

A Model for Straight-Line Pocketing of an Object Ball in Billiards

Zhenming Zhang, Jie Deng*

Shenghua Zizhu Academy, Shanghai, China

**Corresponding Author*

Abstract: In this paper, the problem of speed and aiming for straight-line pocketing of the object ball in billiards will be considered. The simplified motion model will be introduced taking into account the initial speed of the object ball, its effective deceleration and the effective opening half-width. Based on this model, a combined speed and direction criterion for reaching the pocket opening along a straight path is derived. For aligned motion and a zero minimum arrival speed, the initial-speed threshold is proportional to the square root of the normal distance to the pocket opening. The angular tolerance decreases approximately in inverse proportion to this distance in the small-angle limit. The proposed model can inform shot-speed selection and aiming-direction planning for billiards robots.

Keywords: Billiards; Effective Deceleration; Geometric Criterion; Angular Tolerance; Billiards Robot

1. Introduction

1.1 Research Problem

Successful pocketing in billiards depends on shot speed, impact angle, and interactions between the cue ball and the object ball. A comprehensive description may involve sidespin, cushion rebounds, friction during collisions, and ball deformation, resulting in considerable modeling complexity. The present study uses elementary mechanics to develop a simplified analysis of straight-line object-ball pocketing. After being struck by the cue ball, an object ball moving in the correct direction may stop before reaching the pocket. Conversely, an object ball with sufficient initial speed may still miss the pocket because of an angular error. Thus, straight-line pocketing requires both an appropriate direction of motion and sufficient travel distance.

1.2 Related Work

Interrelationships between translational and

rotational motion and friction are important aspects of dynamics of a billiard ball. Witters and Duymelinck [1] studied rolling and sliding resistance, showing that rolling resistance was approximately independent of speed below 0.5 m/s under their experimental conditions. Hierrezuelo and Carnero [2] focused on rolling and sliding as pedagogical concepts, describing the transition from sliding to pure rolling (rolling without slipping) as well as the principles of rolling resistance. Florea and Rosca [3] performed an analysis of the motion of a sphere on a rough horizontal plane. Özkanlar [4] used impulse-momentum and work-energy theorems to analyze the motion of a cue ball after a horizontal cue impact, as well as the transition to pure rolling. All these works provided the physical background for formulating initial conditions and describing effective deceleration. Collisions between billiard balls allow setting initial conditions of subsequent translational and rotational motions of the bodies. Wallace and Schroeder [5] studied influence of friction on two-dimensional billiard ball collisions. Doménech-Carbó [6] offered independent friction-restitution model for determining the linear and angular velocities of the colliding objects. Apart from collisions, one of the important factors for achieving the desired result is the geometric tolerance. Shih [7] studied the influence of aiming errors on the performance of the billiards training system, relating the angular tolerance of the shot to the relative geometry of the cue ball, object ball, and pocket. Geometrically, a longer travel distance of the ball before entering the pocket results in a larger lateral displacement for a given angular deviation, thus leading to a smaller angular tolerance for a fixed pocket-opening width. This consideration was used in the present study, with initial speed and direction of the object-ball motion at the beginning of the modeled rolling segment used as the initial conditions, while the subsequent ball motion was the focus of investigation.

Finally, the game of billiards became a ground for

computational decision-making and robotics development. Smith [8] developed the PickPocket approach to the shot selection in simulated billiards game under uncertainties in the environment. Landry and Dussault [9] used optimization for determination of cue-ball positioning and choice of the optimal shot. Mathavan et al. [10] formulated the ball positioning problem in the robotic billiards in terms of nonprehensile manipulation, with combination of dynamic modeling and shot parameter optimization. Park and Park [11] introduced the assistive system for carom billiards, based on path generation, trajectory prediction, and vision-based detection of ball motion.

1.3 Scope and Approach

This work first develops an equation for the stopping distance of the object ball. The equation uses the ball's initial speed and effective deceleration. The geometry of the pocket opening is then used to calculate the angular tolerance. Together, these results define the speed and direction conditions required for the ball to reach the pocket-opening cross-section on a straight-line path. These conditions apply under the defined assumptions. The model's possible use as a basic building block for a motion-prediction algorithm in billiards robots is also discussed.

2. Motion Model and Analytical Criteria

2.1 Model Scope and Notation

This research considers only one object ball in straight-line pure rolling on a horizontal billiard table. Its initial speed is known at the start of the modeled rolling segment. From that point until the ball stops rolling, the effective deceleration is assumed to remain constant. Cue contact and cushion rebounds are not considered in the model. The same applies to successive ball collisions and airborne motion.

For the geometric analysis, the initial ball center is assumed to lie on the normal through the midpoint of the pocket opening. The distance x is measured along this normal, and θ is the signed deviation of the actual direction of motion from it. The pocket opening is represented by a transverse cross-section, and geometric acceptance is assessed from the position of the ball center at this cross-section. Contact with the pocket jaws and the subsequent drop into the pocket are outside the model.

The notation is summarized in Table 1.

Table 1. Main Variables and Their Definitions

Symbol	Unit	Definition
v_0	$\text{m}\cdot\text{s}^{-1}$	Initial speed at the beginning of the modeled rolling segment
$v(s)$	$\text{m}\cdot\text{s}^{-1}$	Speed after traveling a distance s along the trajectory
v_p	$\text{m}\cdot\text{s}^{-1}$	Speed when the ball center reaches the pocket-opening cross-section
$v_{\min}$	$\text{m}\cdot\text{s}^{-1}$	Prescribed minimum arrival speed
a	$\text{m}\cdot\text{s}^{-2}$	Positive, constant effective deceleration
s, S, L	m	Distance traveled along the trajectory, stopping distance, and trajectory length to the pocket-opening cross-section, respectively
x	m	Initial normal distance from the ball center to the pocket-opening cross-section
y	m	Lateral displacement of the ball center from the midpoint of the opening upon arrival
$\theta, \theta_{\max}$	rad	Signed trajectory angle relative to the opening normal and maximum allowable magnitude of this angle, respectively
R_b, b, w	m	Ball radius, pocket-opening half-width, and effective half-width available to the ball center, respectively

The parameter domain and effective opening half-width are defined as:

$$a > 0, \quad x > 0, \quad b > R_b > 0, \quad v_0 \geq 0, \quad w = b - R_b > 0, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}. \quad (1)$$

The angular restriction ensures that the ball moves toward the opening and that $\cos\theta$ remains positive.

2.2 Speed–Distance Relationship

The modeled motion begins after the collision, with any preceding transition from sliding to pure rolling excluded. Under constant effective deceleration, the speed–distance relationship is:

$$v^2(s) = v_0^2 - 2as, \quad 0 \leq s \leq S. \quad (2)$$

Setting the final speed to zero gives the stopping distance:

$$S = \frac{v_0^2}{2a}. \quad (3)$$

Equation (2) applies only until the ball stops. Motion beyond the stopping distance is not represented by this expression.

2.3 Geometry of Reaching the Pocket Opening

For the centered initial configuration defined in

Section 2.1, the lateral displacement at the pocket-opening cross-section is:

$$y = x \tan \theta. \quad (4)$$

Under the adopted geometric approximation, the ball center must lie within the effective opening interval. Therefore,

$$|y| \leq w \Leftrightarrow |x \tan \theta| \leq w. \quad (5)$$

The actual trajectory length required to reach the cross-section is:

$$L = \frac{x}{\cos \theta}. \quad (6)$$

Thus, an angular deviation changes both the arrival position and the distance traveled.

2.4 Combined Speed and Geometric Criteria

The ball can reach the pocket-opening cross-section only if its stopping distance is at least equal to the required trajectory length. Combining Eqs. (3) and (6) gives:

$$S \geq L \Leftrightarrow \frac{v_0^2}{2a} \geq \frac{x}{\cos \theta} \Leftrightarrow v_0^2 \geq \frac{2ax}{\cos \theta}. \quad (7)$$

The combined criterion for reaching the opening within the prescribed geometric tolerance is therefore:

$$\begin{cases} v_0^2 \geq \frac{2ax}{\cos \theta}, \\ |x \tan \theta| \leq w. \end{cases} \quad (8)$$

Equality in the speed condition means that the ball stops exactly when its center reaches the cross-section. It represents the limiting condition for arrival.

For a trajectory satisfying Eq. (8), the arrival speed is determined by:

$$v_p^2 = v_0^2 - 2aL = v_0^2 - \frac{2ax}{\cos \theta}. \quad (9)$$

If a positive minimum arrival speed is prescribed, the speed requirement becomes:

$$v_p \geq v_{\min} > 0 \Leftrightarrow v_0 \geq \sqrt{v_{\min}^2 + \frac{2ax}{\cos \theta}}. \quad (10)$$

This condition provides a positive speed margin at the opening. Actual pocket capture still depends on the pocket geometry and contact processes excluded from the present model.

Within the angular domain specified in Eq. (1), the geometric constraint gives:

$$\theta_{\max}(x) = \arctan\left(\frac{w}{x}\right), \quad |\theta| \leq \theta_{\max}(x). \quad (11)$$

For a fixed effective opening half-width, differentiation with respect to x yields:

$$\frac{d\theta_{\max}}{dx} = -\frac{w}{x^2 + w^2} < 0. \quad (12)$$

The maximum allowable angle therefore

decreases monotonically as the distance increases. In the small-angle limit,

$$\theta_{\max}(x) \approx \frac{w}{x}, \quad \frac{w}{x} \ll 1. \quad (13)$$

The angular tolerance is thus approximately inversely proportional to distance when expressed in radians.

3. Model Results

Illustrative calculations are performed using the following parameters:

$$\begin{aligned} a &= 0.10 \text{ m/s}^2, & R_b &= 0.025 \text{ m}, \\ b &= 0.045 \text{ m}, & w &= 0.020 \text{ m}. \end{aligned} \quad (14)$$

These assumed values are used to examine the analytical trends of the model. Quantitative application to a particular table requires experimental calibration of the effective deceleration.

The angular-tolerance curve is obtained from Eq. (11). The initial-speed curves are evaluated for motion aligned with the opening normal, corresponding to $\theta = 0$. In this configuration, the limiting initial speed for arrival with zero remaining speed is:

$$v_{0,\text{crit}}(x) = \sqrt{2ax}. \quad (15)$$

For a prescribed minimum arrival speed, the required initial speed is:

$$v_{0,\text{req}}(x; v_{\min}) = \sqrt{v_{\min}^2 + 2ax}. \quad (16)$$

Figure 1 contrasts the angular-tolerance curve with the initial-speed thresholds corresponding to zero arrival speed and a prescribed minimum arrival speed of 0.15 m/s. Some representative values have been presented in Table 2. As the normal distance to the pocket opening increases from 0.5 m to 2.0 m, the maximum allowable angular deviation falls from about 2.291° to 0.573°. Therefore, the greater the normal distance, the more accurate the alignment must be. This process takes place gradually, and no specific distance where only perfectly aligned motion is possible has been found by the model.

Table 2. Angular Tolerance and Initial-Speed Thresholds at Selected Distances

Normal distance x (m)	Maximum angular tolerance $\theta_{\max}$ (°)	Initial-speed threshold for zero arrival speed (m·s ⁻¹)	Initial-speed threshold for an arrival speed of 0.15 m/s (m·s ⁻¹)
0.5	2.291	0.316	0.350
1.0	1.146	0.447	0.472
2.0	0.573	0.632	0.650

Note: The speed thresholds are evaluated for

$$\theta = 0$$

Over the same distance range, the zero-arrival-speed threshold increases from approximately 0.316 to 0.632 m/s, following the square-root relationship in Eq. (15). A positive arrival-speed requirement increases the initial-speed threshold, as described by Eq. (16).

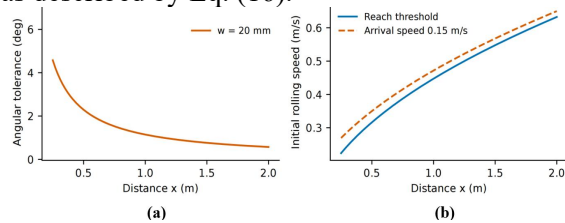


Figure 1. Analytical Results for the Simplified Model: (a) maximum angular tolerance versus the normal distance to the pocket opening; (b) initial-speed thresholds for zero arrival speed (solid line) and a prescribed minimum arrival speed of 0.15 m/s (dashed line)

Note: The speed curves are evaluated for aligned motion, using the parameters in Eq. (14).

4. Discussion and Potential Applications

4.1 Interpretation and Limitations

The model describes the conditions for the object ball to reach the pocket-opening cross-section within the prescribed geometric tolerance. The speed condition ensures that the object ball can travel the required distance, while the geometric condition restricts its arrival position to the effective opening interval. Increasing the initial speed cannot compensate for a trajectory that violates the geometric constraint. Longer shots therefore require both sufficient speed and accurate aiming.

The speed constraint and geometric constraint are linked through the trajectory angle: angular deviation affects the trajectory length by the factor of $1/\cos\theta$. For the given trajectory angle and given effective deceleration, the initial-speed threshold for zero arrival speed increases in proportion to the square root of the normal distance. The angular tolerance reduces monotonically and approaches inverse proportionality to the normal distance in the case of small angles.

Several assumptions set the framework for the applicability of the results. The collision between the cue ball and the object ball is not considered; thus, such factors as restitution, contact friction, and changes in spin due to collision are not taken

into account. The initial state of the object ball is defined at the beginning of the modeled rolling segment. Though pure rolling implies rotation, the rotational state is not regarded as an independent variable, and its influence on the translational deceleration is accounted for in the effective deceleration.

The geometric criterion applies to the centered configuration defined in Section 2.1 and the effective opening interval specified in Eq. (5). It is concerned with reaching the pocket-opening cross-section and disregards contact with the pocket jaws and subsequent pocket capture. General initial configurations require a similar generalization of the geometric considerations.

Similarly, the assumption of constant effective deceleration also implies its own domain of validity. Witters and Duymelinck [1] found the rolling resistance to be almost independent of the speed below 0.5 m/s under their experimental conditions. Thus, the effective deceleration needs to be determined based on the ball, the table surface, and the specific speed range of interest. The results given in Section 3 show the behavior of the assumed model.

The work provides an accessible example of applying basic principles of kinematics and geometry to derive some simple criteria from intuitive knowledge of the shot strength and aiming accuracy. Potential extensions of this analysis would consider the effects of collision restitution and contact friction and the initial spin of the ball.

4.2 Prospects for Robotic Billiards

Robotic billiards is an example of a nonprehensile manipulation task with a quantifiable objective and clear outcomes. A future robotic billiards system will plan its shot based on visual perception of the current situation, motion prediction, shot execution, and feedback about the result.

Initially, the camera will estimate the ball positions, the target pocket position, and the ball velocities. For the centered configuration considered here, the analytical model can find the feasible combinations of initial speed and direction of the object ball. The calibrated model of cue action and collision would then map the desired object-ball states into the corresponding cue speeds and contact points on the cue ball. Comparison of the predicted and observed trajectories would update the effective deceleration and the mapping between control

inputs and ball motion.

The current model can serve as the first step of developing a motion-prediction and geometric-constraint module. The control of the cue stick, modeling of the collision, and feedback-based parameter updates would need further development.

In case of straight-line motion on a fixed billiard table, the candidate shots can be generated analytically, followed by the simulation. It would give a planning procedure where angular tolerances and initial-speed thresholds can be directly calculated. Further work can include calibration for different table-surface conditions and extension of the geometric model to more general initial ball configurations.

As the calibration and feedback data become available, they could be used to update the model parameters and the mapping between control inputs and ball motion.

5. Conclusions

The proposed model combines the speed and geometric constraints for an object ball traveling toward the pocket opening. For straight-line pure rolling with constant effective deceleration, the combined criterion follows from the adopted geometric approximation and is given in Eq. (17):

$$\begin{cases} v_0^2 \geq v_{\min}^2 + \frac{2ax}{\cos \theta}, \\ |x \tan \theta| \leq w. \end{cases} \quad (17)$$

For a fixed trajectory angle, increasing the normal distance raises the initial-speed threshold for zero arrival speed according to a square-root relation. Angular tolerance, meanwhile, decreases monotonically with normal distance when the effective opening half-width is fixed. In the small-angle limit, the angular tolerance is approximately proportional to the reciprocal of that distance.

A zero minimum arrival speed includes the limiting case in which the ball stops as its center reaches the pocket-opening cross-section. With a positive minimum, the ball must arrive with at least the prescribed speed, providing a speed margin at the opening.

These relationships quantify the additional speed and aiming precision needed for longer shots. The model offers a basis for a motion-prediction

module in robotic billiards. Practical implementation would require calibration of the effective deceleration, together with further modeling of collisions, pocket contact, and shot execution.

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