Modeling and Application of MATLAB in Mining Subsidence Prediction

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Abstract: **Accurate prediction of mining subsidence helps lessen ground-settlement-related harm inflicted upon local economic interests and residents' living conditions. Because the traditional prediction work involves cumbersome formulas and a heavy computing workload, a surface subsidence prediction program constructed using the probability-integral algorithm is realized under the MATLAB platform in this paper. In the program, computational models for ground subsidence, tilt, curvature parameters, horizontal displacement and horizontal strain corresponding to arbitrary ground positions are analyzed and coded as separate functional modules, which are then integrated into one prediction procedure. The program was tested on the 7513 working face of a coal mine. The predicted maximum subsidence is 4809 mm, and the predicted basin follows the general law of mining-induced surface movement. Three-dimensional effect diagrams, main section profiles, and contour maps can be generated, and the results can be exported in TXT format for post-processing with Surfer or CAD. This work makes subsidence prediction easier to carry out, shortens the data processing time, and provides a reliable reference for protecting surface buildings and designing mining schemes.**

Keywords: Mining Subsidence; Probability Integral Method; MATLAB

1. Introduction

Coal mining provides a strong impetus for economic development; however, the resulting ground settlement can cause serious damage to farmland and buildings. During the subsidence process of the goaf surface, the original topography and landforms are altered, bringing varying degrees of impact and damage to farmland, buildings, water bodies, and transportation facilities within the influence zone of the subsidence basin, which endangers the human living environment and restricts sustainable economic and social development. Research on mining subsidence in China has been carried out for more than 70 years, and a relatively complete theoretical and technical system for subsidence prediction and prevention has gradually been established [1]. Reliable prediction of this movement and deformation before mining starts is therefore a prerequisite for reasonable mining design, protection of surface structures, and relocation planning of residents.

Of all available prediction techniques, the probability- integral algorithm stands out as the most prevalently applied solution within Chinese research practices, on account of its clear physical meaning and stable parameter performance. With the recent progress of satellite remote sensing and computing technology, combining InSAR observations with this method has also improved the accuracy and spatial coverage of subsidence monitoring and prediction [2]. Its drawback is that the formulas contain many transcendental functions and parameters, so manual calculation is tedious and error-prone. MATLAB provides strong matrix computation and visualization capabilities, which makes it a convenient platform for this kind of model. For this reason, the probability integral method is implemented in MATLAB to perform ground subsidence forecasts in the present investigation. The program simplifies the calculation, delivers ground displacement-deformation values corresponding to arbitrary surface locations, and provides a reliable basis for protecting buildings and transportation facilities and for planning resident relocation.

2. Current Research Status

Chinese scholars have carried out extensive research on mining subsidence prediction methods and their software implementation. In terms of prediction software development, several practical systems have been established [3,4]: Previous research [3] conducted by Shen Zhen, Xu Liangji, Liu Zhe, and Qin Changcai constructed a MATLAB platform for computing prediction parameters and derived precise parameters from surface observation station datasets. Wang Xifu and Ren Zhanying developed an integrated information processing system and related expert systems for mining subsidence prediction in dedicated railway mining areas [4]. Li et al. developed a mining subsidence prediction analysis system that integrates data processing, prediction modeling, and result visualization [5]. When carrying out parameter inversion, Niu et al. employed the GA- BP neural network to perform optimization on the probability- integral algorithm's parameters, leading to higher inversion accuracy [6]; Lou introduced robust ridge estimation into parameter calculation, a method that can suppress outliers and avoid the ill-conditioned normal matrices of least squares fitting [7]. As for model improvement, modified probability integral models have been proposed for special geological and mining conditions [8,9]: Wei et al. put forward a piecewise revised model adapted to mining areas characterized by thick unconsolidated layers, solving the problem that the actual influence range is much larger than the theoretical prediction; For backfill mining of shallow coal seams with hard roof conditions, Huo et al. developed a dedicated forecasting method that delivers technical backing for building- protection practices. For addressing the time- dependent behaviour of ground movement, Li et al. established an analytical subsidence model derived from probability- integral theory, which allows for dynamic forecasts of ground subsidence [10]. In addition, some researchers have combined the probability integral model with other mathematical models and connected the prediction results directly to AutoCAD for engineering drawing output. Even with these achievements, a simple, efficient, and easy-to-use prediction program is still valuable for mining technicians, and this motivated the development of the MATLAB-based program presented here and its application in engineering practice.

3. Brief Analysis of Mining-caused Ground Subsidence Behaviours

Ground subsidence triggered by underground mining is subject to numerous factors, which fall mainly into two groups: topography and rock properties, and mining factors. Surface movement includes vertical settlement and horizontal movement, from which inclination, curvature, and horizontal deformation are derived; these indicators are the basis for evaluating the damage degree of surface buildings and structures. Based on the construction logic of prediction models, current forecasting techniques can be sorted among three separate groups: empirical methodologies, theoretical- model- driven techniques, and approaches built upon influence- function principles.

The influence function method lies between empirical methods and theoretical model methods in terms of effectiveness. Its basic idea is to first determine, through theoretical research or other means, how the extraction of a micro-unit affects the rock strata or the surface, and then to treat the influence of the entire mining as the superposition of the influences of all micro-unit mining in abandoned mining zones. This principle facilitates the computation of rock stratum and ground surface movement and deformation resulting from comprehensive mining exploitation. Several representative forecasting techniques fall into this class: the probability-integral algorithm as well as the Budryka-Knothe and Kochmanski models. Liu and Dai [11] carried out a systematic review on the evolution and theoretical basis of the probability-integral algorithm, a predominant technique commonly applied in Chinese mining-related research. Its prediction parameters are obtained from field observation datasets, maintain fairly stable values within a single mining district, and vary in accordance with definite patterns; with this theoretical basis, the probability- integral algorithm is convenient to use and gives high accuracy, and it has become the dominant prediction method in China.

One notable limitation is that the conventional probability- integral algorithm is capable of describing nothing but the terminal stable condition of ground movement. When describing dynamic subsidence behaviours in the course of working- face advancement, time- dependent functions including the Knothe and Weibull functions are generally incorporated into static models. Coupling these dynamic models with SBAS-InSAR observation datasets helps achieve higher precision for dynamic subsidence-basin forecasts [12].For the purpose of analysing post-mining ultimate ground displacement-deformation behaviours in this study, the static probability-integral algorithm is adopted.

4. Modeling Function Module Description

To run the probability-integral algorithm, several key forecasting parameters are needed: subsidence coefficient q, tangent value of the major influence angle tan β, horizontal displacement coefficient b, major influence radius r, and inflection-point offset s. Guided by these parameter indicators, the program is structured into five separate functional sub-modules responsible for subsidence, inclination, curvature, horizontal movement and horizontal-deformation computation. As shown in Figure 1, this computational framework addresses ground displacement-deformation responses at arbitrary locations; the mathematical expressions for every sub-module are detailed in subsequent content.

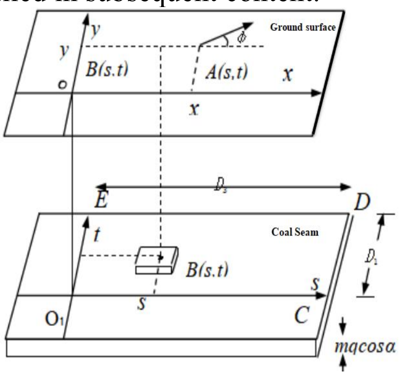

Figure 1. Calculation Model for Surface Potential Movement and Deformation

4.1 Subsidence Corresponding to Any Ground-Surface Point

The subsidence quantity corresponding to any given ground-surface location is obtained from:

$$W(x,y)=\int_{-m_0}^{m_0}\int_{-s_0}^{s_0}\int_{-t_0}^{t_0}\frac{1}{r^2}e^{-\frac{\pi}{r^2}[(x-s)^2+(y-q)^2]}dsdmdq$$
$$=\frac{1}{W_0}W^0(x)W^0(y)=W_0\cdot C(x)\cdot C(y) \tag{1}$$

where C(x) and C(y) stand for coefficient values of subsidence distribution along the principal cross-sections.

4.2 Inclination Corresponding to Arbitrary Ground-Surface Points

For a direction crossing the coordinate axis with an angle φ at any point, the surface inclination is:

$$\mathrm{i}(x,y)_\varphi=\frac{\partial W(x,y)}{\partial x}\cos\varphi+\frac{\partial W(x,y)}{\partial y}\sin\varphi$$
$$=C(y)\mathrm{i}(x)\cos\varphi+C(x)\mathrm{i}(y)\sin\varphi \tag{2}$$

The principal inclination direction and value are:

$$\mathrm{i}(x,y)_\varphi=\frac{\partial W(x,y)}{\partial x}\cos\varphi+\frac{\partial W(x,y)}{\partial y}\sin\varphi$$
$$=C(y)\mathrm{i}(x)\cos\varphi+C(x)\mathrm{i}(y)\sin\varphi \tag{3}$$

4.3 Curvature Corresponding to Arbitrary Ground-Surface Points

The curvature quantity corresponding to arbitrary directions crossing a ground-surface point is given by:

$$\mathrm{k}(x,y)_\varphi=C(y)\mathrm{k}(x)\cos^2\varphi+$$
$$C(x)\mathrm{k}(y)\sin^2\varphi+C_{[x,y]}\sin2\varphi \tag{4}$$

The principal curvature direction and value are:

$$\varphi_k=\frac{1}{2}\mathrm{arctg}\frac{2C_{[x,y]}}{C(y)\mathrm{k}(x)-C(x)\mathrm{k}(y)}$$
$$\mathrm{k}(x,y)_{\varphi_k}=C(y)\mathrm{k}(x)\cos^2\varphi_k+C(x)\mathrm{k}(y)\sin^2\varphi_k$$
$$+C_{[x,y]}\sin2\varphi_k \tag{5}$$

4.4 Horizontal Movement Corresponding to Arbitrary Ground-Surface Points

Horizontal-displacement values along arbitrary directions through any given ground position can be expressed as

$$U(x,y)_\varphi=C(y)U(x)\cos\varphi+C(x)U(y)\sin\varphi \tag{6}$$

The dominant direction and corresponding value of horizontal displacement can be expressed as:

$$\varphi_k=\mathrm{arctg}\frac{C(x)U(y)}{C(y)U(x)}$$
$$U(x,y)_{\varphi_k}=C(y)U(x)\cos\varphi_k+C(x)U(y)\sin\varphi_k \tag{7}$$

4.5 Horizontal Deformation for Arbitrary Positions of the Ground Surface

The horizontal-deformation quantity corresponding to arbitrary directions crossing a ground-surface point is given by:

$$\varepsilon(x,y)_\varphi=C(y)\varepsilon(x)\cos^2\varphi+C(x)\varepsilon(y)\sin^2\varphi+\frac{1}{2}\mathrm{r}_{[x,y]}\sin2\varphi_k \tag{8}$$

The principal horizontal deformation direction and value are:

$$\varphi_{\mathrm{k}} = \frac{1}{2}\mathrm{arctg}\frac{2\mathrm{r}_{[x,y]}}{\mathrm{C}(y)\varepsilon(x) - \mathrm{C}(x)\varepsilon(y)}$$

$$\varepsilon(x,y)_{\varphi_{\mathrm{k}}} = \mathrm{C}(y)\varepsilon(x)\cos^2\varphi_{\mathrm{k}} + \mathrm{C}(x)\varepsilon(y)\sin^2\varphi_{\mathrm{k}} + \frac{1}{2}\mathrm{r}_{[x,y]}\sin 2\varphi_k \quad (9)$$

5. Introduction to MATLAB

Created by MathWorks, MATLAB is a commercial- grade mathematical computing tool. It supplies high- level programming language for technical computation alongside an interactive working environment, which supports algorithm development, data visualization, data analysis and numerical calculation. Compared with the lengthy code of C and FORTRAN, MATLAB code is more intuitive and closer to human thinking habits, giving users a concise program development environment. It also offers powerful scientific computing and data processing capabilities, rich graphics functions, widely used toolboxes, and practical program interfaces and publishing platforms. For mining subsidence prediction, the matrix operation, transcendental function, and two-dimensional and three-dimensional plotting functions of MATLAB can be used directly to implement the probability integral formulas. Functions such as meshgrid, erf, exp, contour, and surf make it possible to build prediction grids, compute subsidence distribution coefficients, and visualize prediction results with only a few lines of code, which greatly reduces the programming workload and facilitates later maintenance and function expansion.

6. Prediction Program Coding

Table 1. Calculation Algorithm for Predicting Surface Subsidence via the Probability Integral Method

Input: parameters of the prediction region and geometric-geological indices of the mining face
Output: calculated surface-movement quantities: subsidence w, tilt i, curvature k, horizontal displacement U, horizontal strain ε
1. Build the prediction grid (x, y) with meshgrid over the prediction area
2. Assign input parameters: Wm ← maximum subsidence value; r ← main influence radius along strike; rs ← main influence radius in dip (downhill) direction; rx ← main influence radius in rise (uphill) direction; lz ← calculated working face length along strike (with inflection offset); lq ← calculated working face length along dip (with inflection offset); b_s ← horizontal movement coefficient along strike; b_d ← horizontal movement coefficient along dip; θ ← prediction direction angle (0 for strike, 90 for dip)
3. Compute inclination for limited mining: $ix \leftarrow Wm/r\cdot[\exp(-\pi x^2/r^2) - \exp(-\pi(x-lz)^2/r^2)]$ (strike direction) $iy \leftarrow Wm/r\cdot[\exp(-\pi y^2/r^2) - \exp(-\pi(y-lq)^2/r^2)]$ (dip direction)
4. Compute curvature for limited mining: $kx \leftarrow -2\pi Wm/r^3\cdot x\cdot\exp(-\pi x^2/r^2) + 2\pi Wm/r^3\cdot(x-lz)\cdot\exp(-\pi(x-lz)^2/r^2)$ $ky \leftarrow -2\pi Wm/r^3\cdot y\cdot\exp(-\pi y^2/r^2) + 2\pi Wm/r^3\cdot(y-lq)\cdot\exp(-\pi(y-lq)^2/r^2)$
5. Compute horizontal movement: $Ux \leftarrow b\_s\cdot r\cdot ix$; $Uy \leftarrow b\_s\cdot r\cdot iy$
6. Compute horizontal deformation: $Ex \leftarrow b\_s\cdot r\cdot kx$; $Ey \leftarrow b\_d\cdot rs\cdot ky$
7. Compute subsidence distribution coefficients in main section: $C_x \leftarrow \frac{1}{2}\cdot[\mathrm{erf}(\sqrt{\pi}\cdot x/r) - \mathrm{erf}(\sqrt{\pi}\cdot(x-lz)/r)]$ (strike) $C_y \leftarrow \frac{1}{2}\cdot[\mathrm{erf}(\sqrt{\pi}\cdot y/rs) - \mathrm{erf}(\sqrt{\pi}\cdot(y-lq)/rx)]$ (dip)
8. Compute subsidence value at any point: $w(x,y) \leftarrow Wm\cdot Cx\cdot Cy$
9. Compute comprehensive values along prediction direction θ: $i\_\theta \leftarrow Cy\cdot ix\cdot\cos\theta + Cx\cdot iy\cdot\sin\theta$ $k\_\theta \leftarrow Cy\cdot kx\cdot\cos^2\theta + Cx\cdot ky\cdot\sin^2\theta + (ix\cdot iy/Wm)\cdot\sin 2\theta$ $U\_\theta \leftarrow Cy\cdot Ux\cdot\cos\theta + Cx\cdot Uy\cdot\sin\theta$ $\varepsilon\_\theta \leftarrow Cy\cdot Ex\cdot\cos^2\theta + Cx\cdot Ey\cdot\sin^2\theta + \frac{1}{2}\cdot(iy\cdot Ux + ix\cdot Uy)/Wm\cdot\sin 2\theta$
10. Generate outputs: draw the contour map of the predicted deformation and export the results to a TXT file for Surfer/CAD post-processing
11. Return w, i_θ, k_θ, U_θ, ε_θ
End Algorithm

According to the basic theory and models of mining subsidence prediction described above, when conducting surface subsidence prediction, the complexity of prediction formulas leads to calculation difficulties. MATLAB provides more intuitive mathematical operation functions that conform to human thinking habits, making it more accessible for scientific and technical personnel without a computer science background. MATLAB programming language is adopted in this work to realize the calculation of prediction formulas. Consequently, prediction outputs can be readily acquired for subsequent analysis concerning surface movement and deformation features. Below is the full workflow adopted by the prediction program: first, the prediction grid is generated according to the prediction area and the prediction parameters are assigned; Subsequently, calculations are conducted for subsidence distribution coefficients corresponding to primary profiles in strike and dip orientations. The subsidence magnitude at any surface location can be derived by superimposing these coefficient values; on this basis, the inclination, curvature, horizontal movement, and horizontal deformation in any prediction direction are computed; finally, the calculation results are visualized and exported. Table 1 gives the detailed steps.

As shown in Table 1, the prediction program takes the geometric and geological parameters of the working face as input, and realizes the unified solution of five types of movement and deformation indicators through modular function calls. The program is divided into three levels: the parameter input module reads the prediction parameters and generates the calculation grid; the core calculation module encapsulates the formulas of each functional module described in Section 4; and the result output module completes graphics rendering and data export. This structured design makes the program easy to debug and extend.

7. Engineering Application

7.1 Data Preparation

This paper takes the 7513 working face of a certain mining area as an example to carry out surface subsidence prediction. The prediction parameters measured at another working face of the same mining area are transferred to the 7513 working face: subsidence coefficient q = 0.74; maximum subsidence value Wm = 4809 mm; main influence radius along strike r = 106 m; r_s=111.3m: primary influence radius along the down-dip orientation; r_x=111.8: calculated working face length along strike considering inflection point offset lz = 800 m; calculated working face length along dip considering inflection point offset lq = 200 m; horizontal movement coefficient along strike b_Strike = 0.29; horizontal movement coefficient along dip bq = 0.29. These parameters are input into the prediction program, and a square prediction grid covering the entire subsidence influence range is constructed by setting the coordinate origin at the mid-point of the working face, allowing movement-and-deformation values to be computed for any ground location.

7.2 Prediction Results Display

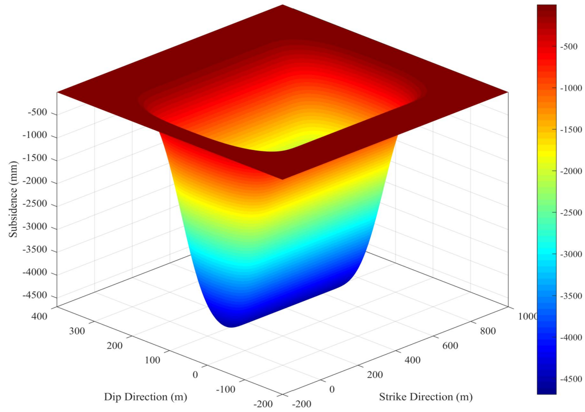

Figure 2. 3D Effect Diagram of Surface Subsidence Prediction for Working Face Mining

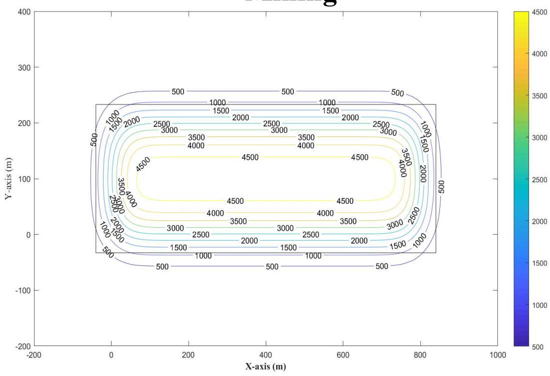

Figure 3. Contour Map of Surface Subsidence Value Prediction for Working Face

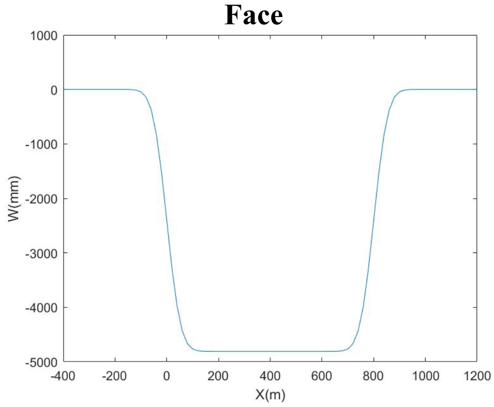

Figure 4. Predicted Surface Subsidence along the Strike Main Section

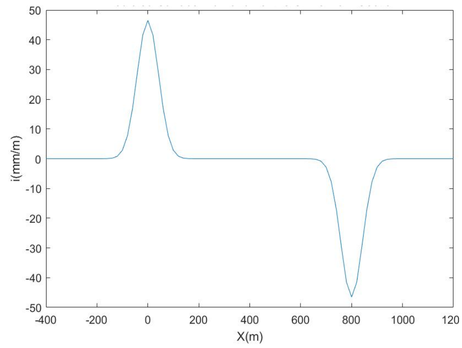

Figure 5. Predicted Surface Inclination along the Major Section along the Strike Direction

Table 2. Maximum Values of Ground-surface Displacement-deformation Quantities Caused by Mining

Prediction Index	Subsidence (mm)	Horizontal Movement (mm)	Horizontal Deformation (mm/m)	Horizontal Deformation (mm/m)	Curvature (mm/m²)	Curvature (mm/m²)	Inclination (mm/m)
Prediction Index	**Subsidence (mm)**	**Horizontal Movement (mm)**	**Tension**	**Compression**	**Convex**	**Concave**	**Inclination (mm/m)**
Strike	4809	1394	19.94	19.94	0.65	-0.65	45.36
Dip	4809	1430	21.53	-21.66	0.67	-0.67	46.51

This system can generate three-dimensional effect diagrams, main section profile diagrams, and contour maps, and can output prediction results in TXT document format. The results can be imported into Surfer, CAD, and other software for subsequent analysis and processing. Taking the obtained subsidence prediction results as an example, Figure 2 gives the three-dimensional effect diagram of the ground subsidence trough generated upon completion of working-face excavation, where the spatial morphology of the basin can be seen intuitively; Figure 3 gives contours of computed ground subsidence magnitudes, which reflects the distribution range and gradient change of subsidence; Figure 4 illustrates ground subsidence results predicted for the primary strike section, and provided in Figure 5 are the calculated ground inclination along the strike main section. Table 2 lists the maximum values of ground motion-deformation quantities generated by working-face excavation.

It can be observed from Table 2 that the peak ground subsidence amounts to 4809 mm upon completion of the 7513 working- face mining; maximum tilt values are 45.36 mm/m for the strike orientation and 46.51 mm/m for the dip orientation; Peak magnitudes of convex-concave curvature are 0.65 mm/m² and -0.65 mm/m² in the strike direction, whereas those for the dip orientation equal 0.67 mm/m² and -0.67 mm/m²; the maximum horizontal ground displacement is 1394 mm along the strike and 1430 mm along the dip; and furthermore, the maximum tensile deformation magnitudes reach 19.94 mm/m along the strike direction and 21.53 mm/m for the dip direction. According to the damage classification standard for buildings in mining areas, these predicted deformation values mean that the surface within the influence range will be severely damaged, so protective measures such as backfill mining, coordinated mining, or building reinforcement should be taken during mining. The predicted subsidence basin in Figures 2 and 3 is basically symmetrical around the mid- point of the working panel; the principal cross- section curves in Figures 4 and 5 are smooth and continuous. Such features agree with the basic patterns of mining- triggered ground movement and verifies the reliability of the prediction program.

8. Summary

This paper presents a study focused on employing MATLAB software to forecast ground subsidence induced by coal extraction. Using the MATLAB language, a surface subsidence prediction program based on the probability-integral algorithm was developed, and displacement and deformation magnitudes at arbitrary ground points over the 7513 working panel were predicted. Obtained computational outcomes include a three-dimensional effect diagram of the surface subsidence basin after stabilization, main section profile diagrams of the major sections in strike- dip orientations, contour maps, and TXT document outputs of the corresponding movement and deformation values, which can be imported into professional drawing software such as Surfer and CAD for further processing and analysis. In summary, the outcomes of the current study decrease the computational burden for ground subsidence prediction for the 7513 working face, shorten the data processing time, improve work efficiency, and have certain practical value for mining operations in this mining area. However, this study still has some limitations: the static probability integral model only predicts the final movement and deformation after the surface reaches stability, and cannot describe the dynamic subsidence process while the working face advances. In future research, time functions will be introduced to construct a dynamic prediction model, and measured data from surface observation stations will be used to invert and optimize the prediction parameters to further improve the prediction accuracy. Integrating the program with GIS platforms and InSAR monitoring data to visualize and dynamically update the prediction results is also a direction worthy of further study.

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References

[1] Sun Q X, Zhang Y, Chen Q T, et al. Seventy-year review and technical outlook of mining subsidence research in China . China Mining Magazine, 2024, 33(3): 86-99. (in Chinese)
[2] Wang Z, Zhao Y, Wang P, et al. Refined subsidence monitoring and dynamic prediction in narrow and long mining areas based on InSAR and probabilistic integral method . Scientific Reports, 2024, 14: 28009.
[3] Shen Z, Xu L, Liu Z, et al. Calculation of mining subsidence prediction parameters based on probability integral method using Matlab . Metal Mine, 2015, 44(9): 170-174. (in Chinese)
[4] Wang X, Ren Z. Integrated information processing system for mining subsidence in dedicated railway areas . China Safety Science Journal, 2004, 14(4): 15-18. (in Chinese)
[5] Li H B, Chai H B, Yu Y. Research on mining subsidence prediction analysis system . Journal of Jiaozuo Institute of Technology, 2003(4): 259-261. (in Chinese)
[6] Niu Y C, Xu L J, Zhang K, et al. Study on the predicted parameters of probability integral method based on GA-BP neural network . Metal Mine, 2019(10): 99-106. (in Chinese)
[7] Lou J. A parameter inversion method for the probability integral method based on robust ridge estimation . Frontiers in Earth Science, 2024, 11: 1330163.
[8] Wei T, Guo G, Li H, et al. A novel probability integral method segmental modified model for subsidence prediction applicable to thick loose layer mining areas . Environmental Science and Pollution Research, 2023, 30(18): 52049-52061.
[9] Huo W, Li H, Guo G, et al. Surface subsidence prediction method for backfill mining in shallow coal seams with hard roofs for building protection . Sustainability, 2023, 15(22): 15791.
[10] Li P, Li Y, Li Q, et al. A novel analytical model of mining subsidence considering time effect based on the probability integral theory . Sustainability, 2022, 14(21): 14443.
[11] Liu B, Dai H. Research development and origin of probability integral method . Journal of Mining and Strata Control Engineering, 2016, 21(2): 1-3. (in Chinese)
[12] Hu J, Yan Y, Dai H, et al. Prediction method for dynamic subsidence basin in mining area based on SBAS-InSAR and time function . Remote Sensing, 2024, 16(11): 1938.

