

Distributed Adaptive Tracking of Random High-Order Nonlinear Multi-Agent Systems with Heterogeneous Powers

Xiaojuan Gong*

School of General Education, Yantai Institute of Science and Technology, Yantai, Shandong, China

**Corresponding Author*

Abstract: This paper studies distributed adaptive tracking for random high-order nonlinear multi-agent systems whose integrator powers may differ across agents and backstepping stages. The powers are known odd rational numbers, whereas a common drift parameter is unknown and the systems are driven by second-order-moment disturbances. A weighted-degree backstepping construction is developed by using the largest network power to close all recursive estimates within one Lyapunov function. The associated tuning function is derived from the coefficient of the parameter-estimation error, and a leakage term prevents parameter drift. For a pinned undirected communication graph, the resulting closed loop is noise-to-state practically stable in the mean-square sense and an explicit ultimate tracking-error bound is obtained. The same recursion yields a containment extension in which the graph determines the convex-combination weights. Two heterogeneous-power examples illustrate the single- and multiple-leader results. The analysis is limited to full-state feedback, fixed topology, known powers, and bounded second moments; no claim of hardware validation is made.

Keywords: Multi-Agent Systems; Heterogeneous Powers; Random Nonlinear Systems; Adaptive Backstepping; Practical Mean-Square Stability

1. Introduction

Distributed tracking is a basic coordination task in which followers reconstruct a leader's motion from local neighbor information. It underlies formation and cooperative-motion problems in robotic and networked systems [1,2]. Representative developments include prescribed-performance control with input quantization [3], distributed optimization based on a small-gain argument [4], and tracking or containment control

for nonlinear agents with nonstandard powers [5,6]. When several leaders are present, containment requires the follower outputs to approach the convex hull generated by the leader outputs rather than a single trajectory.

Random disturbances introduce a second difficulty. Models driven by second-order-moment processes are useful when a disturbance has finite energy and bandwidth but its sample paths need not be differentiable in the stochastic-differential-equation sense. Cooperative control under stochastic disturbances or measurement noise has been considered in [7,8], while [9,10] studied nonlinear networks perturbed by second-order-moment processes. A closely related study by Yao et al. [11] developed adaptive cooperative tracking and containment controllers for a physically derived, fully actuated surface-vessel model under random disturbance. That work and the present paper share the use of graph-weighted errors, adaptive backstepping, and practical mean-square estimates. Consequently, a defensible contribution here cannot be the use of those tools alone.

Recent studies have extended adaptive tracking to heterogeneous nonlinear agents [12], mean-square tracking under switching topologies and multiple measurement noises [13], fixed-time consensus for stochastic nonlinear agents [14], and fixed-time containment of stochastic p -normal systems [15]. These results emphasize the continuing need to distinguish agent heterogeneity, random disturbances, and containment objectives.

The unresolved issue addressed in this paper is the simultaneous presence of a high-order strict-feedback chain and heterogeneous odd-rational powers p_{ij} that vary with both the agent i and the backstepping stage j . Standard quadratic recursion does not align the different powers. The construction below instead uses the network-wide degree $\bar{p} = \max_{i,j} p_{ij}$ and the stage-dependent Lyapunov exponent $\bar{p} - p_{ij} + 2$. This choice

makes the cross terms comparable to a common dissipative power $\bar{p} + 1$ and permits one recursive

estimate for the entire network. The distinction from the closest work is summarized in Table 1.

Table 1. Scope Relative to the Closest Random Cooperative-Control Study.

Aspect	Yao et al. [11]	Present paper
Plant model	Physically derived, fully actuated surface-vessel dynamics	Abstract n th-order strict-feedback agents
Nonlinearity handled by the recursion	Vessel kinematics/dynamics with unknown damping parameters	Agent- and stage-dependent known odd-rational powers p_{ij} and an unknown drift coefficient
Unifying device	Vessel-specific coordinate and Lyapunov construction	Network-wide weighted degree $\bar{p}$ and stage-dependent exponents $\bar{p} - p_{ij} + 2$
Validation	Vessel simulations with PID comparisons	Heterogeneous-power numerical benchmarks; no physical experiment
Common ground	Second-order-moment disturbances, graph-weighted errors, adaptive backstepping, and practical mean-square bounds	

The contributions are therefore stated narrowly as follows. First, a distributed adaptive controller is constructed for random high-order agents with heterogeneous known powers, and an explicit practical mean-square tracking bound is derived. Second, the tuning function is obtained by collecting the coefficient of the parameter-estimation error at every recursive step; its cancellation role and the leakage term are made explicit. Third, the same weighted-degree recursion is shown to extend to a multi-leader graph, with the convex-combination matrix determined by the graph. The containment result is an extension of the main single-leader construction, not a separate methodological contribution.

The method also has clear limitations. It requires full-state feedback, fixed undirected follower links, known odd-rational powers, a common constant drift parameter, and disturbances with uniformly bounded second moments. The computed ultimate bound can be reduced through gain selection only within actuator and noise constraints; it cannot in general be driven to zero when persistent disturbances are present. The simulations are numerical proof-of-concept studies rather than experiments on a physical platform.

Section 2 introduces the notation and comparison tools. Section 3 develops the single-leader controller and stability analysis. Section 4 gives the containment extension. Section 5 presents two numerical examples, and Section 6 concludes the paper.

Notions: The real n -dimensional space is denoted as R^n . $R^{n \times m}$ represents the real $n \times m$ matrix space. R_+ stands for the family of all nonnegative real numbers. The Euclidean norm of a vector x

is denoted as $|x|$ and $|X|$ is the 2-norm of a matrix X . C^i represents the family of all functions with continuous i -th partial derivative. The set of all nonnegative functions $V(x, t)$, which is C^1 in x and C^1 in t , is denoted as $C^{1,1}(R^n \times [t_0, \infty); R_+)$. $\mathcal{K}$ stands for the set of all continuous and strictly increasing functions $\gamma(t)$ and $\gamma(0) = 0$. $\mathcal{K}_\infty$ is the set of all unbounded functions in $\mathcal{K}$. $\mathcal{KL}$ denotes the class of functions $\beta(r, t)$ such that $\beta(\cdot, t) \in \mathcal{K}$ for every fixed $t \geq 0$ and $\beta(r, t) \rightarrow 0$ as $t \rightarrow \infty$ for every fixed $r \geq 0$. For an odd rational number p , the signed power is understood as $x^p = \text{sgn}(x)|x|^p$.

2. Preliminaries

Consider the random nonlinear system

$$\begin{aligned} \dot{x}(t) &= f(x(t), t) + h(x(t), t)\xi(t), \\ x(t_0) &= x_0, \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state and $\xi(t) \in R^l$ is an $\mathcal{F}_t$ -adapted stochastic process defined on the complete filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq t_0}, P)$ with the usual conditions. We assume $\sup_{t_0 \leq s \leq t} E|\xi(s)|^2 \leq K_0$ for a constant $K_0 > 0$. The functions $f(x, t) \in R^n$ and $h(x, t) \in R^{n \times l}$ are locally Lipschitz in x and piecewise continuous in t ; moreover, $|f(0, t)| \vee \|h(0, t)\| \leq D_0$ for a constant $D_0 > 0$.

Definition 2.1. [16] For $m > 0$, system (1) is noise-to-state practically stable in the m th moment (NSpS- m -M) if there exist $\beta_0 \in \mathcal{KL}$, $\gamma_0 \in \mathcal{K}$, and a constant $D_m \geq 0$ such that $E|x(t)|^m \leq \beta_0(|x_0|, t - t_0) + \gamma_0 \left(\sup_{t_0 \leq v \leq t} E|\xi(v)|^2 \right) + D_m, \quad t \geq t_0.$

Lemma 2.1. Suppose that the closed-loop vector field is locally Lipschitz and that its maximal local solution admits a continuously

differentiable, radially unbounded function V satisfying, for some positive constants m, m_1, m_2, c, d and $\delta \geq 0$,

$$m_1|x|^m \leq V(x, t) \leq m_2|x|^m, \quad (2)$$

$$\dot{V}(x(t), t) \leq -cV(x(t), t) + d|\xi(t)|^2 + \delta. \quad (3)$$

Then the solution is global and the closed loop is NSpS- m -M. In particular,

$$EV(t) \leq e^{-c(t-t_0)}EV(t_0) + \frac{dK_0 + \delta}{c}.$$

Proof. Let τ_R be the first exit time from the ball $|x| < R$. Integrating (3) on $[t_0, t \wedge \tau_R]$, taking expectations, and applying the comparison lemma gives a bound independent of R . Coercivity in (2) then implies $\Pr\{\tau_R \leq T\} \rightarrow 0$ as $R \rightarrow \infty$ for every finite T ; hence the maximal solution does not explode. Letting $R \rightarrow \infty$ yields the displayed estimate. Combining it with (2) verifies Definition 2.1.

Lemma 2.2. [6] For any $x, y \in R$,

$$p|x|^m|y|^n \leq q|x|^{m+n} + \frac{n}{m+n} \left(\frac{m+n}{m} q \right)^{\frac{m}{n}} \cdot p^{\frac{m+n}{n}} |y|^{m+n},$$

for any positive constants m, n, p , and q .

Lemma 2.3. [17] For any $\iota_1, \iota_2 \in R$,

$$|\iota_1^\vartheta - \iota_2^\vartheta| \leq \vartheta(2 + 2^{\vartheta-2})|\iota_1 - \iota_2| \cdot |(\iota_1 - \iota_2)^{\vartheta-1} + \iota_2^{\vartheta-1}|,$$

for any odd rational number $\vartheta \geq 1$.

Lemma 2.4. [18] If $D^+\varphi(t) \leq \kappa(t)\varphi(t) + \iota(t)$, then

$$\varphi(t) \leq e^{\int_{t_0}^t \kappa(s) ds} \varphi(t_0) + \int_{t_0}^t \iota(s) e^{\int_s^t \kappa(v) dv} ds,$$

where $\varphi(t)$, $\kappa(t)$ and $\iota(t)$ are continuous functions on $[t_0, \infty)$.

3. Single-Leader Adaptive Tracking

Consider one leader and N followers. The i th follower is described by

$$\begin{cases} \dot{x}_{ij}(t) = x_{ij+1}^{p_{ij}}(t) + \theta f_{ij}(\bar{x}_{ij}(t), t) \\ \quad + h_{ij}(\bar{x}_{ij}(t), t) \xi_{ij}(t), \\ \dot{x}_{in}(t) = u_i^{p_{in}}(t) + \theta f_{in}(x_i(t), t) \\ \quad + h_{in}(x_i(t), t) \xi_{in}(t), \\ y_i(t) = x_{i1}(t), j = 1, \dots, n-1, \\ i = 1, \dots, N, \end{cases} \quad (4)$$

where $x_i(t) = (x_{i1}(t), \dots, x_{in}(t))^T \in R^n$ is the state, $u_i(t) \in R$ is the input, and $y_i(t) \in R$ is the output. For $j = 1, \dots, n$, the functions $f_{ij}(\bar{x}_{ij}, t)$ and $h_{ij}(\bar{x}_{ij}, t)$ are locally Lipschitz in $\bar{x}_{ij}$ and piecewise continuous in t , where $\bar{x}_{ij} = (x_{i1}, \dots, x_{ij})^T \in R^j$ and $x_i = \bar{x}_{in} \in R^n$, and $f_{ij}(0, t)$ and $h_{ij}(0, t)$ are bounded. The $\mathcal{F}_t$ -adapted stochastic process $\xi_i(t) = (\xi_{i1}(t), \dots, \xi_{in}(t))^T$ is

defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq t_0}, P)$ and satisfies $\sup_{t_0 \leq s \leq t} E|\xi_i(s)|^2 \leq K_0$. The scalar θ is an unknown constant, $p_{i1} = 1$, and each known $p_{ij} \geq 1$ for $j \geq 2$ is a ratio of odd integers. We additionally assume that the implemented closed-loop vector field is locally Lipschitz, which ensures a unique local solution before the non-explosion argument in Theorem 3.1.

The objective is to make y_i practically track the leader output y_0 , where $|y_0(t)| \vee |\dot{y}_0(t)| \leq M$, while keeping every closed-loop signal ultimately bounded in mean square.

The follower communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ has node set $\mathcal{V} = \{1, 2, \dots, N\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and the adjacency matrix $A = (a_{ij})_{N \times N}$. The graph is fixed, weighted, and undirected, so $a_{ij} = a_{ji} \geq 0$ and $a_{ii} = 0$. The neighbor set is $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}, j \neq i\}$.

The degree matrix $D = \text{diag}(d_1, \dots, d_N)$ has $d_i =$

$\sum_{j \in \mathcal{N}_i} a_{ij}$, $L = D - A$ denotes the Laplacian matrix of graph $\mathcal{G}$.

The augmented graph $\mathcal{G}_l$ adds leader node 0. The leader-weighted matrix associated with $\mathcal{G}_l$ is denoted as $B_l = \text{diag}(b_1, b_2, \dots, b_N)$. If node 0 is not a neighbor of node i , then $b_i = 0$; otherwise $b_i > 0$. We assume that every connected component of the follower graph contains at least one pinned follower ($b_i > 0$). Equivalently, $\mathcal{G}_l$ contains a leader-rooted spanning forest.

Introduce the coordinate transformation

$$\begin{cases} e_{i1} = b_i(y_i - y_0) + \sum_{s=1}^N a_{is}(y_i - y_s), \\ e_{ij} = x_{ij} - \alpha_{i(j-1)}, j = 2, \dots, n+1, \\ i = 1, \dots, N, \end{cases} \quad (5)$$

where $x_{i(n+1)} = u_i$, $e_{i(n+1)} = 0$ and $\alpha_{i1}, \dots, \alpha_{in}$ are stabilizing functions to be designed.

The distributed adaptive controller is constructed recursively.

Step 1: Let $e_l = (e_{11}, e_{21}, \dots, e_{N1})^T$. From (5),

$$e_l = H_l(y - y_0 1_N). \quad (6)$$

where $H_l = B_l + L$, $1_N = (1, \dots, 1)^T$, and $y = (y_1, \dots, y_N)^T$. The matrix B_l is only positive semidefinite in general. Nevertheless, for any nonzero v ,

$$v^T H_l v = \frac{1}{2} \sum_{i,s=1}^N a_{is}(v_i - v_s)^2 + \sum_{i=1}^N b_i v_i^2 > 0$$

under the pinned-component assumption. Hence H_l is symmetric positive definite and H_l^{-1} is well defined.

We choose a candidate Lyapunov function

$$V_l = \frac{1}{2} e_l^T H_l^{-1} e_l + \frac{1}{2\lambda} \bar{\theta}^2,$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter-estimation error and $\lambda > 0$ is an adaptation gain. It follows from (4) and (6) that

$$\dot{V}_I = e_I^T H_I^{-1} H_I (\dot{y} - \dot{y}_0) - \frac{1}{\lambda} \tilde{\theta} \dot{\hat{\theta}} = \sum_{i=1}^N e_{iI} \quad (7)$$

$$(x_{i2} + \theta f_{iI} + h_{iI} \xi_{iI} - \dot{y}_0) - \frac{1}{\lambda} \tilde{\theta} \dot{\hat{\theta}}.$$

By Young's inequality,

$$e_{iI} h_{iI} \xi_{iI} \leq \frac{1}{4d_{iI}} e_{iI}^2 h_{iI}^2 + d_{iI} |\xi_{iI}|^2, \quad (8)$$

$$-e_{iI} \dot{y}_0 \leq \frac{1}{2d_M} e_{iI}^2 + \frac{d_M}{2} M^2, \quad (9)$$

where $d_{iI} > 0$ and $d_M > 0$ are design constants. Substituting (8) and (9) into (7) leads to

$$\begin{aligned} \dot{V}_I &\leq \sum_{i=1}^N e_{iI} (x_{i2} + \theta f_{iI} + \frac{e_{iI} h_{iI}^2}{4d_{iI}} + \frac{e_{iI}}{2d_M}) \\ &+ \sum_{i=1}^N d_{iI} |\xi_{iI}|^2 - \frac{1}{\lambda} \tilde{\theta} \dot{\hat{\theta}} + \frac{d_M}{2} M^2 = \sum_{i=1}^N e_{iI} \end{aligned} \quad (10)$$

$$\begin{aligned} &(\alpha_{iI} + e_{i2} + \hat{\theta} f_{iI} + \frac{e_{iI} h_{iI}^2}{4d_{iI}} + \frac{e_{iI}}{2d_M}) + \sum_{i=1}^N d_{iI} |\xi_{iI}|^2 \\ &- \tilde{\theta} (\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_I) + \frac{c_\theta}{\lambda} \tilde{\theta} \dot{\hat{\theta}} + \frac{d_M}{2} M^2, \end{aligned}$$

where $\tau_I = \sum_{i=1}^N e_{iI} f_{iI} - c_\theta \hat{\theta}/\lambda$ and $c_\theta > 0$. This form is not postulated independently: after writing $\theta = \hat{\theta} + \tilde{\theta}$, the coefficient multiplying $\tilde{\theta}$ in $\dot{V}_I$ is $\tau_I - \hat{\theta}/\lambda$. The recursive tuning functions collect the corresponding coefficient at each subsequent step, so the final choice $\dot{\hat{\theta}} = \lambda \tau_n$ cancels it. The term $-c_\theta \hat{\theta}/\lambda$ is a leakage term that prevents parameter drift under persistent random excitation. In (10), we choose

$$\alpha_{iI}(x_{i1}, \dots, x_{iN}, y_0, \hat{\theta}) = -c_{iI} e_{iI} - \frac{1}{d_M} e_{iI} - \hat{\theta} f_{iI} - \frac{1}{4d_{iI}} e_{iI} h_{iI}^2,$$

where $c_{iI} > 0$ is a design constant. Then,

$$\begin{aligned} \dot{V}_I &\leq - \sum_{i=1}^N (c_{iI} + \frac{1}{2d_M}) e_{iI}^2 + \sum_{i=1}^N e_{iI} e_{i2} + \sum_{i=1}^N d_{iI} |\xi_{iI}|^2 \\ &- \tilde{\theta} (\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_I) + \frac{c_\theta}{\lambda} \tilde{\theta} \dot{\hat{\theta}} + \frac{d_M}{2} M^2. \end{aligned} \quad (11)$$

Step 2: Let $V_2 = V_I + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+2}/(\bar{p}-p_{i2}+2)$ with $\bar{p} = \max_{1 \leq i \leq N} \{p_{i1}, p_{i2}, \dots, p_{in}\}$, then we have

$$\begin{aligned} \dot{V}_2 &= \dot{V}_I + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} (x_{i2} - \alpha_{iI}) \\ &\leq - \sum_{i=1}^N (c_{iI} + \frac{1}{2d_M}) e_{iI}^2 + \sum_{i=1}^N e_{iI} e_{i2} + \sum_{i=1}^N d_{iI} |\xi_{iI}|^2 \\ &- \tilde{\theta} (\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_I) + \frac{c_\theta}{\lambda} \tilde{\theta} \dot{\hat{\theta}} + \frac{d_M}{2} M^2 + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} (x_{i3}^{p_{i2}} - \alpha_{iI}^I) \\ &+ \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} \alpha_{i2}^{p_{i2}} + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} (\hat{\theta} w_{i2} + h_{i2} \xi_{i2} - \sum_{s=1}^N \frac{\partial \alpha}{\partial x_{sI}} \\ &- \sum_{s=1}^N \frac{\partial \alpha_{iI}}{\partial x_{sI}} h_{sI} \xi_{sI} - \frac{\partial \alpha_{iI}}{\partial y_0} \dot{y}_0 - \frac{\partial \alpha_{iI}}{\partial \hat{\theta}} \lambda \tau_2) \\ &+ \sum_{i=1}^N \frac{\partial \alpha_{iI}}{\partial \hat{\theta}} e_{i2}^{\bar{p}-p_{i2}+1} (\lambda \tau_2 - \dot{\hat{\theta}}), \end{aligned} \quad (12)$$

where $w_{i2} = f_{i2} - \sum_{s=1}^N f_{sI} \partial \alpha_{iI} / \partial x_{sI}$ and $\tau_2 = \tau_I + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} w_{i2}$. By Young's inequality and Lemma 2.2,

$$\begin{cases} e_{iI} e_{i2} \leq \frac{1}{2d_M} e_{iI}^2 + \frac{d_M}{2} e_{i2}^2 \\ \leq \frac{1}{2d_M} e_{iI}^2 + \frac{d_M}{1+\bar{p}} e_{i2}^{\bar{p}+1} + \frac{\bar{p}-1}{2(1+\bar{p})} d_M, \\ -e_{i2}^{\bar{p}-p_{i2}+1} \sum_{s=1}^N \frac{\partial \alpha_{iI}}{\partial x_{sI}} h_{sI} \xi_{sI} \leq e_{i2}^{2(\bar{p}-p_{i2}+1)} \sum_{s=1}^N \frac{1}{4d_{sI}} \\ (\frac{\partial \alpha_{iI}}{\partial x_{sI}} h_{sI})^2 + \sum_{s=1}^N d_{sI2} |\xi_{sI}|^2, \\ e_{i2}^{\bar{p}-p_{i2}+1} h_{i2} \xi_{i2} \leq \frac{1}{4d_{i2}} e_{i2}^{2(\bar{p}-p_{i2}+1)} h_{i2}^2 + d_{i2} |\xi_{i2}|^2, \end{cases} \quad (13)$$

where positive constants d_{sI2} and d_{i2} are to be designed. Substituting (13) into (12) yields that

$$\begin{aligned} \dot{V}_2 &\leq - \sum_{i=1}^N c_{iI} e_{iI}^2 + \frac{d_M}{1+\bar{p}} \sum_{i=1}^N e_{i2}^{\bar{p}+1} \\ &+ \sum_{i=1}^N \sum_{j=1}^2 d_{ij} |\xi_{ij}|^2 + \sum_{i=1}^N d_{iI2} |\xi_{iI}|^2 + \delta_M \\ &- \tilde{\theta} (\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_2) + \frac{c_\theta}{\lambda} \tilde{\theta} \dot{\hat{\theta}} + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} (x_{i3}^{p_{i2}} - \alpha_{i2}^{p_{i2}}) \\ &+ \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} \alpha_{i2}^{p_{i2}} + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} (\hat{\theta} w_{i2} \\ &+ \frac{1}{4d_{i2}} e_{i2}^{\bar{p}-p_{i2}+1} h_{i2}^2 - \sum_{s=1}^N \frac{\partial \alpha_{iI}}{\partial x_{sI}} x_{s2}^{p_{sI}} \\ &+ e_{i2}^{\bar{p}-p_{i2}+1} \sum_{s=1}^N \frac{1}{4d_{sI2}} (\frac{\partial \alpha_{iI}}{\partial x_{sI}} h_{sI})^2 \\ &- \frac{\partial \alpha_{iI}}{\partial y_0} \dot{y}_0 - \frac{\partial \alpha_{iI}}{\partial \hat{\theta}} \lambda \tau_2) + \sum_{i=1}^N \frac{\partial \alpha_{iI}}{\partial \hat{\theta}} e_{i2}^{\bar{p}-p_{i2}+1} (\lambda \tau_2 - \dot{\hat{\theta}}), \end{aligned} \quad (14)$$

where $\delta_M = d_M(\bar{p}-1)/2(1+\bar{p}) + d_M M^2/2$.

By Lemma 2.2, we can derive the following six estimates. They are split into three display blocks so that every line remains legible in one column:

$$\begin{aligned} e_{i2}^{\bar{p}-p_{i2}+1} \hat{\theta} w_{i2} &\leq r_{i2} e_{i2}^{\bar{p}+1} (\hat{\theta} w_{i2})^{\bar{p}} + \bar{\delta}_{i2}, \\ -e_{i2}^{\bar{p}-p_{i2}+1} \sum_{s=1}^N \frac{\partial \alpha_{iI}}{\partial x_{sI}} x_{s2}^{p_{sI}} &\leq r_{i2} e_{i2}^{\bar{p}+1} (\sum_{s=1}^N \frac{\partial \alpha_{iI}}{\partial x_{sI}} x_{s2}^{p_{sI}})^{\bar{p}} + \bar{\delta}_{i2}, \\ -e_{i2}^{\bar{p}-p_{i2}+1} \frac{\partial \alpha_{iI}}{\partial y_0} \dot{y}_0 &\leq r_{i2} e_{i2}^{\bar{p}+1} (\frac{\partial \alpha_{iI}}{\partial y_0} M)^{\bar{p}} + \bar{\delta}_{i2}, \\ -e_{i2}^{\bar{p}-p_{i2}+1} \frac{\partial \alpha_{iI}}{\partial \hat{\theta}} \lambda \tau_2 &\leq r_{i2} e_{i2}^{\bar{p}+1} (\frac{\partial \alpha_{iI}}{\partial \hat{\theta}} \lambda \tau_2)^{\bar{p}} + \bar{\delta}_{i2}, \\ e_{i2}^{\bar{p}-p_{i2}+1} \frac{1}{4d_{i2}} e_{i2}^{\bar{p}-p_{i2}+1} h_{i2}^2 &\leq r_{i2} e_{i2}^{\bar{p}+1} (\frac{1}{4d_{i2}} e_{i2}^{\bar{p}-p_{i2}+1} h_{i2}^2)^{\bar{p}} + \bar{\delta}_{i2}, \\ e_{i2}^{\bar{p}-p_{i2}+1} e_{i2}^{\bar{p}-p_{i2}+1} \sum_{s=1}^N \frac{1}{4d_{sI2}} (\frac{\partial \alpha_{iI}}{\partial x_{sI}} h_{sI})^2 &\leq r_{i2} e_{i2}^{\bar{p}+1} (e_{i2}^{\bar{p}-p_{i2}+1} \sum_{s=1}^N \frac{1}{4d_{sI2}} (\frac{\partial \alpha_{iI}}{\partial x_{sI}} h_{sI})^2)^{\bar{p}} + \bar{\delta}_{i2}, \end{aligned} \quad (15)$$

where $\bar{\delta}_{i2} = p_{i2}(\bar{p}_i r_{i2})^{1/(1-\bar{p}_i)}/(\bar{p}+1)$ with $r_{i2} >$

$0(i = 1, \dots, N)$ being constants to be designed and $\bar{p}_{i2} = (\bar{p} + 1)/(\bar{p} - p_{i2} + 1)$. Together with the definitions of $\bar{p}$ and p_{i2} , we know that $\bar{p}_{i2}$ is a ratio of even integer and odd integer.

Substituting (15) into (14) yields that

$$\begin{aligned} \dot{V}_2 \leq & - \sum_{i=1}^N c_{i1} e_{i1}^2 + \sum_{i=1}^N \bar{\eta}_{i2} e_{i2}^{\bar{p}+1} + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} \alpha_{i2}^{p_{i2}} \\ & + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} (x_{i3}^{p_{i2}} - \alpha_{i2}^{p_{i2}}) + \sum_{i=1}^N \delta_{i2} \\ & + \sum_{i=1}^N \sum_{j=1}^2 d_{ij} |\xi_{ij}|^2 + \sum_{i=1}^N d_{i12} |\xi_{i1}|^2 \\ & + \sum_{i=1}^N \frac{\partial \alpha_{i1}}{\partial \hat{\theta}} e_{i2}^{\bar{p}-p_{i2}+1} (\lambda \tau_2 - \dot{\hat{\theta}}) - \tilde{\theta} \left(\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_2 \right) + \frac{c_{\theta}}{\lambda} \tilde{\theta} \dot{\hat{\theta}} \end{aligned} \quad (16)$$

where

$$\delta_{i2} = \delta_M + \delta \bar{\delta}_{i2},$$

$$\begin{aligned} \bar{\eta}_{i2} = & \frac{d_M}{1 + \bar{p}} + r_{i2} [(\hat{\theta} w_{i2})^{\bar{p}_{i2}} + \left(\sum_{s=1}^N x_{s1}^{p_{s1}} \frac{\partial \alpha_{i1}}{\partial x_{s1}} \right)^{\bar{p}_{i2}} \\ & + (M \frac{\partial \alpha_{i1}}{\partial y_0})^{\bar{p}_{i2}} + (\lambda \tau_2 \frac{\partial \alpha_{i1}}{\partial \hat{\theta}})^{\bar{p}_{i2}} + \left(\frac{e_{i2}^{\bar{p}-p_{i2}+1} h_{i2}^2}{4d_{i2}} \right)^{\bar{p}_{i2}} \\ & + (e_{i2}^{\bar{p}-p_{i2}+1} \sum_{s=1}^N \frac{1}{4d_{s12}} (h_{s1} \frac{\partial \alpha_{i1}}{\partial x_{s1}})^2)^{\bar{p}_{i2}}]. \end{aligned}$$

Selecting

$$\alpha_{i2}(x_{11}, \dots, x_{N1}, x_{12}, \dots, x_{N2}, y_0, \hat{\theta})$$

$$= -e_{i2}(\bar{c}_{i2} + \eta_{i2})^{\frac{1}{\bar{p}_{i2}}} \triangleq -e_{i2}\beta_{i2},$$

where $\eta_{i2} = \bar{\eta}_{i2} + \hat{\eta}_{i2}$ with $\hat{\eta}_{i2}$ being positive design parameter, and $\bar{c}_{i2} = c_{i2}\gamma$ with $c_{i2} > 0$ and $\gamma > 1$ being design constants. Then

$$\begin{aligned} \dot{V}_2 \leq & - \sum_{i=1}^N c_{i1} e_{i1}^2 - \sum_{i=1}^N \bar{c}_{i2} e_{i2}^{\bar{p}+1} - \sum_{i=1}^N \hat{\eta}_{i2} e_{i2}^{\bar{p}+1} \\ & + \sum_{i=1}^N e_{i2}^{\bar{p}-p_{i2}+1} (x_{i3}^{p_{i2}} - \alpha_{i2}^{p_{i2}}) + \sum_{i=1}^N \delta_{i2} \\ & + \sum_{i=1}^N \sum_{j=1}^2 d_{ij} |\xi_{ij}|^2 + \sum_{i=1}^N d_{i12} |\xi_{i1}|^2 \\ & + \sum_{i=1}^N \frac{\partial \alpha_{i1}}{\partial \hat{\theta}} e_{i2}^{\bar{p}-p_{i2}+1} (\lambda \tau_2 - \dot{\hat{\theta}}) - \tilde{\theta} \left(\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_2 \right) + \frac{c_{\theta}}{\lambda} \tilde{\theta} \dot{\hat{\theta}}. \end{aligned} \quad (17)$$

Further, by Lemma 2.2 and Lemma 2.3, we have

$$\begin{aligned} & e_{i2}^{\bar{p}-p_{i2}+1} (x_{i3}^{p_{i2}} - \alpha_{i2}^{p_{i2}}) \\ & \leq m_{i2} |e_{i2}|^{\bar{p}-p_{i2}+1} |x_{i3} - \alpha_{i2}| (x_{i3} - \alpha_{i2})^{p_{i2}-1} + \alpha_{i2}^{p_{i2}-1} \\ & \leq m_{i2} |e_{i2}|^{\bar{p}-p_{i2}+1} |e_{i3}| |e_{i3}^{p_{i2}-1} + e_{i2}^{p_{i2}-1} \beta_{i2}^{p_{i2}-1}| \\ & \leq m_{i2} |e_{i2}|^{\bar{p}-p_{i2}+1} |e_{i3}|^{p_{i2}} \\ & + m_{i2} |e_{i2}|^{\bar{p}} |e_{i3}| \beta_{i2}^{p_{i2}-1} \leq m_{i2} |e_{i2}|^{\bar{p}+1} \\ & + \frac{m_{i2} p_{i2}}{\bar{p} + 1} \frac{\bar{p} - p_{i2} + 1}{\bar{p}_{i2}} |e_{i3}|^{\bar{p}+1} + m_{i2} |e_{i2}|^{\bar{p}+1} \\ & + \frac{m_{i2}}{\bar{p} + 1} \left(\frac{\bar{p} + 1}{\bar{p}} \right)^{-\bar{p}} (\beta_{i2}^{p_{i2}-1})^{\bar{p}+1} \cdot |e_{i3}|^{\bar{p}+1} \\ & \leq \hat{\eta}_{i2} e_{i2}^{\bar{p}+1} + \rho_{i3} e_{i3}^{\bar{p}+1}, \end{aligned} \quad (18)$$

where $\hat{\eta}_{i2} = 2m_{i2}$, $m_{i2} = p_{i2}(2 + 2^{p_{i2}-2})$ and $\rho_{i3} =$

$m_{i2}[1 + (\beta_{i2}^{p_{i2}-1})^{\bar{p}+1}]$. Therefore, (14) can be rewritten as

$$\begin{aligned} \dot{V}_2 \leq & - \sum_{i=1}^N c_{i1} e_{i1}^2 - \sum_{i=1}^N \bar{c}_{i2} e_{i2}^{\bar{p}+1} \\ & + \sum_{i=1}^N \rho_{i3} e_{i3}^{\bar{p}+1} + \sum_{i=1}^N \delta_{i2} + \sum_{i=1}^N \sum_{j=1}^2 d_{ij} |\xi_{ij}|^2 \\ & + \sum_{i=1}^N d_{i12} |\xi_{i1}|^2 + \sum_{i=1}^N \frac{\partial \alpha_{i1}}{\partial \hat{\theta}} e_{i2}^{\bar{p}-p_{i2}+1} (\lambda \tau_2 - \dot{\hat{\theta}}) \\ & - \tilde{\theta} \left(\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_2 \right) + \frac{c_{\theta}}{\lambda} \tilde{\theta} \dot{\hat{\theta}}. \end{aligned} \quad (19)$$

Step k ($k = 3, \dots, n$): Suppose that there exists a series of smooth functions $\alpha_{ij} = -e_{ij}\beta_{ij}$ ($i = 1, \dots, N; 1 \leq j \leq k-1$) and τ_j ($1 \leq j \leq k-1$) such that

$$\begin{aligned} \dot{V}_{k-1} \leq & - \sum_{i=1}^N c_{i1} e_{i1}^2 - \sum_{i=1}^N \sum_{j=2}^{k-1} \bar{c}_{ij} e_{ij}^{\bar{p}+1} + \sum_{i=1}^N \rho_{ik} e_{ik}^{\bar{p}+1} \\ & + \sum_{i=1}^N \sum_{j=1}^{k-1} d_{ij} |\xi_{ij}|^2 + \sum_{i=1}^N \sum_{j=1}^{k-2} \sum_{l=j+1}^{k-1} d_{ijl} |\xi_{ijl}|^2 \\ & + \sum_{i=1}^N \sum_{j=1}^{k-2} \frac{\partial \alpha_{ij}}{\partial \hat{\theta}} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} (\lambda \tau_{k-1} - \dot{\hat{\theta}}) \\ & - \tilde{\theta} \left(\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_{k-1} \right) + \frac{c_{\theta}}{\lambda} \tilde{\theta} \dot{\hat{\theta}} + \sum_{i=1}^N \delta_{i(k-1)}, \end{aligned} \quad (20)$$

where

$$V_{k-1} = V_{k-2} + \sum_{i=1}^N e_{i(k-1)}^{\bar{p}-p_{i(k-1)}+2} / (\bar{p} - p_{i(k-1)} + 2).$$

In this step, we prove that (20) also holds for

$$V_k = V_{k-1} + \sum_{i=1}^N \frac{1}{\bar{p} - p_{ik} + 2} e_{ik}^{\bar{p}-p_{ik}+2}.$$

It follows from (4), (5) and (20) that

$$\begin{aligned} \dot{V}_k \leq & - \sum_{i=1}^N c_{i1} e_{i1}^2 - \sum_{i=1}^N \sum_{j=2}^{k-1} \bar{c}_{ij} e_{ij}^{\bar{p}+1} + \sum_{i=1}^N \rho_{ik} e_{ik}^{\bar{p}+1} \\ & + \sum_{i=1}^N \sum_{j=1}^{k-1} d_{ij} |\xi_{ij}|^2 + \sum_{i=1}^N \sum_{j=1}^{k-2} \sum_{l=j+1}^{k-1} d_{ijl} |\xi_{ijl}|^2 + \frac{c_{\theta}}{\lambda} \tilde{\theta} \dot{\hat{\theta}} \\ & + \sum_{i=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{ij}}{\partial \hat{\theta}} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} (\lambda \tau_k - \dot{\hat{\theta}}) \\ & - \tilde{\theta} \left(\frac{1}{\lambda} \dot{\hat{\theta}} - \tau_k \right) + \sum_{i=1}^N \delta_{i(k-1)} + \sum_{i=1}^N e_{ik}^{\bar{p}-p_{ik}+1} (x_{i(k+1)}^{p_{ik}} + \hat{\theta} w_{ik}) \\ & + h_{ik} \xi_{ik} - \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} x_{s(j+1)}^{p_{sj}} - \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} h_{sj} \xi_{sj} \\ & - \frac{\partial \alpha_{i(k-1)}}{\partial y_0} \dot{y}_0 - \frac{\partial \alpha_{i(k-1)}}{\partial \hat{\theta}} \lambda \tau_k - \sum_{i=1}^N \sum_{j=1}^{k-2} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} \lambda w_{ik}, \end{aligned} \quad (21)$$

where $w_{ik} = f_{ik} - \sum_{s=1}^N \sum_{j=1}^{k-1} f_{sj} \partial \alpha_{i(k-1)} / \partial x_{sj}$

and $\tau_k = \tau_{k-1} + \sum_{i=1}^N e_{ik}^{\bar{p}-p_{ik}+1} w_{ik}$. Young's inequality also gives

$$\begin{aligned}
& -e_{ik}^{\bar{p}-p_{ik}+1} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} h_{sj} \xi_{sj} \\
& \leq e_{ik}^{2(\bar{p}-p_{ik}+1)} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{1}{4d_{sjk}} \left(\frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} h_{sj} \right)^2 \quad (22)
\end{aligned}$$

$$\begin{aligned}
& + \sum_{s=1}^N \sum_{j=1}^{k-1} d_{sjk} |\xi_{sj}|^2, \\
& e_{ik}^{\bar{p}-p_{ik}+1} h_{ik} \xi_{ik} \leq \frac{1}{4d_{ik}} e_{ik}^{2(\bar{p}-p_{ik}+1)} h_{ik}^2 d_{ik} |\xi_{ik}|^2, \quad (23)
\end{aligned}$$

where $d_{sjk} > 0$ and $d_{ik} > 0$ are constants. Substituting (22)-(23) into (21) yields that

$$\begin{aligned}
\dot{V}_k \leq & - \sum_{i=1}^N c_{il} e_{il}^2 - \sum_{i=1}^N \sum_{j=2}^{k-1} \bar{c}_{ij} e_{ij}^{\bar{p}+1} + \sum_{i=1}^N \rho_{ik} e_{ik}^{\bar{p}+1} \\
& + \sum_{i=1}^N \sum_{j=1}^k d_{ij} |\xi_{ij}|^2 + \sum_{i=1}^N \sum_{j=1}^{k-1} \sum_{l=j+1}^k d_{ijl} |\xi_{ijl}|^2 + \frac{c_\theta}{\lambda} \bar{\theta} \hat{\theta} \\
& + \sum_{i=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{ij}}{\partial \theta} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} (\lambda \tau_k - \hat{\theta}) \quad (24)
\end{aligned}$$

$$\begin{aligned}
& - \bar{\theta} \left(\frac{1}{\lambda} \hat{\theta} - \tau_k \right) + \sum_{i=1}^N \delta_{i(k-1)} \\
& + \sum_{i=1}^N e_{ik}^{\bar{p}-p_{ik}+1} (x_{i(k+1)}^{p_{ik}} - \alpha_{ik}^{p_{ik}}) \\
& + \sum_{i=1}^N e_{ik}^{\bar{p}-p_{ik}+1} \alpha_{ik}^{p_{ik}} + \sum_{i=1}^N e_{ik}^{\bar{p}-p_{ik}+1} (\hat{\theta} w_{ik} \\
& + \frac{e_{ik}^{\bar{p}-p_{ik}+1}}{4d_{ik}} h_{ik}^2 - \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} x_{s(j+1)}^{p_{sj}} \\
& - \frac{\partial \alpha_{i(k-1)}}{\partial y_0} y_0 + e_{ik}^{\bar{p}-p_{ik}+1} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{1}{4d_{sjk}} \\
& \left(\frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} h_{sj} \right)^2 - \frac{\partial \alpha_{i(k-1)}}{\partial \theta} \lambda \tau_k - \sum_{i=1}^N \sum_{j=1}^{k-2} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1}
\end{aligned}$$

By Lemma 2.3, we have

$$\begin{aligned}
& e_{ik}^{\bar{p}-p_{ik}+1} \hat{\theta} w_{ik} \leq r_{ik} e_{ik}^{\bar{p}+1} (\hat{\theta} w_{ik})^{\bar{p}_{ik}} + \bar{\delta}_{ik}, \\
& -e_{ik}^{\bar{p}-p_{ik}+1} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} x_{s(j+1)}^{p_{sj}} \leq r_{ik} \\
& e_{ik}^{\bar{p}+1} \left(\sum_{s=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} x_{s(j+1)}^{p_{sj}} \right)^{\bar{p}_{ik}} + \bar{\delta}_{ik}, \\
& -e_{ik}^{\bar{p}-p_{ik}+1} \frac{\partial \alpha_{i(k-1)}}{\partial y_0} y_0 \\
& \leq r_{ik} e_{ik}^{\bar{p}+1} \left(\frac{\partial \alpha_{i(k-1)}}{\partial y_0} M \right)^{\bar{p}_{ik}} + \bar{\delta}_{ik}, -e_{ik}^{\bar{p}-p_{ik}+1} \frac{\partial \alpha_{i(k-1)}}{\partial \theta} \lambda \tau_k \\
& \leq r_{ik} e_{ik}^{\bar{p}+1} \left(\frac{\partial \alpha_{i(k-1)}}{\partial \theta} \lambda \tau_k \right)^{\bar{p}_{ik}} + \bar{\delta}_{ik}, e_{ik}^{\bar{p}-p_{ik}+1} \frac{e_{ik}^{\bar{p}-p_{ik}+1}}{4d_{ik}} h_{ik}^2 \\
& \leq r_{ik} e_{ik}^{\bar{p}+1} \left(\frac{1}{4d_{ik}} e_{ik}^{\bar{p}-p_{ik}+1} h_{ik}^2 \right)^{\bar{p}_{ik}} + \bar{\delta}_{ik},
\end{aligned}$$

$$\begin{aligned}
& e_{ik}^{2(\bar{p}-p_{ik}+1)} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{1}{4d_{sjk}} \left(\frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} h_{sj} \right)^2 \\
& \leq r_{ik} e_{ik}^{\bar{p}+1} (e_{ik}^{\bar{p}-p_{ik}+1} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{1}{4d_{sjk}} \\
& \left(\frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} h_{sj} \right)^2)^{\bar{p}_{ik}} + \bar{\delta}_{ik}, \quad (25)
\end{aligned}$$

$$\begin{aligned}
& -e_{ik}^{\bar{p}-p_{ik}+1} \sum_{i=1}^N \sum_{j=1}^{k-2} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} \lambda w_{ik} \\
& \leq r_{ik} e_{ik}^{\bar{p}+1} (e_{ik}^{\bar{p}-p_{ik}+1})
\end{aligned}$$

$$\begin{aligned}
& \sum_{i=1}^N \sum_{j=1}^{k-2} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} \lambda w_{ik})^{\bar{p}_{ik}} + \bar{\delta}_{ik},
\end{aligned}$$

where $\bar{\delta}_{ik} = p_{ik}(\bar{p}_{ik} r_{ik})^{1/(1-\bar{p}_{ik})}/(\bar{p}+1)$ with $r_{ik} > 0$ ($i = 1, \dots, N$) and $\bar{p}_{ik} = (\bar{p}+1)/(\bar{p}-p_{ik}+1)$.

Substituting (25) into (24) leads to where

$$\begin{aligned}
\bar{\eta}_{ik} = & \rho_{ik} + r_{ik} \left[\left(\sum_{s=1}^N \sum_{j=1}^{k-1} x_{s(j+1)}^{p_{sj}} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}} \right)^{\bar{p}_{ik}} + (\lambda \tau_k \frac{\partial \alpha_{i(k-1)}}{\partial \theta})^{\bar{p}_{ik}} \right. \\
& + (M \frac{\partial \alpha_{i(k-1)}}{\partial y_0})^{\bar{p}_{ik}} + (\hat{\theta} w_{ik})^{\bar{p}_{ik}} \\
& + (e_{ik}^{\bar{p}-p_{ik}+1} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{1}{4d_{sjk}} (h_{sj} \frac{\partial \alpha_{i(k-1)}}{\partial x_{sj}})^2)^{\bar{p}_{ik}} \\
& \left. + \left(\sum_{i=1}^N \sum_{j=1}^{k-2} e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} \lambda w_{ik} \right)^{\bar{p}_{ik}} + \left(\frac{h_{ik}^2 e_{ik}^{\bar{p}-p_{ik}+1}}{4d_{ik}} \right)^{\bar{p}_{ik}} \right], \quad (26)
\end{aligned}$$

$$\delta_{ik} = 7\bar{\delta}_{ik} + \delta_{i(k-1)} > 0.$$

Choosing the stabilizing function

$$\alpha_{ik}(x_{1l}, \dots, x_{Nl}, \dots, x_{1k}, \dots, x_{Nk}, y_0, \hat{\theta}) = -e_{ik}(\bar{c}_{ik} + \eta_{ik})^{\frac{1}{\bar{p}_{ik}}} \triangleq -e_{ik}\beta_{ik}, \quad (27)$$

where $\eta_{ik} = \bar{\eta}_{ik} + \hat{\eta}_{ik}$ with $\hat{\eta}_{ik}$ being a design parameter, and $\bar{c}_{ik} = c_{ik}\gamma$ with $\bar{c}_{ik} > 0$ being a design constant. Then, (26) can be changed into

$$\begin{aligned}
\dot{V}_k \leq & - \sum_{i=1}^N c_{il} e_{il}^2 - \sum_{i=1}^N \sum_{j=2}^k \bar{c}_{ij} e_{ij}^{\bar{p}+1} - \sum_{i=1}^N \hat{\eta}_{ik} e_{ik}^{\bar{p}+1} \\
& + \sum_{i=1}^N e_{ik}^{\bar{p}-p_{ik}+1} (x_{i(k+1)}^{p_{ik}} - \alpha_{ik}^{p_{ik}}) + \sum_{i=1}^N \sum_{j=1}^k d_{ij} |\xi_{ij}|^2 \\
& + \sum_{i=1}^N \sum_{j=1}^{k-1} \sum_{l=j+1}^k d_{ijl} |\xi_{ijl}|^2 + \sum_{i=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{ij}}{\partial \theta} \\
& e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} (\lambda \tau_k - \hat{\theta}) - \bar{\theta} \left(\frac{1}{\lambda} \hat{\theta} - \tau_k \right) + \frac{c_\theta}{\lambda} \bar{\theta} \hat{\theta} + \sum_{i=1}^N \delta \quad (28)
\end{aligned}$$

Meanwhile, from Lemma 2.2 and Lemma 2.3, we also have

$$\begin{aligned}
& e_{ik}^{\bar{p}-p_{ik}+1} (x_{i(k+1)}^{p_{ik}} - \alpha_{ik}^{p_{ik}}) \leq m_{ik} |x_{i(k+1)} - \alpha_{ik}| \\
& |(x_{i(k+1)} - \alpha_{ik})^{p_{ik}-1} + \alpha_{ik}^{p_{ik}-1}| |e_{ik}|^{\bar{p}-p_{ik}+1}
\end{aligned}$$

$$\begin{aligned}
&\leq m_{ik}|e_{i(k+1)}| \leq m_{ik}|e_{i(k+1)}| \\
&|e_{i(k+1)}^{p_{ik}-1} + e_{ik}^{p_{ik}-1} \beta_{ik}^{p_{ik}-1}||e_{ik}|^{\bar{p}-p_{ik}+1} \\
&\leq m_{ik}(|e_{ik}|^{\bar{p}-p_{ik}+1}|e_{i(k+1)}|^{p_{ik}} \\
&+ \beta_{ik}^{p_{ik}-1}|e_{ik}|^{\bar{p}}|e_{i(k+1)}|) \\
&\leq m_{ik}|e_{ik}|^{\bar{p}+1} \\
&+ \frac{m_{ik}p_{ik}}{\bar{p}+1} \frac{\bar{p}-p_{ik}+1}{p_{ik}} |e_{ik}|^{\bar{p}+1} + m_{ik}|e_{ik}|^{\bar{p}+1} \\
&+ \frac{m_{ik}}{\bar{p}+1} \left(\frac{\bar{p}+1}{\bar{p}}\right)^{-\bar{p}} (\beta_{ik}^{p_{ik}-1})^{\bar{p}+1} \\
&|e_{i(k+1)}|^{\bar{p}+1} \\
&\leq \hat{\eta}_{ik} e_{ik}^{\bar{p}+1} + \rho_{i(k+1)} e_{i(k+1)}^{\bar{p}+1},
\end{aligned} \quad (29)$$

where $\hat{\eta}_{ik} = 2m_{ik}$, $m_{ik} = p_{ik}(2 + 2^{p_{ik}-2}) > 0$ and $\rho_{i(k+1)} = m_{ik}[1 + (\beta_{ik}^{p_{ik}-1})^{\bar{p}+1}]$. Hence, (28) can be rewritten as

$$\begin{aligned}
\dot{V}_k &\leq - \sum_{i=1}^N c_{il} e_{il}^2 - \sum_{i=1}^N \sum_{j=2}^k \bar{c}_{ij} e_{ij}^{\bar{p}+1} \\
&+ \sum_{i=1}^N \rho_{i(k+1)} e_{i(k+1)}^{\bar{p}+1} + \sum_{i=1}^N \sum_{j=1}^k d_{ij} |\xi_{ij}|^2 \\
&+ \sum_{i=1}^N \sum_{j=1}^{k-1} \sum_{l=j+1}^k d_{ijl} |\xi_{ijl}|^2 + \sum_{i=1}^N \sum_{j=1}^{k-1} \frac{\partial \alpha_{ij}}{\partial \hat{\theta}} \\
&e_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} (\lambda \tau_k - \hat{\theta}) \\
&- \hat{\theta} \left(\frac{1}{\lambda} \hat{\theta} - \tau_k \right) + \frac{c_{\theta}}{\lambda} \hat{\theta} \dot{\hat{\theta}} + \sum_{i=1}^N \delta_{ik}.
\end{aligned} \quad (30)$$

Let $k = n$, we select the distributed tracking controller

$$u_i = \alpha_{in} = -e_{in}(\bar{c}_{in} + \eta_{in}) \frac{1}{p_{in}} \triangleq -e_{in} \beta_{in}, \quad (31)$$

and the adaptive law

$$\dot{\hat{\theta}} = \lambda \tau_n, \quad \tau_n = \tau_{n-1} + \sum_{i=1}^N e_{in}^{\bar{p}-p_{in}+1} w_{in}, \quad (32)$$

then (28) can be turned into

$$\begin{aligned}
\dot{V}_n &\leq - \sum_{i=1}^N c_{il} e_{il}^2 - \sum_{i=1}^N \sum_{j=2}^n \bar{c}_{ij} e_{ij}^{\bar{p}+1} + \frac{c_{\theta}}{\lambda} \hat{\theta} \dot{\hat{\theta}} \\
&+ \sum_{i=1}^N \delta_{in} + \sum_{i=1}^N \bar{d}_i \sum_{j=1}^n |\xi_{ij}|^2 \\
&\leq -c_l \sum_{i=1}^N e_{il}^2 - \sum_{i=1}^N \sum_{j=2}^n c_{ij} \gamma e_{ij}^{\bar{p}+1} \\
&- \frac{c_{\theta}}{2\lambda} \hat{\theta}^2 + \sum_{i=1}^N \delta_{in} + \frac{c_{\theta}}{2\lambda} \theta^2 + \bar{d} \sum_{i=1}^N |\xi_i|^2,
\end{aligned} \quad (33)$$

where $e_{i(n+1)} = 0$ is considered, $c_l = \min_{1 \leq i \leq N} \{c_{il}\}$, $\bar{d}_i = \max\{d_{in}, \max_{1 \leq j \leq n-1} \{d_{ij} + \sum_{l=j+1}^n d_{ijl}\}\}$ and $\bar{d} = \max_{1 \leq i \leq N} \{\bar{d}_i\}$.

Finally, we discuss the stability of multi-agent

system (4) with one leader.

Theorem 3.1. Under the stated regularity and pinned-component assumptions, controller (31) with adaptive law (32) gives a unique global solution that is NSpS-2-M. All closed-loop signals are ultimately bounded in mean square, and every follower satisfies

$$\lim_{t \rightarrow \infty} E|y_i(t) - y_0(t)|^2 \leq \frac{2\bar{d}NK_0 + 2\delta_0}{c\lambda_{\min}(H_l)}.$$

The bound is tunable but remains nonzero under persistent disturbances.

Proof. First, let the Lyapunov function

$$\begin{aligned}
V &= V_n = \frac{1}{2} e_l^T H_l^{-1} e_l \\
&+ \sum_{i=1}^N \sum_{j=2}^n \frac{1}{\bar{p} - p_{ij} + 2} e_{ij}^{\bar{p}-p_{ij}+2} + \frac{1}{2\lambda} \hat{\theta}^2, \\
H_l &= B_l + L,
\end{aligned}$$

then V satisfies that

$$\begin{aligned}
m_l |\tilde{e}|^2 &\leq \frac{1}{2} \lambda_{\min}(H_l^{-1}) \sum_{i=1}^N e_{il}^2 \\
&+ \frac{1}{\bar{p}+2} \sum_{i=1}^N \sum_{j=2}^n (e_{ij}^{\frac{\bar{p}-p_{ij}+2}{2}})^2 + \frac{1}{2\lambda} \hat{\theta}^2 \\
&\leq \frac{1}{2} e_l^T H_l^{-1} e_l \\
&+ \sum_{i=1}^N \sum_{j=2}^n \frac{1}{\bar{p} - p_{ij} + 2} e_{ij}^{\bar{p}-p_{ij}+2} + \frac{1}{2\lambda} \hat{\theta}^2 \\
&\leq \frac{1}{2} \lambda_{\max}(H_l^{-1}) \sum_{i=1}^N e_{il}^2 \\
&+ \frac{1}{2} \sum_{i=1}^N \sum_{j=2}^n (e_{ij}^{\frac{\bar{p}-p_{ij}+2}{2}})^2 + \frac{1}{2\lambda} \hat{\theta}^2 \leq m_2 |\tilde{e}|^2,
\end{aligned} \quad (34)$$

where $\tilde{e} = (\tilde{e}_1, \dots, \tilde{e}_N, \tilde{\theta})^T$ with $\tilde{e}_i = (e_{i1}, e_{i2}^{(\bar{p}-p_{i2}+2)/2}, \dots, e_{in}^{(\bar{p}-p_{in}+2)/2})$ and $i = 1, 2, \dots, N$, $m_l = \min\{\lambda_{\min}(H_l^{-1})/2, 1/(\bar{p}+2), 1/2\lambda\}$, $m_2 = \max\{\lambda_{\max}(H_l^{-1})/2, 1/2, 1/2\lambda\}$.

On the other hand, we also have

$$\begin{aligned}
\frac{1}{2} \lambda_{\min}(H_l^{-1}) \sum_{i=1}^N e_{il}^2 &\leq \frac{1}{2} e_l^T H_l^{-1} e_l \\
&\leq \frac{1}{2} \lambda_{\max}(H_l^{-1}) \sum_{i=1}^N e_{il}^2, \\
e_{ij}^{\bar{p}-p_{ij}+2} &\leq \frac{\bar{p} - p_{ij} + 2}{\bar{p} + 1} \gamma e_{ij}^{\bar{p}+1} \\
&+ \frac{p_{ij} - 1}{\bar{p} + 1} \gamma \frac{\bar{p}-p_{ij}+2}{p_{ij}-1} \leq \gamma e_{ij}^{\bar{p}+1} + \gamma_0^{-1}, \\
p_{ij} &> 1,
\end{aligned} \quad (35)$$

$$\begin{aligned}
&+ \frac{p_{ij} - 1}{\bar{p} + 1} \gamma \frac{\bar{p}-p_{ij}+2}{p_{ij}-1} \leq \gamma e_{ij}^{\bar{p}+1} + \gamma_0^{-1}, \\
p_{ij} &> 1,
\end{aligned} \quad (36)$$

$$e_{ij}^{\bar{p}-p_{ij}+2} = e_{ij}^{\bar{p}+1} \leq \gamma e_{ij}^{\bar{p}+1} \quad (37)$$

$$< \gamma e_{ij}^{\bar{p}+1} + \gamma_0^{-1}, \quad p_{ij} = 1,$$

where $\gamma_0 = \gamma^{\bar{p}-p_{ij}+2/(p_{ij}-1)}$, Lemma 2.2 and $\gamma > 1$ are used in (36). Hence, (33), together with (35)-(37), can be further rewritten as

$$\begin{aligned} \dot{V} &\leq -\frac{2c_l}{\lambda_{\max}(H_l^{-1})} \frac{1}{2} e_l^T H_l^{-1} e_l \\ &\quad - \sum_{i=1}^N \sum_{j=2}^n \frac{c_{ij}(\bar{p} - p_{ij} + 2)}{\bar{p} - p_{ij} + 2} e_{ij}^{\bar{p}-p_{ij}+2} \\ &\quad - \frac{c_\theta}{2\lambda} \bar{\theta}^2 + \bar{d} \sum_{i=1}^N |\xi_i|^2 + \delta_0 \\ &\leq -cV + \bar{d} \sum_{i=1}^N |\xi_i|^2 + \delta_0, \end{aligned} \quad (38)$$

where $\delta_0 = \sum_{i=1}^N \sum_{j=2}^n c_{ij}/\gamma_0 + \sum_{i=1}^N \delta_{in} + c_\theta \theta^2/2\lambda > 0$ and

$$c = \min \left\{ \min_{1 \leq i \leq N} \min_{2 \leq j \leq n} c_{ij}(\bar{p} - p_{ij} + 2), \frac{2c_l}{\lambda_{\max}(H_l^{-1})}, c_\theta \right\}.$$

We now verify the hypotheses of Lemma 2.1 explicitly. The local closed-loop solution exists by the regularity assumption following (4). Inequality (34) supplies both coercive bounds with $m = 2$, and (38) is precisely the derivative inequality (3) with input $\Xi = (\xi_1^T, \dots, \xi_N^T)^T$, gain $d = \bar{d}$, and residual $\delta = \delta_0$. Moreover, $E|\Xi(t)|^2 = \sum_i E|\xi_i(t)|^2 \leq NK_0$. Lemma 2.1 therefore proves non-explosion, uniqueness of the global continuation, and NSpS-2-M.

Second, it follows from (38) that for any $\varepsilon > 0$, $E(V|_{t=t+\varepsilon}) - E(V|_{t=t})$

$$\leq \int_t^{t+\varepsilon} (-cEV + \bar{d} \sum_{i=1}^N E|\xi_i(v)|^2 + \delta_0) dv, \quad t \geq t_0,$$

from which we can derive that

$$D^+(EV) \leq -cEV + \bar{d} \sum_{i=1}^N E|\xi_i(v)|^2 + \delta_0 \quad (39)$$

$$\leq -cEV + \bar{d}NK_0 + \delta_0.$$

From Lemma 2.4, together with (34), (39) indicates that

$$\begin{aligned} m_l E|\tilde{e}(t)|^2 &\leq EV \leq e^{-c(t-t_0)} V \\ |_{t=t_0} + \frac{\bar{d}}{c} NK_0 + \frac{\delta_0}{c} &\leq m_2 |\tilde{e}(t_0)|^2 \\ e^{-c(t-t_0)} + \frac{\bar{d}}{c} NK_0 + \frac{\delta_0}{c}, \end{aligned} \quad (40)$$

which implies that

$$\lim_{t \rightarrow \infty} E|\tilde{e}(t)|^2 \leq \frac{\bar{d}}{cm_l} NK_0 + \frac{\delta_0}{cm_l}. \quad (41)$$

Thus the transformed errors and parameter estimate are ultimately bounded in mean square. Because the backstepping transformations are triangular, bounded reference signals and transformed coordinates recursively imply bounded plant states and control signals on every finite interval.

Finally,

$$\begin{aligned} EV &\geq \frac{1}{2} E(e_l^T H_l^{-1} e_l) = \frac{1}{2} \\ E[(y - y_0 1_N)^T H_l^T H_l^{-1} H_l (y - y_0 1_N)] \\ &\geq \frac{1}{2} \lambda_{\min}(H_l) E|y - y_0 1_N|^2, \end{aligned}$$

together with (40), we have

$$\frac{1}{2} \lambda_{\min}(H_l) E|y - y_0 1_N|^2 \leq EV$$

$$\leq m_2 |\tilde{e}(t_0)|^2 e^{-c(t-t_0)} + \frac{\bar{d}}{c} NK_0 + \frac{\delta_0}{c},$$

that is, for $i = 1, \dots, N$,

$$\begin{aligned} E|y_i(t) - y_0(t)|^2 &\leq E|y - y_0 1_N|^2 \\ &\leq \frac{2m_2}{\lambda_{\min}(H_l)} |\tilde{e}(t_0)|^2 e^{-c(t-t_0)} \\ &\quad + \frac{2\bar{d}}{c\lambda_{\min}(H_l)} NK_0 + \frac{2\delta_0}{c\lambda_{\min}(H_l)}. \end{aligned} \quad (42)$$

Letting $t \rightarrow \infty$ in (42), we can obtain that

$$\begin{aligned} \lim_{t \rightarrow \infty} E|y_i(t) - y_0(t)|^2 \\ \leq \frac{2\bar{d}}{c\lambda_{\min}(H_l)} NK_0 + \frac{2\delta_0}{c\lambda_{\min}(H_l)}. \end{aligned} \quad (43)$$

Equation (43) makes the performance tradeoff explicit. Increasing the dissipative gains and reducing the Young-inequality weights can decrease the certified residual, but these choices also increase control effort and are coupled through the recursively defined coefficients. Thus the theorem asserts practical regulation, not exact tracking or an unconstrained arbitrarily small error under persistent noise.

4. Containment Extension with Multiple Leaders

This section reuses the weighted-degree recursion for system (4) with Q leaders, where $y_j^0(t)$ ($j = 1, \dots, Q$) represents the j -th leader's output signal and satisfies $\max_{1 \leq j \leq Q} \{|y_j^0(t)|, |\dot{y}_j^0(t)|\} \leq M$.

For this purpose, we introduce an augmented graph $\mathcal{G}_2 = (\mathcal{V}_2, \mathcal{E}_2, A_2)$ to describe the communication topology with N followers and Q leaders, where the set of edges $\mathcal{E}_2 \subseteq \mathcal{V}_2 \times \mathcal{V}_2$, the set of nodes $\mathcal{V}_2 = \{I^0, \dots, Q^0; 1, 2, \dots, N\}$ and the

adjacency matrix $A_2 = A$. The leader-weighted matrix associated with $\mathcal{G}_2$ is $B_2 = \text{diag}(\sum_{j=1}^Q b_{1j}, \dots, \sum_{j=1}^Q b_{Nj})$ with $b_{ij} > 0$ if follower i receives leader j ; otherwise $b_{ij} = 0$. Every connected component of the follower graph is assumed to receive information from at least one leader. This is the multi-leader counterpart of the pinned-component assumption in Section 3.

The objective is to design a distributed containment controller such that the i th follower output $y_i(t)$ practically tracks $y_{qi}(t) = \sum_{j=1}^Q k_{ij} y_j^0(t)$ with $\sum_{j=1}^Q k_{ij} = 1$ and $k_{ij} \geq 0$ and all closed-loop signals remain ultimately bounded in mean square.

Remark 4.1. The signal $y_{qi}(t) = \sum_{j=1}^Q k_{ij} y_j^0(t)$ is a convex combination because $k_{ij} \geq 0$ and $\sum_j k_{ij} = 1$. Its weights are fixed by the communication graph rather than selected by the controller. If all rows of $\mathcal{K}$ coincide, all followers have the same containment target. When $Q = 1$, $y_{qi} = y_1^0 \triangleq y_0$, and the problem reduces to single-leader tracking.

We now reuse the recursion of Section 3.

Step 1: Let

$$z_{i1} = \sum_{s=1}^N a_{is} (y_i - y_s) + \sum_{j=1}^Q b_{ij} (y_i - y_j^0),$$

$z_{i2} = x_{i2} - \tilde{\alpha}_{i1}$, $i = 1, \dots, N$, where $\tilde{\alpha}_{i1}$ is a stabilizing function. Let $z_1 = (z_{11}, z_{21}, \dots, z_{N1})^T$. Then

$$z_1 = H_2 y - B_0 y^0. \quad (44)$$

where $H_2 = B_2 + L$ is symmetric positive definite by the same quadratic-form argument used for H_1 , $y = (y_1, \dots, y_N)^T \in \mathbb{R}^N$, $B_0 = (b_{ij})_{N \times Q}$ and $y^0 = (y_1^0, \dots, y_Q^0)^T \in \mathbb{R}^Q$.

It follows from (44) that

$$H_2^{-1} z_1 = y - H_2^{-1} B_0 y^0 \triangleq y - \mathcal{K} y^0, \quad (45)$$

in which $\mathcal{K} = H_2^{-1} B_0 = (k_{ij})_{N \times Q}$. The matrix H_2 is a nonsingular M-matrix, so H_2^{-1} is nonnegative. Together with B_0 being a nonnegative matrix, we know that $\mathcal{K} = H_2^{-1} B_0 = (k_{ij})_{N \times Q}$ is a nonnegative matrix, i.e., $k_{ij} \geq 0$. On the other hand,

$$\begin{aligned} H_2 1_N &= (B_2 + L) 1_N = B_2 1_N \\ &= \left(\sum_{j=1}^Q b_{1j}, \dots, \sum_{j=1}^Q b_{Nj} \right)^T = B_0 1_Q, \end{aligned} \quad (46)$$

where $L 1_N = 0 1_N$ is used and $1_Q = (1, 1, \dots, 1) \in \mathbb{R}^Q$.

$\mathbb{R}^Q$. Thus, together with (45), (46) implies that $1_N = H_2^{-1} B_0 1_Q = \mathcal{K} 1_Q$, that is to say, $\sum_{j=1}^Q k_{ij} = 1$ holds for any $i (i = 1, \dots, N)$.

Choosing a candidate Lyapunov function

$$\tilde{V}_1 = \frac{1}{2} z_1^T H_2^{-1} z_1 + \frac{1}{2\lambda} \tilde{\theta}^2, \quad (47)$$

where $\tilde{\theta} = \theta - \hat{\theta}$ with $\hat{\theta}$ being the estimation of unknown parameter θ and λ is positive. By Young's inequality, together with (45) and $\sum_{j=1}^Q k_{ij} = 1$, we have

$$\begin{aligned} \dot{\tilde{V}}_1 &= z_1^T (\dot{y}(t) - \mathcal{K} \dot{y}^0(t)) - \frac{1}{\lambda} \tilde{\theta} \dot{\hat{\theta}} \leq \sum_{i=1}^N z_{i1} \\ &\quad \left(x_{i2} + \theta f_{i1} + h_{i1} \xi_{i1} - \sum_{j=1}^Q k_{ij} \dot{y}_j^0 \right) - \frac{1}{\lambda} \tilde{\theta} \dot{\hat{\theta}} \end{aligned} \quad (48)$$

$$\begin{aligned} &\leq \sum_{i=1}^N z_{i1} (\tilde{\alpha}_{i1} + z_{i2} + \hat{\theta} f_{i1} + \frac{z_{i1} h_{i1}^2}{4\tilde{d}_{i1}} + \frac{z_{i1}}{2\tilde{d}_M}) \\ &\quad + \sum_{i=1}^N \tilde{d}_{i1} |\xi_{i1}|^2 - \tilde{\theta} \left(\frac{1}{\lambda} \dot{\hat{\theta}} - \tilde{\tau}_1 \right) + \frac{\tilde{c}_\theta}{\lambda} \tilde{\theta} \hat{\theta} + \frac{\tilde{d}_M}{2} M^2, \end{aligned}$$

where $\tilde{\tau}_1 = \sum_{i=1}^N z_{i1} f_{i1} - \tilde{c}_\theta \hat{\theta} / \lambda$ and positive constants $\tilde{d}_{i1}$, $\tilde{c}_\theta$ and $\tilde{d}_M$ are to be designed. Selecting the stabilizing function

$$\tilde{\alpha}_{i1} = -\tilde{c}_{i1} z_{i1} - \frac{1}{\tilde{d}_M} z_{i1} - \hat{\theta} f_{i1}$$

$$- \frac{1}{4\tilde{d}_{i1}} z_{i1} h_{i1}^2,$$

where $\tilde{c}_{i1} > 0$ is a design parameter. Then (48) becomes

$$\begin{aligned} \dot{\tilde{V}}_1 &\leq - \sum_{i=1}^N \left(\tilde{c}_{i1} + \frac{1}{2\tilde{d}_M} \right) z_{i1}^2 \\ &\quad + \sum_{i=1}^N z_{i1} z_{i2} + \sum_{i=1}^N \tilde{d}_{i1} |\xi_{i1}|^2 \\ &\quad - \tilde{\theta} \left(\frac{1}{\lambda} \dot{\hat{\theta}} - \tilde{\tau}_1 \right) + \frac{\tilde{c}_\theta}{\lambda} \tilde{\theta} \hat{\theta} + \frac{\tilde{d}_M}{2} M^2. \end{aligned} \quad (49)$$

Step k ($k = 2, \dots, n$): Let $z_{ij} = x_{ij} - \tilde{\alpha}_{i(j-1)}$, $i = 1, \dots, N$; $j = k, \dots, n+1$, where $z_{i(n+1)} = 0$, $x_{i(n+1)} = u_i$ and $\tilde{\alpha}_{i1}, \dots, \tilde{\alpha}_{in}$ are stabilizing functions. Using the construction in Section 3, define

$$\tilde{\alpha}_{ij} = -z_{ij} (\tilde{c}_{ij} \gamma + \tilde{\eta}_{ij})^{1/p_{ij}}$$

$$\triangleq -z_{ij} \tilde{\beta}_{ij} \quad (2 \leq j \leq k)$$

and tuning functions $\tilde{\tau}_j = \tilde{\tau}_{j-1} +$

$\sum_{i=1}^N z_{ij}^{\bar{p}-p_{ij}+1} \tilde{w}_{ij} (2 \leq j \leq k)$ such that

$$\dot{\tilde{V}}_k \leq - \sum_{i=1}^N \tilde{c}_{i1} z_{i1}^2 - \sum_{i=1}^N \sum_{j=2}^k \tilde{c}_{ij} \gamma z_{ij}^{\bar{p}+1} \quad (50)$$

$$\begin{aligned}
& + \sum_{i=1}^N \tilde{\rho}_{i(k+1)} z_{i(k+1)}^{\bar{p}+1} + \sum_{i=1}^N \sum_{j=1}^k \tilde{d}_{ij} |\xi_{ij}|^2 \\
& + \sum_{i=1}^N \sum_{j=1}^{k-1} \sum_{l=j+1}^k \tilde{d}_{ijl} |\xi_{ij}|^2 + \sum_{i=1}^N \sum_{j=1}^{k-1} \frac{\partial \tilde{\alpha}_{ij}}{\partial \hat{\theta}} \\
& z_{i(j+1)}^{\bar{p}-p_{i(j+1)}+1} (\lambda \tilde{\tau}_k - \hat{\theta}) \\
& - \tilde{\theta} \left(\frac{1}{\lambda} \hat{\theta} - \tilde{\tau}_k \right) + \frac{\tilde{c}_\theta}{\lambda} \tilde{\theta} \hat{\theta} + \sum_{i=1}^N \tilde{\delta}_{ik}
\end{aligned}$$

where

$$\begin{aligned}
\tilde{V}_k &= \tilde{V}_{k-1} + \sum_{i=1}^N \frac{z_{ik}^{\bar{p}-p_{ik}+2}}{\bar{p} - p_{ik} + 2}, \\
\bar{p} &= \max_{1 \leq i \leq N} \{p_{i1}, \dots, p_{in}\}, \\
\tilde{\eta}_{ik} &= \tilde{\rho}_{ik} + 2m_{ik} \\
&+ r_{ik} [(\hat{\theta} \tilde{w}_{ik})^{\bar{p}_{ik}} \\
&+ (\sum_{s=1}^N \sum_{j=1}^{k-1} x_{s(j+1)}^{p_{sj}} \frac{\partial \tilde{\alpha}_{i(k-1)}}{\partial x_{sj}}) \bar{p}_{ik} \\
&+ (\sum_{j=1}^Q M \frac{\partial \tilde{\alpha}_{i(k-1)}}{\partial y_j^0}) \bar{p}_{ik} + (\lambda \tilde{\tau}_k \frac{\partial \tilde{\alpha}_{i(k-1)}}{\partial \hat{\theta}}) \bar{p}_{ik} \\
&+ (z_{ik}^{\bar{p}-p_{ik}+1} \sum_{s=1}^N \sum_{j=1}^{k-1} \frac{1}{4\tilde{d}_{sjk}} \\
&(h_{sj} \frac{\partial \tilde{\alpha}_{i(k-1)}}{\partial x_{sj}})^2 \bar{p}_{ik} + (\sum_{i=1}^N \sum_{j=1}^{k-2} e^{\bar{p}-p_{i(j+1)}+1} \lambda \tilde{w}_{ik}) \bar{p}_{ik} \\
&+ (\frac{z_{ik}^{\bar{p}-p_{ik}+1} h_{ik}^2}{4\tilde{d}_{ik}}) \bar{p}_{ik}] \\
\tilde{\delta}_{ik} &= \tilde{\delta}_{i(k-1)} + 7\tilde{\delta}_{ik}, \\
\tilde{\rho}_{i(k+1)} &= m_{ik} [1 + (\tilde{\beta}_{ik}^{p_{ik}-1})^{\bar{p}+1}].
\end{aligned}$$

Here $\tilde{c}_{ij}$, $\tilde{d}_{ij}$, and $\tilde{d}_{ijl}$ are positive design constants, $i = 1, \dots, N$.

Let $k = n$, then we have the distributed tracking controller

$$\begin{aligned}
u_i &= \tilde{\alpha}_{in} = -z_{in}(\tilde{c}_{in}\gamma + \tilde{\eta}_{in})^{\frac{1}{\bar{p}_{in}}} \triangleq \\
&- z_{in} \tilde{\beta}_{in},
\end{aligned} \quad (51)$$

and the adaptive law

$$\hat{\theta} = \lambda \tilde{\tau}_n, \quad \tilde{\tau}_n = \tilde{\tau}_{n-1} + \sum_{i=1}^N z_{in}^{\bar{p}-p_{in}+1} \tilde{w}_{in}, \quad (52)$$

Meanwhile, the derivative of

$$\tilde{V}_n = \frac{1}{2} z_1^T H_2^{-1} z_1 + \sum_{i=1}^N \sum_{j=2}^n \frac{z_{ij}^{\bar{p}-p_{ij}+2}}{\bar{p} - p_{ij} + 2} + \frac{\tilde{\theta}^2}{2\lambda}$$

in (48) is changed into

$$\begin{aligned}
\dot{\tilde{V}}_n &\leq - \sum_{i=1}^N \tilde{c}_{il} z_{il}^2 - \sum_{i=1}^N \sum_{j=2}^n \tilde{c}_{ij} \gamma z_{ij}^{\bar{p}+1} + \frac{\tilde{c}_\theta}{\lambda} \tilde{\theta} \hat{\theta} \\
&+ \sum_{i=1}^N \tilde{\delta}_{in} + \sum_{i=1}^N \tilde{d}_i \sum_{j=1}^n |\xi_{ij}|^2
\end{aligned} \quad (53)$$

$$\leq - \tilde{c}_l \sum_{i=1}^N z_{il}^2 - \sum_{i=1}^N \sum_{j=2}^n \tilde{c}_{ij} \gamma z_{ij}^{\bar{p}+1}$$

$$- \frac{\tilde{c}_\theta}{2\lambda} \tilde{\theta}^2 + \sum_{i=1}^N \tilde{\delta}_{in} + \frac{\tilde{c}_\theta}{2\lambda} \tilde{\theta}^2 + \tilde{d} \sum_{i=1}^N |\xi_i|^2,$$

where $\tilde{c}_l = \min_{1 \leq i \leq N} \{\tilde{c}_{il}\}$, $\tilde{d} = \max_{1 \leq i \leq N} \{\tilde{d}_i\}$ and

$$\tilde{d}_i = \max\{\tilde{d}_{in}, \max_{1 \leq j \leq n-1} \{\tilde{d}_{ij} + \sum_{l=j+1}^n \tilde{d}_{ijl}\}\}.$$

The same estimates as in Section 3 give

$$\begin{aligned}
\dot{\tilde{V}}_n &\leq - \frac{2\tilde{c}_l}{\lambda_{\max}(H_2^{-1})} \frac{1}{2} z_1^T H_2^{-1} z_1 \\
&- \sum_{i=1}^N \sum_{j=2}^n \tilde{c}_{ij} z_{ij}^{\bar{p}-p_{ij}+2} - \frac{\tilde{c}_\theta}{2\lambda} \tilde{\theta}^2 \\
&+ \tilde{d} \sum_{i=1}^N |\xi_i|^2 + \tilde{\delta}_0 \leq - \tilde{c} \tilde{V}_n + \tilde{d} \sum_{i=1}^N |\xi_i|^2 + \tilde{\delta}
\end{aligned} \quad (54)$$

where $\tilde{\delta}_0 = \sum_{i=1}^N \sum_{j=2}^n \tilde{c}_{ij}/\gamma_0 + \sum_{i=1}^N \tilde{\delta}_{in} + \tilde{c}_\theta \tilde{\theta}^2 / 2\lambda > 0$ and

$$\tilde{c} = \min \left\{ \min_{1 \leq i \leq N} \min_{2 \leq j \leq n} \tilde{c}_{ij} (\bar{p} - p_{ij} + 2), \frac{2\tilde{c}_l}{\lambda_{\max}(H_2^{-1})}, \tilde{c}_\theta \right\}.$$

In addition, we also have

$$\tilde{m}_1 |\tilde{z}|^2 \leq \tilde{V}_n \leq \tilde{m}_2 |\tilde{z}|^2, \quad (55)$$

where $\tilde{z} = (\tilde{z}_1, \dots, \tilde{z}_N, \tilde{\theta})^T$ with $\tilde{z}_i = (z_{i1}, z_{i2}^{(\bar{p}-p_{i2}+2)/2}, \dots, z_{in}^{(\bar{p}-p_{in}+2)/2})$ and $i = 1, 2, \dots, N$, $\tilde{m}_1 = \min\{1/(\bar{p}+2), 1/2\lambda, \lambda_{\min}(H_2^{-1})/2\}$, $\tilde{m}_2 = \max\{1/2, 1/2\lambda, \lambda_{\max}(H_2^{-1})/2\}$.

For $\tilde{V} = \tilde{V}_n$, (55) gives the two coercive bounds required by Lemma 2.1, while (54) is its derivative condition with $m = 2$, $d = \tilde{d}$, and $\delta = \tilde{\delta}_0$. The local-Lipschitz assumption and $E|\Xi|^2 \leq NK_0$ verify the remaining conditions. Hence the closed loop has a unique global solution and is NSpS-2-M. The comparison estimate is

$$\begin{aligned}
\tilde{m}_1 E|\tilde{z}(t)|^2 &\leq E\tilde{V} \\
&\leq \tilde{m}_2 |\tilde{z}(t_0)|^2 e^{-\tilde{c}(t-t_0)} + \frac{\tilde{d}}{\tilde{c}} NK_0 + \frac{\tilde{\delta}_0}{\tilde{c}},
\end{aligned} \quad (56)$$

which means that

$$\lim_{t \rightarrow \infty} E|\tilde{z}(t)|^2 \leq \frac{\tilde{d}}{\tilde{c}\tilde{m}_1} NK_0 + \frac{\tilde{\delta}_0}{\tilde{c}\tilde{m}_1}.$$

Thus the transformed errors and parameter

estimate are ultimately bounded in mean square; the triangular coordinate maps give the corresponding boundedness of the plant variables. On the other hand, we also have

$$\begin{aligned} E\tilde{V} &\geq \frac{1}{2}E(z_1^T H_2^{-1} z_1) = \frac{1}{2} \\ E[(H_2 y - B_0 y^0)^T H_2^{-1} (H_2 y - B_0 y^0)] \\ &\geq \frac{1}{2} \lambda_{\min}(H_2) E|y - \mathcal{K}y^0|^2, \end{aligned}$$

together with (45) and (56), we can further derive that

$$\begin{aligned} \frac{1}{2} \lambda_{\min}(H_2) E|y - \mathcal{K}y^0|^2 &\leq E\tilde{V} \\ &\leq \tilde{m}_2 |\tilde{z}(t_0)|^2 e^{-\tilde{c}(t-t_0)} + \frac{\tilde{d}}{\tilde{c}} NK_0 + \frac{\tilde{\delta}_0}{\tilde{c}}, \end{aligned}$$

that is, for $i = 1, \dots, N$,

$$\begin{aligned} E|y_i(t) - \sum_{j=1}^q k_{ij} y_j^0(t)|^2 &\leq E|y - \mathcal{K}y^0|^2 \\ &\leq \frac{2\tilde{m}_2}{\lambda_{\min}(H_2)} |\tilde{z}(t_0)|^2 e^{-\tilde{c}(t-t_0)} + \frac{2\tilde{d}}{\tilde{c} \lambda_{\min}(H_2)} NK_0 + \frac{2\tilde{\delta}_0}{\tilde{c} \lambda_{\min}(H_2)} \end{aligned} \quad (57)$$

Letting $t \rightarrow \infty$ in (57), we have

$$\lim_{t \rightarrow \infty} E|y_i(t) - \sum_{j=1}^q k_{ij} y_j^0(t)|^2 \leq \frac{2\tilde{d}}{\tilde{c} \lambda_{\min}(H_2)} NK_0 + \frac{2\tilde{\delta}_0}{\tilde{c} \lambda_{\min}(H_2)}. \quad (58)$$

The bound (58) has the same gain-versus-control-effort interpretation as (43). It guarantees practical containment rather than exact convergence under persistent random excitation. Based on the above analysis, we conclude the following stability theorem.

Theorem 4.1. Under the stated assumptions, controller (51) and adaptive law (52) give a unique global solution that is NSpS-2-M. All closed-loop signals are ultimately bounded in mean square, and the i th follower satisfies

$$\lim_{t \rightarrow \infty} E|y_i(t) - \sum_{j=1}^q k_{ij} y_j^0(t)|^2 \leq \frac{2\tilde{d}NK_0 + 2\tilde{\delta}_0}{\tilde{c} \lambda_{\min}(H_2)}.$$

5. Numerical Studies and Engineering Interpretation

The examples are designed to test the heterogeneous-power recursion and the single-versus multiple-leader graph constructions. They are numerical benchmarks, not a substitute for experiments on a physical plant.

Example 1 (single-leader tracking): Consider three followers and one leader. The unit-weight communication graph $\mathcal{G}_I$ is shown in Figure 1, and the follower dynamics are

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) + \theta f_{i1}(\bar{x}_{i1}(t), t) \\ \quad + h_{i1}(\bar{x}_{i1}(t), t) \xi_{i1}(t), \\ \dot{x}_{i2}(t) = u_i^3(t) + \theta f_{i2}(\bar{x}_{i2}(t), t) \\ \quad + h_{i2}(\bar{x}_{i2}(t), t) \xi_{i2}(t), \\ y_i(t) = x_{i1}(t), \quad i = 1, 2, 3, \end{cases} \quad (59)$$

where $f_{ij}(\bar{x}_{ij}(t), t) = i \sin x_{ij}$ and $h_{ij}(\bar{x}_{ij}(t), t) = x_{ij}$ for $i = 1, 3$ and $j = 1, 2$, and $f_{ij}(\bar{x}_{ij}(t), t) = \cos x_{ij}$ and $h_{ij}(\bar{x}_{ij}(t), t) = j x_{ij}$ with $i = 2$ and $j = 1, 2$. The stochastic processes $\xi_{ij}(t) (i = 1, 2, 3; j = 1, 2)$ are generated by

$$\begin{aligned} \ell_{ij} \dot{\xi}_{ij}(t) &= w_{ij}(t) - \xi_{ij}(t), \quad \xi_{ij}(0) = 0, \\ i &= 1, 2, 3, j = 1, 2, \end{aligned}$$

where $\ell_{i1} = 0.4$ and $\ell_{i2} = 2$. Here $w_{ij}(t)$ is zero-mean white noise with power spectral density 0.2. The stationary second moments are therefore $E\xi_{i1}^2(t) = 0.25$ and $E\xi_{i2}^2(t) = 0.05$, $i = 1, 2, 3, j = 1, 2$.

The leader output is $y_0 = \sin(2t) + 0.5\cos(t + 0.5\pi)$. The initial values are

$$\begin{aligned} x_{11}(0) &= -0.2, \quad x_{12}(0) = 0.3, \quad x_{21}(0) = 0.1, \\ x_{22}(0) &= -0.4, \quad x_{31}(0) = 0, \quad x_{32}(0) = -0.2, \\ \text{and } \theta &= 0.8. \text{ We define } e_{i1} = y_i - y_0 \text{ and select} \\ d_M &= 1, \quad M = 4, \quad \lambda = 2, \quad c_\theta = 1, \end{aligned}$$

$$\begin{aligned} c_{i1} &= c_{i2} = 20, \quad d_{11} = d_{12} = d_{i12} = 0.5, \\ d_{21} &= d_{22} = d_{31} = d_{32} = 0.1, \quad r_{i2} = 1, \\ i &= 1, 2, 3. \end{aligned}$$

Figure 2 reports the control inputs, including the initial transients generated by the high-power feedback. Figure 3 shows that the outputs enter a small neighborhood of the leader trajectory and that the adaptive estimate remains bounded, consistently with Theorem 3.1.



Figure 1. Communication Topology $\mathcal{G}_I$.

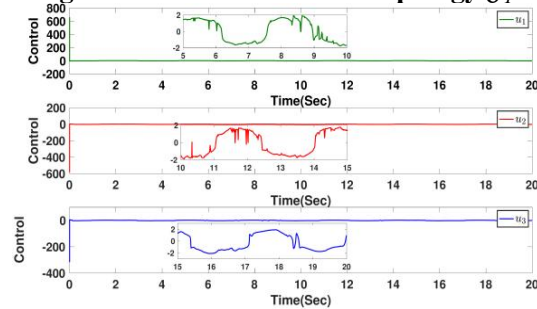


Figure 2. Control Inputs in Example 1.

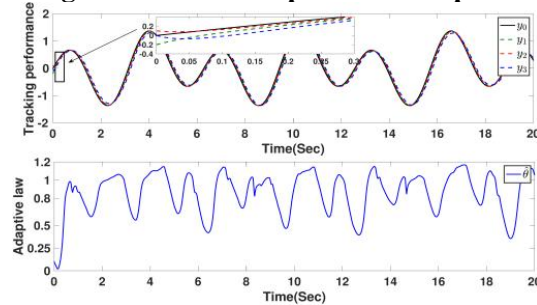


Figure 3. Tracking Outputs and Adaptive Estimate in Example 1.

Example 2 (multi-leader containment): Consider three followers and two leaders. The unit-weight graph $\mathcal{G}_2$ is shown in Figure 4, and the follower dynamics are

$$\begin{cases} \dot{x}_{i1}(t) = x_{i1}^{p_{i1}}(t) \\ + \theta f_{i1}(\bar{x}_{i1}(t), t) \\ + h_{i1}(\bar{x}_{i1}(t), t) \xi_{i1}(t), \\ \dot{x}_{i2}(t) = x_{i2}^{p_{i2}}(t) \\ + \theta f_{i2}(\bar{x}_{i2}(t), t) \\ + h_{i2}(\bar{x}_{i2}(t), t) \xi_{i2}(t), \\ \dot{x}_{i3}(t) = x_{i3}^{p_{i3}}(t) \\ + \theta f_{i3}(\bar{x}_{i3}(t), t) \\ + h_{i3}(\bar{x}_{i3}(t), t) \xi_{i3}(t), \\ y_i(t) = x_{i1}(t), \quad i = 1, 2, 3, \end{cases} \quad (60)$$

where the powers and nonlinearities are $p_{i1} = p_{22} = 1$,

$$p_{12} = p_{32} = p_{23} = p_{33} = 3, \quad p_{13} = 5,$$

$$f_{i1} = h_{i1} = h_{i2} = 0, \quad f_{12} = x_{12},$$

$$f_{22} = x_{21}, \quad f_{32} = x_{32},$$

$$f_{13} = x_{11} + x_{12}, \quad f_{23} = x_{22},$$

$$f_{33} = x_{32} + x_{33},$$

$$h_{13} = x_{13}, \quad h_{23} = x_{23}, \quad h_{33} = x_{31}.$$

The stochastic processes $\xi_{ij}(t)$ ($i = 1, 2, 3$; $j = 1, 2, 3$) are generated by

$$\tilde{\ell}_{ij} \dot{\xi}_{ij}(t) = w_{ij}(t) - \xi_{ij}(t), \quad \xi_{ij}(0) = 0,$$

$$i = 1, 2, 3, j = 1, 2, 3,$$

where $\tilde{\ell}_{i1} = 0.5$, $\tilde{\ell}_{i2} = 1$, and $\tilde{\ell}_{i3} = 0.8$. The zero-mean white-noise input has power spectral density 0.1, giving stationary second moments $E\xi_{i1}^2(t) = 0.1$, $E\xi_{i2}^2(t) = 0.05$, and $E\xi_{i3}^2(t) = 0.0625$, $i = 1, 2, 3, j = 1, 2, 3$.

The leader outputs are $y_1^0 = 1.5 \sin t$ and $y_2^0 = 2 \sin 2t$, the initial values

$$x_{11}(0) = -0.1, \quad x_{12}(0) = 0.1,$$

$$x_{13}(0) = -0.2,$$

$$x_{21}(0) = 0.1, \quad x_{22}(0) = 0.7,$$

$$x_{23}(0) = -0.4,$$

$$x_{31}(0) = -0.1, \quad x_{32}(0) = -0.4,$$

$$x_{33}(0) = 0.5,$$

and $\theta = 0.1$. We define $z_{i1} = y_i - \sum_{j=1}^2 k_{ij} y_j^0$ representing the tracking error of i -th follower where $k_{11} = k_{21} = k_{32} = 1$ and $k_{12} = k_{22} = k_{31} = 0$ are determined by Figure 4, and select the design parameters

$$\tilde{a}_M = 1, \quad M = 2, \quad \lambda = 10, \quad \tilde{c}_\theta = 10,$$

$$\tilde{c}_{11} = 24, \quad \tilde{c}_{21} = 20, \quad \tilde{c}_{31} = 40,$$

$$\tilde{c}_{12} = 12, \quad \tilde{c}_{13} = 15, \quad \tilde{c}_{22} = \tilde{c}_{23} = 10,$$

$$\tilde{c}_{32} = \tilde{c}_{33} = 20, \quad \tilde{d}_{13} = \tilde{d}_{23} = 0.1,$$

$$\tilde{d}_{33} = 0.01,$$

$$r_{ij} = 1.5, \quad i = 1, 2, 3, \quad j = 2, 3.$$

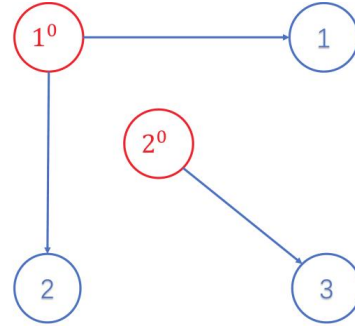


Figure 4. Communication Topology $\mathcal{G}_2$.

Figure 5 shows the control inputs, and Figure 6 shows practical containment and bounded internal signals, in agreement with Theorem 4.1. The large input and high-power-state excursions also illustrate why actuator constraints must be considered before a physical implementation.

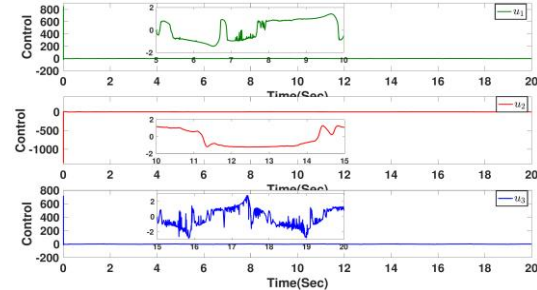


Figure 5. Control Inputs in Example 2.

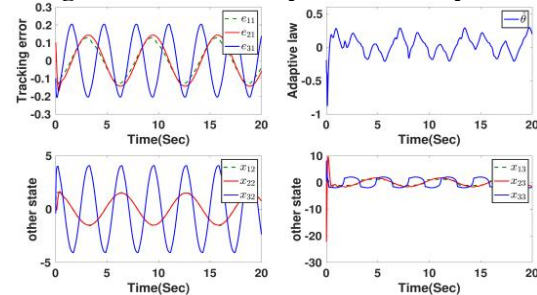


Figure 6. Containment Errors, Adaptive Estimate, and Internal States in Example 2.

6. Conclusions

This paper developed a distributed adaptive backstepping construction for random high-order nonlinear agents with heterogeneous known odd-rational powers. The network-wide degree $\bar{p}$ aligns the recursive Lyapunov estimates, while the tuning function cancels the accumulated parameter-estimation term and leakage limits parameter drift. For pinned fixed undirected graphs, the closed loop is NSpS-2-M and the single-leader and containment errors satisfy explicit ultimate bounds.

The result complements, but does not subsume, the surface-vessel study [11]: the present strength is the heterogeneous high-order power structure, whereas [11] provides a physically derived vessel

model and application-oriented validation. The present numerical examples verify the internal consistency of the proposed recursion only. Future work should address output feedback, switching or directed graphs, unknown powers, input saturation, and validation on a specific physical platform.

References

- [1] H. W. Zhang, F. Lewis. Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics. *Automatica*, 2012, 48(7): 1432-1439.
- [2] H. G. Zhang, J. Duan, Y. C. Wang, Z. Y. Gao. Bipartite fixed-time output consensus of heterogeneous linear multiagent systems. *IEEE Transactions on Cybernetics*, 2021, 51(2): 548-557.
- [3] H. J. Liang, Y. H. Zhang, T. W. Huang, H. Ma. Prescribed performance cooperative control for multiagent systems with input quantization. *IEEE Transactions on Cybernetics*, 2020, 50(5): 1810-1819.
- [4] T. F. Liu, Z. Y. Qin, Y. G. Hong, Z. P. Jiang. Distributed optimization of nonlinear multi-agent systems: a small-gain approach. *IEEE Transactions on Automatic Control*, 2022, 67(2): 676-691.
- [5] H. J. Li, W. Q. Li, J. Z. Gu. Distributed output tracking control of nonlinear multi-agent systems with unknown time-varying powers. *International Journal of Control*, 2022, 95(11): 2960-2971.
- [6] W. Q. Li, L. Liu, G. Feng. Containment control with multiple leaders for nonlinear multi-agent systems with unstabilizable linearizations. *Neurocomputing*, 2020, 380: 43-50.
- [7] M. F. Hu, L. X. Guo, A. H. Hu, Y. Q. Yang. Leader-following consensus of linear multi-agent systems with randomly occurring nonlinearities and uncertainties and stochastic disturbances. *Neurocomputing*, 2015, 149: 884-890.
- [8] Y. Y. Zhang, R. F. Li, W. Zhao, X. M. Huo. Stochastic leader-following consensus of multi-agent systems with measurement noises and communication time delays. *Neurocomputing*, 2018, 282: 136-145.
- [9] W. Q. Li, L. Liu, G. Feng. Cooperative control of multiple nonlinear benchmark systems perturbed by second-order moment processes. *IEEE Transactions on Cybernetics*, 2020, 50(3): 902-910.
- [10] H. Wang, W. Q. Li, M. Q. Tang. Distributed output tracking of nonlinear multi-agent systems perturbed by second-order moment processes. *Neurocomputing*, 2021, 452: 789-795.
- [11] L. Q. Yao, Q. X. Xu, L. K. Feng, Z. J. Wu. Adaptive cooperative tracking control for multiple surface vessel systems with random disturbance. *Ocean Engineering*, 2023, 286: 115528.
- [12] Y. Fan, Y. Zhang, Z. Li. Distributed adaptive tracking consensus control for a class of heterogeneous nonlinear multi-agent systems. *Mathematics and Computers in Simulation*, 2025, 227: 420-441.
- [13] W. Wan, Z.-Y. Li, K. Zhang. Cooperative output feedback tracking control of heterogeneous multi-agent systems with semi-Markovian switching topologies and multiple measurement noises. *Nonlinear Analysis: Hybrid Systems*, 2025, 57: 101605.
- [14] L. Yan, Z. Liu, C. L. P. Chen, Y. Zhang, Z. Wu. Adaptive fuzzy fixed-time bipartite consensus control for stochastic nonlinear multi-agent systems with performance constraints. *Fuzzy Sets and Systems*, 2025, 514: 109401.
- [15] J. OuYang, H. Yu. Fixed-time containment control for stochastic multi-agent systems with state constraints. *European Journal of Control*, 2025, 84: 101241.
- [16] Z. J. Wu. Stability criteria of random nonlinear systems and their applications. *IEEE Transactions on Automatic Control*, 2015, 60(4): 1038-1049.
- [17] H. J. Li, W. Q. Li, and Y. Liu. Adaptive tracking control of stochastic nonlinear systems with unknown powers. 2020 Chinese Control And Decision Conference, 2020, 4526-4531.
- [18] T. M. Flett. *Differential analysis*. Cambridge, UK: Cambridge University Press, 1980.