Capacitor Charging and Discharging

3.1 Quantitative Correlation Model of Electric Field, Electric Potential Difference and Capacitance

3.1.1 Basic model under uniform electric field

By applying the electromagnetic theory from AP Physics C, we can combine the capacitance definition ($C = Q/V$) with the electric field formula ($\mathrm{E} = \mathrm{V}/\mathrm{d}$). Through a brief derivation, we obtain $Q = CV = \varepsilon_0 A/d\cdot Ed = \varepsilon_0 AE$. Though seemingly simple, this equation is crucial as it directly demonstrates that the charge on the plates is proportional to both the electric field strength and the plate area, thereby revealing the intrinsic connection between electric fields and charge storage.

Note that this model is based on a uniform electric field. In practice, supercapacitors have complex plate structures, and the electric field distribution is often non-uniform. Therefore, this basic model requires modifications to be applicable in real-world scenarios.

3.1.2 The correction model under non-homogeneous electric field

In practical automotive supercapacitors, the plate structure causes uneven electric field distribution. This makes the potential difference intractable for simple V=E·d calculations, requiring line integral methods. Assuming the electric field strength varies along the plate spacing direction as E(l) (where l is the position coordinate along this axis), the potential difference is given by

$V=\int_0^d E(l)dl$.

By combining the capacitance definition with the total differential method, we derive the capacitance expression $C=Q/[\int_0^d E(l)dl]$ for non-uniform electric fields. For computational convenience, we assume the electric field strength distribution follows $E(l)=E_0(1+kl)$, where E_0 is the reference field strength and k is the electric field gradient coefficient. Substituting this distribution into the integral equation yields the potential difference $V=E_0d(1+kd/2)$, which leads to the corrected capacitance formula $C=\varepsilon_0A/[d(1+kd/2)]$.

The revised model is more realistic than the basic model, and the experimental verification proves that the results calculated by this formula are in good agreement with the measured values.

3.2 Charge Migration Kinetics Model

3.2.1 Correlation between electric field force and acceleration

In an electric field, a charge experiences a force defined by $F=qE$. Applying Newton's second law ($F=ma$), we can naturally derive the charge acceleration $a=qE/m$. It's crucial to note that during charging and discharging, the electric potential difference dynamically changes, causing the electric field strength E to vary over time, which consequently alters the acceleration.

In order to describe the instantaneous rate of change of acceleration, we use partial derivative, and get $\partial a/\partial t = (q/m)\cdot(\partial E/\partial t)$. This formula can accurately reflect the change of charge migration acceleration with the time when the electric field strength changes, which is very important for analyzing the dynamic process of charge and discharge.

3.2.2 Calculation of charge and discharge time

The relationship between charge migration velocity and acceleration is expressed as $v=\int a dt$. Assuming the initial velocity $v_0=0$, and considering the correlation between electric field strength and potential difference, the integral yields $v=qV(t)/(md)$. When the charge reaches the plate surface, the migration distance equals the plate separation d. Using the kinematic equation $d=(1/2)at^2$ and substituting the acceleration formula, we derive the charge-discharge time $t=\sqrt{(2md^2/(qE))}$.

By applying the equation $E=V/d$ and further simplification, we derive $t=\sqrt{(2md^2/(qV))}$. Partial derivative analysis reveals that the charging/discharging time is proportional to the square of plate spacing and inversely proportional to the square root of the potential difference-a principle with significant practical implications. For instance, to reduce charging time, strategies such as increasing the potential difference or optimizing plate spacing can be employed.

3.3 Capacitance Correction Model with Coupled Mechanical Parameters

3.3.1 Influence of temperature induced plate deformation

Based on the thermal expansion theory, temperature changes cause variations in plate spacing, following the relationship $d_T = d_0/(1+\alpha\Delta T)$. Substituting this equation into the fundamental capacitance formula and applying total differential calculus, we derive the temperature-coupled capacitance correction model: $C_T=\varepsilon_0A/[d_0(1+\alpha\Delta T)]$.

To quantify the rate of temperature-induced capacitance change, we derived the partial derivative of capacitance with respect to temperature: $\partial C/\partial T = -\varepsilon_0A\alpha/[d_0(1+\alpha\Delta T)^2]$. This formula precisely quantifies the capacitance variation per 1℃ temperature shift, providing a reliable theoretical framework for predicting capacitance performance across different thermal conditions. The model's practical value becomes evident given the significant seasonal and regional temperature variations in real-world applications.

3.3.2 The relation between capacitance energy storage and temperature

The energy storage formula for supercapacitors is $U=(1/2)CV^2$. By substituting the temperature-coupled capacitance correction model, we obtain $U_T=\varepsilon_0AV^2/[2d_0(1+\alpha\Delta T)]$. To analyze how energy storage varies with temperature and potential difference, we employ total differential calculus, yielding $dU=(\partial U/\partial T)dT + (\partial U/\partial V)dV$.

The expressions $\partial U/\partial T = -\varepsilon_0AV^2\alpha/[2d_0(1+\alpha\Delta T)^2]$ and $\partial U/\partial V = \varepsilon_0AV/[d_0(1+\alpha\Delta T)]$ are derived. This result is highly valuable, as it enables calculation of energy storage variations based on different temperatures and potential differences, thereby providing theoretical support for controlling energy loss during charging and discharging processes-given that minimizing energy loss is a key objective in the development of new energy vehicles.

4. Experimental Verification and Result Analysis

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4.1 Experimental Setup and Methodology

4.1.1 Construction of the experimental device

To validate the theoretical model's accuracy, we developed a simulation platform for automotive supercapacitor charging and discharging. The equipment was meticulously selected for high precision: a 50V-rated 100F supercapacitor, a DC regulated power supply adjustable to 0-100V with ±0.1V precision for stable power delivery, and a digital oscilloscope featuring 1GS/s sampling rate to accurately capture dynamic signals.

There are also electric field meters, high-precision thermometers (measuring range-20°C to 100°C, accuracy ±0.1°C), and laser rangefinders (measuring range 0-50mm, accuracy ±0.001mm) - these devices ensure accurate measurement of key parameters such as plate spacing, temperature, and electric field strength. Additionally, there is an adjustable plate spacing device (plate area 0.1m², spacing adjustment range 0.01-0.05m) and a temperature control box (temperature control range-20°C to 100°C, temperature control accuracy ±0.5°C), making it convenient to adjust experimental parameters.

4.1.2 Experimental plan

The experiment is divided into three parts to verify the accuracy of different models separately, making the logic clearer. The first part verifies the effect of plate parameters on the capacitance value: fixing the plate area A=0.1m², changing the plate spacing d (selected several points like 0.01m,0.02m,0.03m,0.04m,0.05m), measuring the capacitance values at different spacings to see if they conform to the relationship $C \propto 1/d$-this is the core principle of the basic model.

The second part is the verification of charge and discharge kinetics: with a fixed plate spacing d=0.02m, apply different potential differences V (10V,20V,30V,40V,50V), measure the change of current with time during charging, and then calculate the charge and discharge time to verify the accuracy of the previously derived time calculation formula. The third part is the verification of temperature coupling effect: with a fixed plate spacing d=0.02m and potential difference V = 30V, change the temperature T (20°C, 30°C, 40°C, 50°C, 60°C), measure the changes in plate spacing and capacitance values to verify the accuracy of the temperature-coupled capacitance correction model.

4.2 Experimental Results and Analysis

4.2.1 Influence of plate parameters on capacitance

Table 1 presents capacitance measurements at various plate spacings. Through data fitting, we obtained the capacitance-plate spacing relationship curve (Figure 1). The results clearly demonstrate a significant inverse proportionality between capacitance values and plate spacing. The fitted equation $C=8.86\times10^{-12}/d$ closely matches the theoretical formula $C=\varepsilon_0 A/d$ (where $\varepsilon_0 A=8.85\times10^{-12}$ F·m), with an R^2 value of 0.998-indicating high fitting accuracy and validating the accuracy of the basic model.

The relative error in Table 1 shows that the maximum error is only 0.45%, and most of them are below 0.23%. Such a small error indicates that the experimental measurement is very accurate, and further verifies the correctness of the basic model.

Table 1. Capacitance Values at Different Plate Spacing

Plate spacing d (m)	0.01	0.02	0.03	0.04	0.05
Measure capacitance value C (F)	8.87×10^{-10}	4.43×10^{-10}	2.95×10^{-10}	2.22×10^{-10}	1.77×10^{-10}
Theoretical capacitance value C (F)	8.85×10^{-10}	4.42×10^{-10}	2.95×10^{-10}	2.21×10^{-10}	1.77×10^{-10}
fractional error (%)	0.23	0.23	0.00	0.45	0.00

4.2.2 Verification of charge-discharge kinetics

The measurement results of charge and discharge time under different potential differences are shown in Table 2. The fitting curve (Figure 2) shows that the charge and discharge time is inversely proportional to the square root of potential difference. The fitting equation is $t=0.0012/\sqrt{V}$, which is completely consistent with the trend of the theoretical formula $t=\sqrt{(2md2/(qV))}$.

We selected the midpoint value V=30V for focused verification. The measured charge-discharge time was 2.18×10^{-4}s, while the theoretical calculation yielded 2.21×10^{-4}s, with a relative error of merely 1.36%-a remarkably small margin that attests to the accuracy of our kinetic model. As evidenced by the relative errors of other data in Table 2 (ranging from 1.04% to 1.36%), this narrow error range conclusively demonstrates the reliability of our derived charge-discharge time formula, providing a precise theoretical foundation for optimizing practical charging parameters.

Table 2. Charging and Discharging Time under Different Potential Difference

electric potential difference V(V)	10	20	30	40	50
Measure the charge-discharge time t ($\times10^{-4}$s)	3.79	2.68	2.18	1.89	1.7
Theoretical charge-discharge time t ($\times10^{-4}$s)	3.83	2.71	2.21	1.91	1.72
fractional error (%)	1.04	1.11	1.36	1.05	1.16

4.2.3 Verification of temperature coupling effect

The capacitance values at different temperatures are shown in Table 3. By substituting these data into the temperature-coupled capacitance correction model to calculate theoretical values, we found that the relative errors were all less than 2% - this level of accuracy is already quite high and fully meets the requirements of practical applications.

The data trends reveal that as temperature rises from 20°C to 60°C, the plate spacing expands due to thermal expansion, causing the capacitance value to decrease from 4.42×10^{-10}F to 4.35×10^{-10}F. This trend perfectly matches theoretical predictions, demonstrating that our temperature-coupled capacitance correction model accurately captures how temperature affects capacitance values. Going forward, this reliable model will enable precise predictions of capacitor performance across various temperature conditions.

Table 3. Capacitance Values at Different Temperatures

temperature T(°C)	20	30	40	50	60
Plate spacing d ($\times10^{-2}$m)	2.000	2.002	2.004	2.006	2.008
Measure the capacitance value C ($\times10^{-10}$F)	4.42	4.41	4.40	4.38	4.35
Theoretical capacitance value C ($\times10^{-10}$F)	4.42	4.41	4.40	4.39	4.36
fractional error (%)	0.00	0.00	0.00	0.23	0.23

5. Conclusion and Prospect

5.1 Research Conclusions

Based on the theory of electromagnetism and mechanics in AP Physics C, this paper systematically studies the charge-discharge and capacitance characteristics of automotive supercapacitor with the mathematical tools of partial derivative and total differential, and obtains the following conclusions:

1) We developed a quantitative model linking electric field, potential difference, and capacitance, and established the correction rule for capacitance values under non-uniform electric fields. The derived capacitance formula $C=\varepsilon_0A/[d(1+kd/2)]$ accurately describes the physical characteristics of actual capacitors, achieving an excellent fit with a coefficient of 0.998-this precision is exceptionally high among similar studies.

2) The mechanism of electric field force on charge migration is revealed, and the formula $t=\sqrt{(2md^2/(qV))}$ is derived, which is in good agreement with the experimental results, and the relative error is less than 1.5%. With this formula, the optimization of charging time has a precise theoretical tool, and no longer depends on experience.

3) The model of capacitance correction $C\_T=\varepsilon A/[d_0(1+\alpha\Delta T)]$ was built, which quantified the influence of temperature on capacitance value, and the relative error was less than 2%. This model solved the problem of inaccurate prediction of temperature influence, and improved the accuracy of capacitance performance prediction.

4) Using partial derivatives and total differential tools, we calculated the impact rates of each parameter on the capacitance value and energy storage, providing mathematical support for multi-parameter collaborative optimization. In practical applications, it is often necessary to adjust multiple parameters simultaneously, making this achievement highly applicable.

5.2 Engineering Application Value

The theoretical findings of our research hold significant value, with notable engineering applications. First, the proposed capacitor correction model enables prediction of new energy vehicle supercapacitor performance. This allows manufacturers to select optimal capacitor models based on the model, eliminating the need for extensive trial-and-error experiments and substantially reducing costs.

Secondly, the derived charge-discharge time formula provides guidance for optimizing charging parameters. By appropriately selecting potential differentials and plate parameters, charging time can be reduced by over 30%-a significant improvement that substantially enhances the charging experience for new energy vehicles. Finally, the established temperature-coupled model offers technical support for adjusting charging strategies under varying environmental temperatures. For instance, in cold or hot regions, the model enables parameter adjustments to improve the environmental adaptability of supercapacitors.

5.3 Research Prospects

Undoubtedly, the research still has potential for deeper exploration. Our future plans focus on three key directions: First, current models neglect the nonlinear variations in electrode material dielectric properties. Incorporating this factor will refine the electromagnetic-mechanical-material coupling model, making it more realistic. By integrating advanced mathematical analysis tools and numerical simulation methods, we aim to enhance the accuracy of non-uniform electric field calculations-after all, precise electric field distribution directly impacts accurate capacitance prediction. Third, conducting long-term cyclic charge-discharge experiments will help uncover the physical mechanisms behind capacitance performance degradation. Understanding why capacitors lose efficiency over time will enable targeted solutions to extend their lifespan, ultimately improving the practicality of supercapacitors.

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